A TREATISE ON HYDRAULICS

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A TREATISE

ON

HYDRAULICS

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PREFACE

THIS book is intended as a text-book for technical schools and colleges on certain parts of the broad subject of hydraulics, viz. water pressure, the stability of simple structures subjected to water pressure, the flow of water, the measurement of flow, and the fundamental principles of hydraulic motors.

The design of hydraulic motors, and other hydraulic machinery, and the design of water power plants are subjects too extensive to be properly treated here, and too important to be given but a passing notice; they have therefore not been attempted. Certain drawings of water motors have been inserted by way of illustrating fundamental principles, which are of interest to all students of engineering.

Teachers and engineers are frequently confronted with the fact that beginners have too much confidence in the numerical results obtained by solving hydraulic formulas, that they do not sufficiently realize the possibilities of difficulties, and that in practice, problems are rarely as free from complications as classroom examples. Moreover, while it is easy to make clear that from the standpoint of practical hydraulics experiments and experience furnish the only sound basis of practice, the difficulty always arises that the range of practice is not completely covered by experience. To fill in the gaps experimental results are classified, and from them are derived empirical coefficients and formulas, which are indispensable; but they should not be used blindly. There is, perhaps, no better way to prevent improper use of empirical formulas than to keep before students the experimental results upon which such formulas are based, and these should always be available for the few who will study them in detail.

In so far as the limits of one volume will permit, the authors have tried to meet the following requirements:

To present as directly and simply as possible the recognized methods of solving hydraulic problems;

To point out difficulties that arise in practice, and how to meet them;

To present such empirical coefficients and formulas as are in good use, pointing out their limitations in so many words if feasible, but especially by giving at the same time experimental values, and the limits of experiments, not only for guidance in using empirical values, but to train the judgment of the user;

To provide tables of suitable precision and extent, and diagrams, to save time in computations;

To indicate as an essential part of the subject the present state of practice, which is changing rapidly; for this reason a few matters will be found which are on the point of becoming obsolete.

The arrangement of the subject matter has been planned with these ends in view, and for that reason it has seemed desirable to connect as closely as possible related formulas, coefficients, and the practice of water measurements, and to finish, in a manner, each subdivision of the subject as it is taken up. Bernoulli's theorem, modified to include the effects of resistances to flow, is taken to be the basis for formulas of flow in general; and in the present state of experimental knowledge is a good working hypothesis. This premise once stated, the equations of flow through different apertures or channels are assumed to be applications of Bernoulli's theorem.

Piezometers that will give a true measure of head or intensity of pressure are essential to accurate results both in practical and experimental hydraulics; many important and otherwise very accurate experiments stand with a much diminished value because defective apparatus was used in measuring head or pressure. Certain experiments on Pitot tubes show very well the fundamental difficulties in making a true piezometer; for this reason piezometers are taken up in connection with Pitot tubes. In the chapters on Venturi meters and nozzles are also shown good forms of piezometer rings and gauges.

Somewhat more than usual attention has been given to rod float measurements, and especially to current meter measurements; because of their practice they are of sufficient importance, if taken up at all, to deserve more than passing notice.

Logarithmic diagrams to show the discharge through nearly all forms of apertures are relatively simple, and very useful for

office computations; they may easily be made on a scale sufficiently open to take off results as accurately as hydraulic coefficients warrant; several kinds of these diagrams are given. Logarithmic diagrams and methods are unusually valuable as a means of analyzing experimental results; and it is not improbable that with the general increase in the use of these methods more satisfactory expressions may be found for computing the loss of head in flowing water.

An extended study of pipe experiments will be found in Table XLIX; elsewhere well-known tables of coefficients are also given; and it is suggested that, in using the latter, experimental results should always be consulted. We have retained the Chezy formula because it is still in very good use; and we have not elaborated the very satisfactory exponential formula of Williams and Hazen because it can be so easily procured in the most economical form, that is, a set of tables which entirely cover the needs of ordinary practice. This study of pipes includes the experiments on old pipes, and has distinctly strengthened the opinion of the authors that empirical rules, for determining the probable effect of ageing, should be used very cautiously, and that in general such allowance should be made in the light of experimental results.

The authors gratefully acknowledge their obligations:

To the many authors whose names are herein given, for the use of their experimental results;

To The Pitometer Co., The Builders Iron Foundry, James Leffel & Co., The Pelton Water Wheel Co., The Abner Doble Co., The Allis-Chalmers Co., Buff and Buff, Dr. Rudolf Camerer, for data and the use of drawings;

To Professor A. E. Norton, and Mr. C. H. Paige of Harvard University, for valuable aid;

To Professor E. V. Huntington of Harvard University for valuable suggestions, and for Tables LXIV to LXXVIII inclusive, most of which he made especially for this book.

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CAMBRIDGE, MASS.
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July, 1911.

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A TREATISE ON HYDRAULICS

CORRECTIONS

"A Treatise on Hydraulics," by HUGHES and SAFFORD

Page 261, section 306, column 6,
second item should read 23 (instead of 3).

Page 316, equation (47) should read:

$$\text{Log}\, m = \log H_f' - n\,(\log V'), \text{ or } = \log H_f'' - n\,(\log V''), \text{ or } = \log H_f''' - n\,(\log V''').$$

Page 359, Table LIV, column (8),
seventh item should read 102.4 (instead of 02.4).

A TREATISE ON HYDRAULICS

HYDRAULICS

CHAPTER I

INTRODUCTORY

1. **Hydraulics** strictly means the science of fluids in motion. Hydrostatics is the science of fluids at rest. The term hydraulics, as used by engineers, includes the principles of hydrostatics; and, in effect, means practical mechanics of water.

The science of hydraulics comprises: pressures due to water; the stability of structures subjected to water pressures; the flow of water; the measurement of the flow of water; the proportioning of channels to convey water; the conversion of energy contained in water (by virtue of its velocity, elevation, or pressure) into useful work; the raising of water by pumps; the transmission of energy by water; the storage of water; the purification of water for domestic and commercial uses; the design of vessels to move in water; and various special applications which have been developed by experiment and practice.

The subject matter of this book will be confined chiefly to questions of water pressure, the stability of simple structures subjected to water pressure, the flow of water, the measurement of the flow of water, and the fundamental principles of water motors.

2. Limited application of theoretic formulas. Theoretic hydromechanics, which treats of fluids without viscosity, hence without friction, while leading to definite and reliable results when dealing with fluids at rest, may in treating of fluids in motion yield results not only useless to the engineer, but also positively misleading. The formulas expressing the laws of fluids in motion, derived from such purely theoretic consideration, have

become in many instances merely a framework on which · hung the results of experiment. While the formulas are ratic in form and serve to define the limits of application, they have account of the wide range of numerical coefficients, become in strictly empirical. In measuring the flow of water the engi must have recourse to experiments to determine the valu modifying elements for the problem under consideration.

UNITS OF MEASURE AND VOLUME

3. Primary units. In American and English hydraulic prac the customary primary units are the foot, pound, second, c foot, and cubic foot per second. In Continental practice where the metric system is used, the corresponding units are metre, kilogram, second, cubic metre, and cubic metre second.

Table I gives the principal units of measure, volume, press and energy common to the American and the metric systems, conversion factors, with their logarithms.

TABLE I

CONVERSION TABLE FOR THE PRINCIPAL UNITS

	FACTOR	
American to Metric.		
Feet to metres	0.3048	
Square feet to square metres	0.0929	
Square inches to square centimetres	6.4516	
Cubic feet to cubic metres	0.0283	
Cubic inches to cubic centimetres	16.3872	
Cubic feet per foot to cubic metres per metre	0.0929	
Pounds avoirdupois to kilograms	0.4536	
Pounds per cubic foot to kilograms per cubic metre	16.0184	
Foot pounds to kilogram metres	0.1383	
Horse power to metric horse power	1.0139	
Horse power to kilowatts	0.7460	
Pounds per square foot to kilograms per square metre	4.8824	
Pounds per square inch to kilograms per square centimetre	0.0703	
Inches of mercury to centimetres (barometer)	2.5400	
Degrees Fahrenheit to Centigrade, $C = (F - 32) \times \frac{5}{9}$.		

TABLE I. — (*Continued*)

	Factor	Log
Metric to American.		
1 meter = 39.37 inches (flat), U. S. Bureau of Standards		
Metres to feet	3.2808	0.5160
Square metres to square feet	10.7639	1.0320
Square centimetres to square inches	0.1550	$\bar{1}.1903$
Cubic metres to cubic feet	35.3145	1.5479
Cubic centimetres to cubic inches	0.0610	$\bar{2}.7853$
Cubic metres per metre to cubic feet per foot	10.7639	1.0320
Kilograms to pounds avoirdupois	2.2046	0.3433
Kilograms per cubic metre to pounds per cubic foot . .	0.0624	$\bar{2}.7954$
Kilogram metres to foot pounds	7.2330	0.8593
Metric horse power to horse power	0.9863	$\bar{1}.9940$
Kilowatts to horse power	1.3406	0.1273
Kilograms per square metre to pounds per square foot .	0.2048	$\bar{1}.3113$
Kilograms per square centimetre to pounds per square inch	14.2234	1.1530
Centimetres of mercury to inches	0.3937	$\bar{1}.5952$
Degrees Centigrade to Fahrenheit, $F = \frac{9}{5} + C\ 32$.		

4. Special units. Certain other units of volume and of discharge are frequently used, viz.:

The United States gallon of 231 cubic inches. One gallon of water may be taken to weigh $8\frac{1}{3}$ pounds. 1 cubic foot = $\frac{1728}{231}$ gallons = 7.48052 gallons.

Cubic feet per minute; used in waterwheel manufacturers' catalogues.

Gallons per minute; used in waterworks practice, the discharge of pumps, of play pipes, and of mains.

Gallons or million gallons per day of 24 hours; used in water supply and sewerage practice.

Acre-foot (one foot in depth over one acre); used in American irrigation practice.

Miner's inch; a measure of discharge used in western American hydraulic practice, originally the discharge through an orifice one inch square, the head and other important conditions varying in different localities. Each state, by law, now defines the miner's inch in equivalent cubic feet per second; the value in different states varying from about .02 to .028 cubic feet per second.

A mill power, a measure of water power, varying locally; in Lowell, for example, it is equivalent to 25 cubic feet per second on a fall of 30 feet.

Rainfall, evaporation, and run-off are frequently stated in terms of inches in depth per unit of drainage area.

Table II gives conversion factors for the more common units, with their logarithms.

TABLE II

Conversion Table for Common American Units

	Cubic Feet		U.S. Gallons		Cubic Feet per Second		U.S. Gallons per Minute		U.S. Gallons per Day of 24 Hours	
	Factor	Log	Factor	Log	Factor	Log	Factor	Log	Factor	Log
Cubic feet per second ("second feet") . .	1.0000	0.0000	7.481	0.8739			448.83	2.6521	646317	5.8105
Cubic feet per minute .	1.0000	0.0000	7.481	0.8739	0.01667	$\bar{2}$.2218	7.481	0.8739	10772	4.0323
U.S. gallons per minute.	0.1337	$\bar{1}$.1261	1.0000	0.0000	0.00223	$\bar{3}$.3483			1440	3.1584
1,000,000 gallons per 24 hours	133681	5.1261	1000000	6.0000	1.547	0.1895	694.4	2.8416	1000000	6.0000
Imperial gallons . . .	0.1603	$\bar{1}$.2049	1.201	0.0795						
Inches of rain per square mile in 24 hours . .	2323200	6.3661			26.89	1.4296	12066.0	4.0817	17378744	7.2400
Inches of rain per square mile in 365 days . .	2323200	6.3661	17378744	7.2400	0.0737	$\bar{2}$.8675	33.06	1.5194	47618	4.6777
Inches of rain per acre in 24 hours	3630	3.5599			0.0420	$\bar{2}$.6232	18.86	1.2755	27154	4.4338
Feet per acre (acre feet) in 24 hours	43560	4.6391			0.504	$\bar{1}$.7026	226.26	2.3547	325851	5.5130
Miner's inches (Nevada)					0.02	$\bar{2}$.3010	8.98	0.9531	12926	4.1115
Miner's inches (Colorado) nearly . .					0.026	$\bar{2}$.4157	11.69	1.0677	16831	4.2261

The Weight of Water

5. The weight of water. The weight of a cubic foot of water for the range of temperatures used in hydraulics may usually be taken as 62.4 pounds (for rough calculations 62.5); and of sea water, 64.0 pounds. The weight of a cubic metre of water is commonly taken as 1000 kilograms at 4° C.

Variation in weight of water with change of temperature. Above 39.3° F., the temperature of maximum density, water decreases in density as the temperature rises; and below 39.3° F., decreases as the temperature falls to freezing.

Table III gives the weight of distilled water at temperatures ranging from 32° to 212° F.

TABLE III*

Temperature, Degrees F.	Weight of a Cubic Foot, w	Temperature, Degrees F.	Weight of a Cubic Foot, w
32	62.416	75	62.261
35	62.421	80	62.217
39.3	62.424	85	62.169
45	62.419	90	62.118
50	62.408	95	62.061
55	62.390	100	61.998
60	62.366	150	61.203
65	62.336	200	60.135
70	62.300	212	59.843

The specific gravity of distilled water at 39.3° F. being 1.000, the specific gravity of sea water is about 1.025.

6. Increase of weight of water due to impurities. Water entirely free from foreign substances is rarely found in nature; and unless water contains considerable air its specific gravity is always higher than that of distilled water of the same temperature. When dealing with fresh water, except in the most refined calculations, and at unusual temperatures, variation both in temperature and in specific gravity may usually be disregarded. Sewage may weigh as much as ½ per cent and in extreme cases 1 per cent

* H. Smith, Jr., *Hydr*

more than distilled water; yet such differences are commonly neglected, chiefly because the degree of precision attainable in hydraulic engineering does not warrant the consideration of these variations.

Atmospheric Pressure

7. The pressure of the atmosphere. Atmospheric pressure, which is due to the weight of the air, varies with meteorological conditions and with differences in altitude. For hydraulic problems, it is sufficiently accurate to use, as a constant, the mean value of atmospheric pressure at sea level, or 14.7 pounds per square inch (p_a), commonly designated as one "atmosphere."

Units of Pressure

8. American units. In American and English practice, intensity of pressure may be stated in pounds per square inch; in

TABLE IV

Conversion Factors for Units of Pressure

	Feet of Water	Log	Inches of Mercury	Log	Pounds per Square Inch	Log	Pounds per Square Foot	Log
Pounds per square inch to	2.308	0.3632	2.037	0.3090	1.0000	0.0000	144.00	2.1584
Pounds per square foot to	0.01603	$\bar{2}.2048$	0.01414	$\bar{2}.1506$	0.00694	$\bar{3}.8416$	1.000	0.0000
Inches in height of mercury to	1.133	0.0542	1.000	0.0000	0.4910	$\bar{1}.6910$	70.699	1.8494
Feet in height of fresh water to	1.000	0.0000	0.8826	$\bar{1}.9458$	0.4333	$\bar{1}.6368$	62.4	1.7952
Feet in height of sea water to	1.025	0.0107	0.9047	$\bar{1}.9565$	0.4442	$\bar{1}.6475$	64.0	1.8062
Atmospheres to	33.923	1.5305	29.942	1.4763	14.70	1.1673	2116.8	3.3257
Atmospheres to	Sea water 33.096							

Specific gravities used in this table are: distilled water, 1.000; sea water, 1.025; mercury, 13.5956.

For rough calculations the weight of fresh water is frequently taken as 62.5 pounds per cubic foot; and one atmosphere equivalent to 34 feet of fresh water, 33 feet of sea water, or 30 inches of mercury.

pounds per square foot; in the height in inches of a column of mercury; in the height (called "head") of a column of fresh or sea water; or in terms of the number of atmospheres. The intensity of pressure may be determined by direct measurement of the head, or by means of a gauge calibrated to indicate pressure in pounds per square inch measured from atmospheric pressure.

Vacuum is usually stated in head *below* atmospheric pressure in inches of mercury. It is taken as the difference between atmospheric pressure and absolute intensity of pressure in a closed vessel rather than the absolute intensity of pressure, measured from zero.

Table IV gives certain factors, with their logarithms, for the conversion of units of pressure. Table LXI, in the Appendix, is a more extended table.

9. Metric units. In the metric system, intensity of pressure may be stated in kilograms per square centimetre; in kilograms per square metre; in the height of a mercury column in centimetres, or in the height of a column of water in metres.

THE ACCELERATION DUE TO GRAVITY

10. The value of *g*. In American and English practice, velocity is usually expressed in feet per second, and acceleration in feet per second *per second;* in the metric system, in metres per second, and in metres per second *per second.* The acceleration due to the force of gravity (g) is a minimum at sea level at the equator; and it increases with an increase of latitude, and decreases with a rise in altitude. The following expressions* give very nearly the value of g for any latitude (ϕ), and any elevation on land (h feet) above sea level. ρ is the radius of the earth in feet.

(1) In feet per second per second,

$$g = 32.1740(1 - .002662 \cos 2\,\phi)\left(1 - \frac{5}{4}\frac{h}{\rho}\right).$$

* *Travaux et Mémoires du Bureau International des poids et mesures*, Vol. 12 (1902); *Comptes rendus de la troisième conférence générale des poids et mesures réunie à Paris en 1901*, p. 68.

For each 100 feet increase in elevation, the correction (diminution) $= \frac{500}{4\rho} = .000193$.

(2) In centimetres per second per second,

$$g = 980.665\ (1 - .002662 \cos 2\phi)\left(1 - \frac{5}{4}\frac{h}{\rho}\right).$$

For each 100 feet increase in elevation the correction (diminution) $= \frac{500}{4\rho} = .00588$.

For example, in latitude 42°, at sea level,

$$g = 32.163 \text{ feet}; \qquad g = 9.8033 \text{ metres}.$$

Functions of g. The value of g used in this book will be 32.16, which is practically the value for latitude 42°. Common functions of this value and their equivalents in metres are here given:

TABLE V

	In Feet		In Metres	
	Number	Log	Number	Log
g	32.16	1.5073	9.803	0.9914
$2g$	64.32	1.8083	19.607	1.2924
$(2g)^{\frac{1}{2}}$	8.02	0.9042	4.428	0.6462
$\frac{2}{3}(2g)^{\frac{1}{2}}$	5.347	0.7281	2.952	0.4701
$\frac{1}{2g}$	0.01555	$\bar{2}.1917$	0.051	$\bar{2}.7076$

11. Precision in computation. In this book, four-place logarithms are usually stated, because they give a sufficiently high degree of precision for nearly all problems considered. In the appendix is a four-place table. The precision of the data upon which a problem is based should indicate the required nicety in calculation. In general, logarithms should be used to one more place than the number of significant figures desired in the result. Answers should be computed only to the number of significant figures justified by the original data. Work should invariably be checked, not only to eliminate numerical mistakes, but also to make certain above all things that the answer is reasonable.

Problems

1. Convert a discharge (volume of flow) of 15 cubic metres per second into its equivalent in cubic feet per second.

2. Convert a discharge of 14 cubic feet per second into its equivalent: (*a*) in U. S. gallons per minute; (*b*) in gallons per day of 24 hours.

3. Convert a discharge of 3,500,000 gallons per day into its equivalent: (*a*) in cubic feet per second; (*b*) in litres per second.

4. Convert intensity of pressure of 115 pounds per square inch into its equivalent in kilograms per square centimetre.

5. Convert intensity of pressure of 5.5 atmospheres into its equivalent: (*a*) in pounds per square inch; (*b*) pounds per square foot; (*c*) head in feet of fresh water; (*d*) head in feet of sea water; (*e*) inches of mercury; (*f*) metres of fresh water.

6. A tank 16 feet internal diameter and 40 feet high is filled with water at 40° F.; (*a*) How many U. S. gallons will it contain? (*b*) How many pounds? (*c*) How many litres? (*d*) If the temperature were 190° F., how many pounds?

7. How many pounds of water at ordinary temperature will be contained in a 48-inch (internal diameter) pipe 1 mile long?

8. Compute the value of g, (*a*) for latitude 20° N. at sea level; (*b*) for latitude $42\frac{1}{2}°$ N. at sea level; and (*c*) at an elevation of 4000 feet at the same latitudes.

CHAPTER II

FLUID PRESSURE

12. **Fluids** are substances the parts of which oppose little or no resistance to distortion of form, and include gases, liquids, and vapors.

Properties of a fluid. A perfect fluid, that is, a frictionless fluid, can not exert resistance to shearing or tensional stresses, and takes the exact form of a vessel in which it may be contained. Actual fluids do exert more or less resistance to change of form; and the measure of this opposition is termed the viscosity. The lower the viscosity, the more nearly the actual approaches the perfect fluid. In fluids considered in engineering, the viscosity is slight. With fluids in motion, the resistance to motion increases with an increase of viscosity. Water at rest may in engineering be treated, with negligible error, as a perfect fluid.

13. **Gases.** A gas is a compressible fluid; practically, a fluid in which slight changes of pressure produce very sensible changes in volume. The law of gaseous pressures may be briefly stated: *if the temperature is constant, the pressure multiplied by the volume is a constant.*

14. **Liquids.** A liquid is a fluid which changes volume very slightly even with considerable variation in pressure, and when the pressure is entirely removed does not sensibly dilate; but, like all elastic substances, is actually compressible. The liquid chiefly considered in hydraulics is water.

15. **Vapors.** A liquid may be changed to a gas by the application of heat; and a gas changed to liquid form by the abstraction of heat. A gas on the point of becoming a liquid, or a liquid on the point of becoming a gas, is called vapor. Rain clouds are water vapor.

16. Compressibility of water. Water at its maximum density has a coefficient of compressibility of about .00005 * for each atmosphere of added pressure; and a modulus of elasticity (E) in compression of about 296,000. The coefficient of compressibility decreases and the modulus increases with a rise in temperature; at 212° F. they are about .00004, and about 360,000. Thus, a cubic foot of water, which weighs 62.4 pounds under one atmosphere, under a pressure of 11 atmospheres weighs $62.4\,(1 + .00005 \times 10)$ or 62.43 pounds; a very slight increase. This correction for compression, like that for temperature, implies a higher degree of precision than is ordinarily required in hydraulic problems. Water is, therefore, considered as incompressible.

17. The free surface of a liquid. That surface of a body of liquid at rest which is in contact with a gas or vapor is the free surface. It, therefore, is a surface of equal pressure normal to the line of action of the force of gravity at any point on the surface of the earth, and for ordinary engineering work may be considered horizontal.

The free surface (so called) of a body of water in steady motion will be normal at any point to the resultant of (a) the force required to reduce a particle of water at that point to rest or to uniform motion in a straight line, and (b) the force of gravity in this particle.

18. Pascal's law. The basic theorem of the laws of fluid pressure is Pascal's law. **In a fluid, the pressure at a given point is normal to the surface on which it acts, and of equal intensity in all directions.**

19. Intensity of pressure. The intensity of pressure (p) is the pressure per unit of area between contiguous fluid surfaces, or between a fluid and the face of a solid, and is the weight of a column of the fluid supported by a unit area. The intensity at any point is therefore measured by the vertical distance from the point to the free surface of the fluid, called *the head* (h), and *the weight* (w) of a cubic unit of the fluid. Intensity of pressure is independent of the shape, area, or direction of the surface presented to the liquid. If all factors are given in corresponding units of

* *Smithsonian Physical Tables*, p. 83.

the same system, intensity of pressure may be expressed by the following fundamental formula:

$$p = hw\text{; therefore, } h = \frac{p}{w}. \tag{1}$$

Conversion formula for intensity of pressure. The terms commonly used to express intensities of pressure are in several different units; such, for example, as feet or inches of water or of mercury, or of oil of various specific gravities, in pounds per square inch or square foot, kilograms per square metre or square centimetre. The following formula may be used in converting expressions for intensities of pressure into units of the same system or other systems:

$$p = h\gamma\text{; therefore, } h = \frac{p}{\gamma}. \tag{2}$$

h = the head corresponding to an intensity of pressure p; h being the height of a column of any given liquid, and measured in any linear unit.

p = the intensity of pressure that will support a column of the given liquid of height h; p being stated in any unit of weight and any unit of area.

γ = the weight of a prism of the given liquid in the same units of weight as p, having for its height one unit of length in which h is measured, and for the cross-sectional area, the unit area in which p is stated.

Intensity of water pressure. The weight of a cubic foot of water may be taken as 62.4 pounds (see § 5); therefore, the relation between intensity of pressure (p), and head (h) in feet of water is as follows:

By (1) p, in pounds per square foot, $= h\ 62.4$; and $h = \frac{p}{62.4}$. (3)

By (2) p, in pounds per square inch, $= h\frac{62.4}{144} = h\ 0.4333$; and

$$h = \frac{p}{.4333} = p\ 2.308. \tag{4}$$

Table IV gives necessary factors with their logarithms for converting units of pressure; a more extended table (LXI) is given in the Appendix.

20. The term "head" in hydraulics. The word head has various special meanings in hydraulics, but their distinctions have become dulled by a necessarily varied usage; and in any particular case the intended meaning must be clearly specified. The following special meanings may be distinguished:

The velocity head of water moving with a given velocity is the equivalent height through which a body must fall to acquire the same velocity.

The pressure head is the difference in intensity of pressure between two points in a body of water; or merely the intensity of pressure at any point.

The head due to elevation is the difference in elevation between any point in a body of water and any other point or reference plane either within or without the body.

The head due to atmospheric pressure.

Lost head in flowing water represents the energy irrecoverably expended in overcoming resistances.

Heads or pressures are measured from some reference point usually fixed by the conditions of a problem. Intensities of pressure, unless otherwise stated, are measured from atmospheric pressure as zero.

21. Differences in intensity of pressure; effective pressure. In computing the effective intensity of pressure at a given point in a fluid, the modifying effect of the action of external forces, such as atmospheric pressure, compressed air, rarefied air, weights, or mechanical force, must be thoroughly considered. Obviously there are numerous instances where the effect of atmospheric pressure may be disregarded without error because it acts equally on opposite sides, yet there are many cases where the fluid pressure on one side of a surface is greatly intensified by the production of a vacuum on the other which brings the atmospheric pressure into action. Effective pressure is determined, therefore, by differences in absolute pressure and elevation.

Example. A vessel, shown in figure 1, is filled with water to a point 100 feet above the axis of the cylinder. It is open only at D to the atmosphere.

At any point on the line AA' the *absolute* intensity of pressure

on the inside $(p) = 100 \times .433 + p_a = 43.3 + 14.7 = 58$ pounds per square inch.

This pressure acts with equal intensity in all directions.

At A the *effective* intensity of pressure on the shell or the dif-

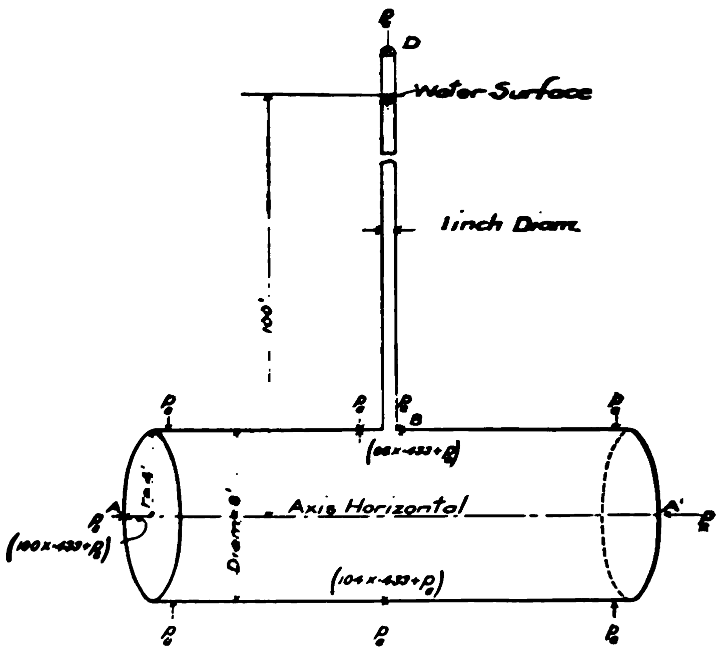


FIG. 1.

ference between the inside or outside pressures $= 100 \times .433 + p_a - p_a$, or simply 43.3 pounds per square inch.

At B the absolute intensity of pressure on the inside $= 96 \times .433 + 14.7 = 56.3$ pounds per square inch.

The intensity of pressure in the small pipe at B is equal to that in the large cylinder at the same point.

22. Total normal pressure. Intensity of pressure should be considered a means of determining a force rather than a force itself. **Intensity of pressure multiplied by the area of a surface,** however, **is the total pressure on a given surface,** which is a force. To define a force its *magnitude*, *direction*, and ***point of application*** must be known.

The total normal fluid pressure on an immersed surface or solid is the summation of all the forces due to fluid pressure on its faces. On irregular surfaces, the total normal pressure *per se* has no significance because it can not be defined. In investigating the effects of fluid pressures on irregular surfaces or solids they should be divided into suitably small areas, presenting as nearly as may be regular plane surfaces. The magnitude, direction, and point of application of the total normal pressure or force on each area should be computed and considered a single force. The problem then consists in summing up these individual forces, to determine the total resultant pressure in any required direction.

23. Uniform and non-uniform pressures. It is desirable to consider (1) surfaces subjected to pressures of uniform intensity, and (2) surfaces subjected to pressures of varying intensity.

Uniform Fluid Pressures

24. Pressures strictly uniform. When all points of a surface immersed in a fluid are equally distant from the free surface, the intensity of pressure is uniform over the whole surface.

25. Pressures assumed as uniform. In engineering practice, many cases occur in which, without impairing necessary precision, the intensity of pressure may be assumed to be uniform. The following groups include the important cases:

(1) When the surface is in contact with a gas or a vapor; for example, a compressed air reservoir.

(2) When the surface under liquid pressure is very small, *i.e.*, an elementary area.

(3) When all parts of the surface are under such high intensities of liquid pressure that the assumption of a uniform pressure will not introduce error in excess of the limits fixed by the other factors of a problem. This group includes water pipes under pressure, certain forms of steam boilers, of gates and valves, and many receptacles for liquids. The uniform intensity which is usually assumed is the effective pressure either on the lowest point of the area, or on its center of gravity.

Notation. In Chapters II, III, and IV the following symbols will be used:

p = intensity of pressure.
P = the total normal pressure.
H = the horizontal component of P.
V = the vertical component of P.

Directions will be designated by the total number of degrees from 0, measured as in trigonometry; that is, contra-clockwise.

26. An immersed plane surface parallel to the free surface of the water. Consider an irregular plane surface $BCDE$, area A, lying

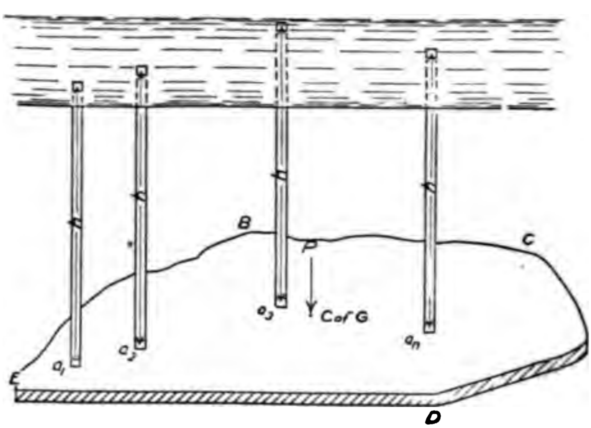


FIG. 2.

in a horizontal plane h feet below the free surface of the water. See figure 2.

Let a_1, a_2, $\cdots$ a_n, be elementary areas.

The intensity of pressure at any point, equation (1), $= p = hw$.

The magnitude of P. The normal pressure on any element (§ 22) $= hwa_n$.

The total normal pressure (P) is the sum of the normal pressures on all elementary areas.

Since Limit $\Sigma(a_1 + a_2 + \cdots a_n) =$ area (A),

$$P = \text{Limit } \Sigma(a_1 + a_2 + \cdots a_n)p = p\int_{\prime\prime}^{\prime\prime}\int_{\prime\prime}^{\prime\prime} \delta x \delta y = pA = hwA.$$

Example. If $h = 20$ feet and $A = 10$ square feet, $P = 20 \times 62.4 \times 10 = 12{,}480$ pounds.

The direction of P. Since the pressure on every elementary area must be normal to the surface, it is therefore vertical and downward, *i.e.* its direction is 270°; and P, which is the resultant of this system of parallel forces, must have the same direction, 270°.

The point of application of P. The point of application of P, called the center of pressure, must, since the intensity of pressure is uniform over the area A, and P is the resultant of a system of parallel forces normal to A, be the center of gravity of the area A.

27. A plane surface subjected to uniform intensity of pressure. Consider an irregular plane surface $BCDE$, area A, subjected on one side to a uniform intensity of pressure, p. See figure 3.

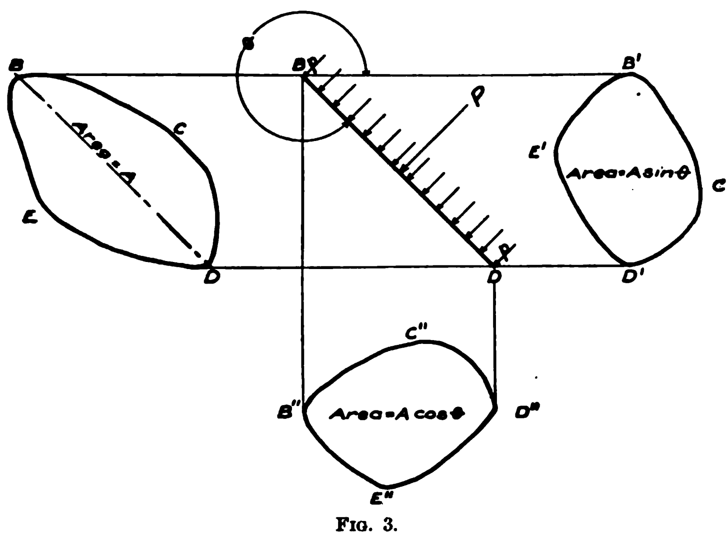


Fig. 3.

The magnitude of P. The total normal pressure $P = pA$.

The direction of P. The direction of P is normal to BD, *i.e.* at an angle of $(\theta - 90°)$.

The point of application of P. The point of application of P, or center of pressure, is at the center of gravity of $BCDE$, since the intensity of pressure is uniform.

The components of P. H, the horizontal component of P, $= P \cos(\theta - 90°) = pA \sin\theta$.

V, the vertical component of P, $= P \sin(\theta - 90°) = pA \cos\theta$.

Example. If $A = 50$ square inches; $p = 100$ pounds per square inch; $\theta = 300°$. Then

$$P = 100 \times 50 = 5000 \text{ pounds.}$$

$$(\theta - 90°) = 300° - 90° = 210°.$$

$$H = 100 \times 50 \times \cos 210° = 5000 \times .866 = 4333 \text{ pounds.}$$

$$V = 100 \times 50 \times \sin 210° = 5000 \times .500 = 2500 \text{ pounds.}$$

RULE. **The total normal pressure on an immersed plane surface subjected to pressure of uniform intensity equals the intensity of pressure multiplied by the area; and the total component in any direction equals the intensity of pressure multiplied by the area of the projection of the surface on a plane perpendicular to the given direction.**

28. A curved surface subjected to uniform intensity of pressure; a general case of an irregular curved surface. The total normal pressure on a curved surface, whether regular or irregular, may be said to be the sum of all the normal pressures on all its parts; but this statement is without practical significance. Although pressures normal to a curved surface can not for practical purposes be combined into a total resultant pressure, yet the components in any direction may be computed. The total horizontal pressure is the sum of all the horizontal components of the normal pressures on every part of the surface; and the total vertical pressure is the sum of all the vertical components. An irregular curved surface, representing a general case for curved surfaces, will be considered. See figure 4.

Let an irregular curved surface $BDEG$, of area A, be subjected to a pressure of uniform intensity, p, on its entire under face. Let the planes CF and RL represent rectangular planes of reference. The plane of the paper is a vertical plane of reference. Compute the total horizontal pressure to the right of CF, toward R; also to the left toward L, both parallel to RL; and the total upward and downward vertical pressures.

Let the planes $H'I'J'K'$ and $H''I''J''K''$ be perpendicular to the horizontal plane of reference and to the line RL. Let the plane $H'''I'''J'''K'''$ be perpendicular to the plane of the paper.

Divide the whole surface into elements of area $a_1, a_2, \cdots a_n$, which being very small are practically planes.

Magnitude of the total horizontal pressure toward the left. The total normal pressure on $a_1 = pa_1$, on $a_2 = pa_2$, and in like manner for every element of area; each of these pressures is the normal pressure on a plane area.

The projection of the element a_1 on $H'I'J'K'$ is a'_1; and the horizontal component of pa_1 toward L is pa'_1 (§ 27). In a similar

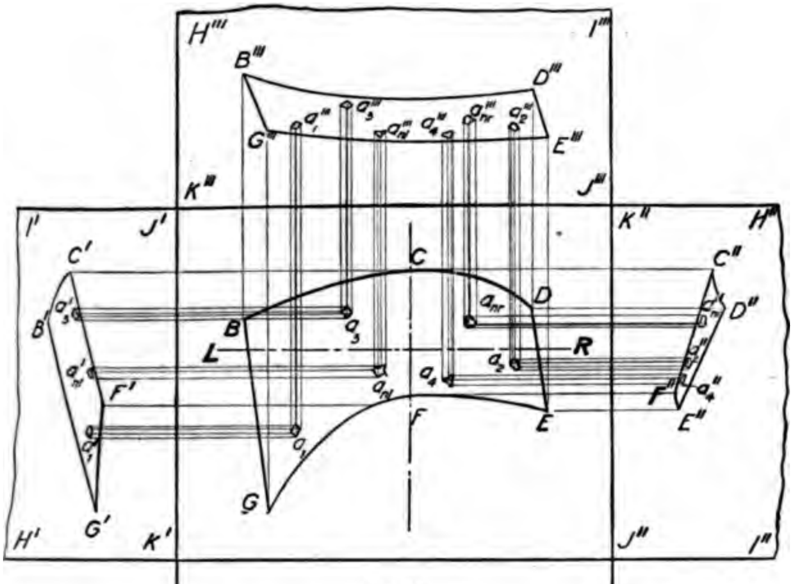


FIG. 4.

manner for every element in the surface $BCFG$, its horizontal component is p multiplied by its projection on $H'I'J'K'$.

The sum of the projected areas $=$ Limit $\Sigma\,(a'_1 + a'_3 + \cdots a'_{nl})$ $=$ area $B'C'F'G'$; and therefore, **the total horizontal pressure toward the left** $=$ Limit $\Sigma p(a'_1 + a'_3 + \cdots a'_{nl}) = p \times$ area $B'C'F'G'$.

Point of application. Since p is uniform, the point of application must be on the surface $BCFG$, on a horizontal line through the center of gravity of, and normal to, its projection $B'C'F'G'$.

Magnitude of the total horizontal pressure toward the right. By similar reasoning, it may be shown that:

The sum of the projected areas $=$ Limit $\Sigma(a''_2 + a''_4 + \cdots a''_{nr})$ $=$ area $C''D''E''F''$; and therefore **the total horizontal pressure to the right** $=$ Limit $\Sigma p(a''_2 + a''_4 + \cdots a''_{nr}) = p \times$ area $C''D''E''F''$.

Point of application is on a horizontal line through the center of gravity of $C''D''E''F''$.

Magnitude of the total vertical pressure upward. The projection of $BDEG$ on the plane of reference is $B'''D'''E'''G'''$; and by reasoning similar to that of computing the horizontal pressures, **the total vertical pressure upward** $= p \times$ area $B'''D'''E'''G'''$.

Point of application. Its point of application is on the under side of the surface $BDEG$, on a vertical line through the center of gravity of its projection, $B'''D'''E'''G'''$.

There are no components vertically downward.

Example. Let the area of the surface $BDEG$ be 200 square inches: the areas of its projections be: $B'C'F'G'$, 35.8 square inches; $C''D''E''F''$, 10.7 square inches; and $B'''D'''E'''G'''$, 71.6 square inches; and $p = 200$ pounds per square inch.

The total horizontal pressure to the left $= 200 \times 35.8 = 7160$ pounds.
The total horizontal pressure to the right $= 200 \times 10.7 = 2140$ pounds.
The total vertical pressure upward $= 200 \times 71.6 = 14{,}320$ pounds.
The total vertical pressure downward $=$ 0 pounds.

The point of application in any case must be determined from the shape of the projections: if regular figures, by mathematical formulas; if irregular, by approximate methods.

The total normal pressure pA, while obviously $200 \times 200 = 40{,}000$ pounds, has no definable direction, and is therefore of no value in computations.

RULE. **On any curved surface subjected to pressure of uniform intensity, the total component pressure in any direction equals the intensity of pressure multiplied by the area of the projection of the surface on a plane perpendicular to the given direction;** and its point of application is on a line perpendicular to the projection and through the center of gravity of that projection.

29. Internal pressures of uniform intensity in thin pipes and other cylindrical vessels. Hoop tension. Thickness of shell. Consider a pipe or cylindrical vessel (figure 5), subjected to an internal water pressure of uniform intensity, p.

Let the shell be thin and of homogeneous material, and have the following dimensions: $D =$ the internal diameter in inches, and r the radius; $t =$ the uniform thickness of shell in inches.

p = the effective intensity of pressure, or the difference between the absolute interior and exterior intensities of pressure, in pounds per square inch.

Since the shell is homogeneous and symmetrical, and the pressure uniform, the forces tending to rupture the material in a plane parallel to the axis of the pipe are symmetrical about any diameter. Consider a strip 1 inch long; and let AB be any diameter.

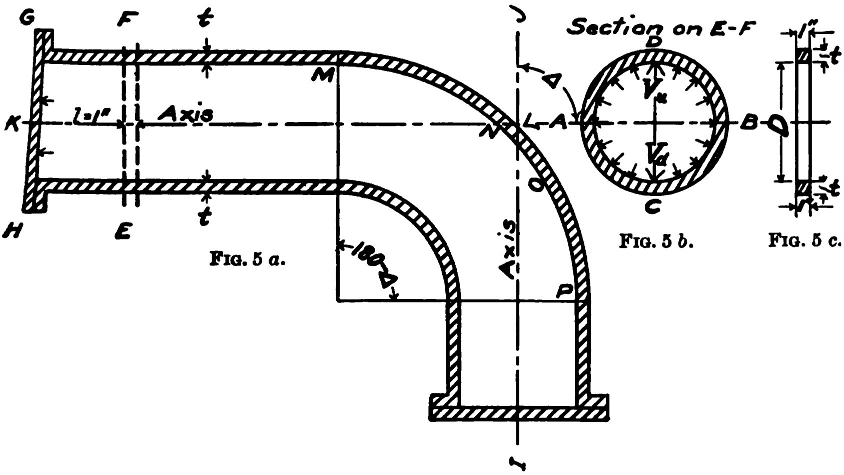


FIG. 5 a. FIG. 5 b. FIG. 5 c.

The total upward pressure (V_u) perpendicular to AB (see figure 5 b), due to the normal pressures on the area $ADB \times 1 = p \times D \times 1$, since $D \times 1$ is the horizontal projection of one half the circumference ADB. There is also an equal perpendicular force V_d opposite in direction, due to the normal pressure on $ACB \times 1$.

The tendency of these two forces to pull the pipe apart at A and B is resisted by two strips of material having a cross-sectional area of $2\,t \times 1$ (see figure 5 c). Since V_u is uniformly distributed on the projection of ABD, the stresses at A and B are equal.

If S is the tensile stress in pounds per square inch produced in the material at A and B, the total resistance of the material, or hoop tension at A and B, is $2 \times S \times t \times 1$, which, if the pipe does not rupture, must equal the force tending to pull the pipe apart at A and B.

Therefore $p\,2\,r = 2\,St$; or $pr = St$.

The tensile stress, $S, = \frac{pr}{t}$ pounds per square inch.

The requisite thickness, $t, = \frac{pr}{S_a}$ inches; S_a being in this case the allowable tensile stress in pounds per square inch.

The bursting intensity of pressure, $p_b, = \frac{S_u t}{r}$; S_u in this case being ultimate tensile strength in pounds per square inch.

30. Formula in practical use for the thickness of cast-iron pipes. The following formula is used in determining the thickness of shell in the specifications of the New England Water Works Association :

$$t = \frac{(p + p')r}{3300} + 0.25 \text{ inch.}$$

p = intensity of internal pressure in pounds per square inch.

p' = additional pressure in pounds per square inch allowed for water hammer.

3300 = allowable unit tensile stress for cast iron, being $\frac{1}{5}$ of 16,500, the ultimate tensile strength of cast iron in pounds per square inch.

0.25 inch = extra allowance for deterioration in use, or eccentricity of casting.

Values of p' used in allowing for water hammer: *

Nominal Diameter of Pipe in Inches	p' in Pounds per Square Inch
4, 6, 8, and 10	120
12 and 14	110
16 and 18	100
20	90
24	85
30	80
36	75
42 to 60	70

Note that the allowance for water hammer is in most cases greater than the actual fluid pressure. In Chapter XVII will be found two formulas for computing water hammer.

* From "Standard Specifications of the New England Water Works Association," adopted Sept. 10, 1902. *Journal N. E. W. W. Assoc.*, Vol. 16, 1902, pp. 89, 343.

Example. Compute the requisite thickness of a 24-inch cast-iron pipe for a pressure head of 100 feet.

$$t = \frac{(100 \times .433 + 85)12}{3300} + 0.25 = 0.72 \text{ inch.}$$

Compute the intensity of pressure required to burst this pipe.

$$p_b = \frac{16500 \times .72}{12} = 990 \text{ pounds per square inch.}$$

The table on the following page, computed by the N. E. W. W. formula, represents prevailing American practice as to dimensions of cast-iron water pipes.

Class A is computed for a pressure head of 50 feet; class B for 100 feet; each succeeding class advancing by an increase in head of 50 feet.

31. Formula for determining the thickness of riveted steel pipes.

$$t = \frac{(p + p')r}{S_a}.$$

S_a = allowable unit tensile stress, usually 15,000 pounds per square inch, $\frac{1}{4}$ of the ultimate strength, on the net cross-sectional area, that is, the area after deducting the rivet holes; or 15,000 multiplied by the efficiency of riveted joints, which varies from .50 to 1.00.

NOTE ON p'. Although the practice in allowing for water hammer is not as well defined for steel as for cast-iron pipes, there is usually some provision against it. The values of p' given for cast-iron pipes are recommended as a safe guide for steel pipes, but are excessive. The student should bear in mind that steel as a material is more reliable under tensile stress than cast iron.

Minimum thickness. When t as computed by this formula gives a value less than $\frac{3}{16}$ inch to $\frac{1}{4}$ inch, a minimum t, determined by practice, must be used instead of the calculated t; as the ordinary wear would destroy a thinner pipe in a few years.

Example. Compute the requisite thickness of a 24-inch riveted steel pipe for a pressure head of 100 feet, making the same allowance for water hammer as for cast iron. Efficiency of riveted joints taken as .80; S_a for steel, 15,000.

$$t = \frac{(100 \times .433 + 85)12}{15000 \times .8} = .128 \text{ inch.}$$

TABLE VI

Standard Thicknesses and Weights of Cast-iron Pipes *

(12 feet in length exclusive of socket)

Nominal Diameter of Pipe	Class A		Class B		Class C		Class D		Class E		Class F		Class G		Class H		Class I		Class K	
	Thickness of Shell	Weight per Length	Thickness of Shell	Weight per Length	Thickness of Shell	Weight per Length	Thickness of Shell	Weight per Length	Thickness of Shell	Weight per Length	Thickness of Shell	Weight per Length	Thickness of Shell	Weight per Length	Thickness of Shell	Weight per Length	Thickness of Shell	Weight per Length	Thickness of Shell	Weight per Length
	Inches	Pounds	Inches	Pounds	Inches	Pounds	Inches	Pounds	Inches	Pounds	Inches	Pounds	Inches	Pounds	Inches	Pounds	Inches	Pounds	Inches	Pounds
4	.31	200			.36	215			.39	230			.42	250			.45	265	.48	280
6	.38	330			.42	350			.46	380			.50	420			.51	445		
8	.42	475			.48	530			.53	575			.58	640			.63	690		
10	.47	650	.50	680	.53	720	.56	760	.60	810	.63	850	.67	890	.70	935				
12	.49	810	.53	855	.57	910	.61	970	.65	1040	.69	1100	.73	1160	.77	1220				
14	.53	1010	.57	1080	.61	1150	.66	1220	.70	1310	.75	1390	.79	1460	.83	1530				
16	.55	1215	.60	1300	.65	1390	.70	1490	.75	1610	.80	1710	.85	1810	.90	1900				
18	.57	1400	.63	1520	.69	1660	.75	1780	.80	1910	.86	2040								
20	.60	1610	.66	1760	.72	1920	.79	2090	.85	2260	.92	2420								
24	.61	2050	.72	2290	.80	2550	.88	2780	.95	3000	1.03	3240								
30	.71	2860	.81	3230	.91	3600	1.01	3950	1.10	4340	1.20	4700								
36	.79	3800	.90	4270	1.02	4840	1.13	5310	1.25	5900	1.37	6400								
42	.87	4920	1.00	5560	1.13	6270	1.27	6970	1.40	7720	1.53	8360								
48	.95	6130	1.10	6970	1.25	7920	1.40	8780	1.55	9740	1.70	10600								
54	1.03	7510	1.20	8600	1.37	9800	1.54	10900	1.72	12400	1.90	13500								
60	1.10	8900	1.30	10300	1.50	11900	1.70	13300	1.90	15100	2.10	16500								

* From "Standard Specifications of the New England Water Works Association," adopted Sept. 10, 1902. *Jour. N. E. W. W. Assoc.*, Vol. 16, 1902, pp. 89, 343.

As the requisite thickness is somewhat too thin, it should be made about $\frac{1}{4}$ inch.

Compute the bursting pressure if the ultimate strength of steel is taken as 60,000 pounds per square inch.

$$p_b = \frac{.8 \times 60000 \times 0.25}{12} = 1000 \text{ pounds per square inch.}$$

32. The formula for pipe thickness holds for any cylindrical vessel, or such parts thereof as may be, or may be assumed to be, under uniform intensity of pressure. Allowance for water hammer need not be made when provision is made to preclude its occurrence.

33. The total pressure in a cylindrical shell on an end closed by a plate, or on the surface of a bend. The total pressure parallel to the axis on the flat plate $GH = p\pi r^2$.

The projection of the curved surface MNO (see figure 5 a) on a plane perpendicular to the axis is πr^2; hence the total pressure parallel to the axis on this curved surface is $p\pi r^2$.

There is therefore a thrust on the end, and on the bend, of $p\pi r^2$ pounds, which, if not suitably resisted, will set up a longitudinal tensile stress in the shell, pull apart the joints, or cause the pipe to creep; in some cases special external devices are required.

34. The longitudinal tensile stress in a cylindrical shell. If t is the thickness, the cross-sectional area of the shell is

$$\pi(r+t)^2 - \pi r^2 = 2\pi rt + \pi t^2 = 2\pi rt\left(1 + \frac{t}{2r}\right).$$

(See figure 6.)

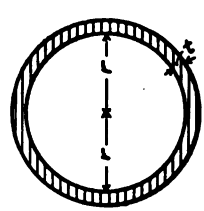


FIG. 6.

If S = actual unit tensile stress, and S_a = allowable unit tensile stress in the material of the shell,

The resistance of the material

$$S\,2\pi rt\left(1 + \frac{t}{2r}\right) = \text{the thrust } p\pi r^2.$$

Hence, $$S = \frac{p\pi r^2}{2\pi rt\left(1 + \frac{t}{2r}\right)} = \frac{pr}{2t\left(1 + \frac{t}{2r}\right)};$$

or, $$S = \frac{\pi r^2 p}{2\pi r t} = \frac{pr}{2t} \text{ (very nearly)};$$

and $$t = \frac{pr}{2S_a}.$$

The intensity of bursting pressure,

$$p_b = \frac{2S_u t}{r},$$

if S_u is the ultimate unit strength of the material.

35. A hollow sphere subjected to internal pressure. The formulas for longitudinal stress in a cylindrical shell apply to a sphere; viz.:

$$P = p\pi r^2 = S_a 2\pi r t;\ S = \frac{pr}{2t};\ \text{and } t = \frac{pr}{2S_a}.$$

The longitudinal stress in a pipe, or the stress on any diameter of a hollow sphere, is obviously but one half the "hoop tension." (See § 29.)

36. Resultant of the axial thrusts on a bend. The total axial pressure, parallel to KL on the curved surface MNO (see figure 5 a), is $p\pi r^2$.

At right angles along the axis IJ there is an equal force, $p\pi r^2$.

NOTE that figure 5 a is a special case of a right angled bend; for a general case, see figure 7.

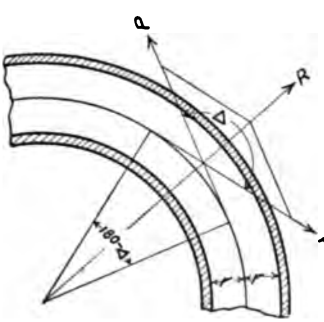


FIG. 7.

The single force along the bisector of Δ which may replace these two equal forces (P), *i.e.* their resultant, is

$$R = P\cos\frac{\Delta}{2} + P\cos\frac{\Delta}{2} = 2P\cos\frac{\Delta}{2}.$$

Example. Given a bent pipe: $p = 100$ pounds per square inch; $D = 24$ inches; $\Delta = 120°$.

Then $R = 2 \times 452.4 \times 100 \times \cos 60° = 45{,}240$ pounds.

37. Collapsing pressures on cylindrical shells. When the external exceeds the internal intensity of pressure, the two opposite forces normal to any diameter tend to crush the shell, or cause it to collapse.

The formula for a true cylindrical shell, if t is given, derived by reasoning similar to that used in determining hoop tension, is

$$S_c = \frac{pr_0}{t}.$$

S_c = actual compressive unit stress in the material in pounds per square inch.

If S_{ca} = allowable compressive stress, t = the necessary thickness; and r_0 = the outside radius of the cylinder,

$$t = \frac{pr_0}{S_{ca}}.$$

The formula differs only from the hoop tension formula in that the stress is compressive instead of tensile; and would indicate that S_c is independent of the length; this conclusion, however, is not confirmed by experience. The formula, moreover, is applicable only so long as the body maintains its true cylindrical form. If inaccurately constructed, or locally deformed, the stresses set up not only become complex but also intensify with increase in deformation; as shown by numerous cases of the collapse of cylindrical vessels, where the stresses due to the maximum possible intensity of pressure as computed by rational formulas have indicated an ample factor of safety.

38. Formulas deduced from experiments. Fairbairn's experiments on thin cylinders. Sir William Fairbairn, in 1858, published * a formula for collapsing pressure,

$$p_c = 9{,}672{,}000\,\frac{t^{2.19}}{lD_0}.$$

Limited by the author to lengths of 1.5 feet to 10 feet.

p_c = intensity of collapsing pressure in pounds per square inch;
t = thickness of shell in inches;
l = length of shell in inches;
D_0 = outside diameter in inches.

This formula, entirely empirical, was based upon two series of experiments. The first series, of about twenty-five, was made on sheet-iron tubes .043 inch thick, riveted and soldered, in diameters from 4 to 12 inches, and in lengths from 15 inches to 60 inches; the second series, less than 10 in all, on cylinders varying

* W. C. Unwin, *Proceedings Inst. of Civ. Engrs.*, Vol. 46, p. 225.

in thickness from $\frac{1}{8}$ inch to $\frac{3}{8}$ inch, in diameters from about 8 to 42 inches, and in lengths from 21 inches to 420 inches. Based on this meager data, with later additional facts, still very meager, some half score formulas bearing the names of distinguished engineers or scientists have been published in the last half century. Experience, however, has proved no one of them to be satisfactory, and most of them virtually inapplicable.

39. Stewart's experiments on steel lap-welded tubes. Professor R. T. Stewart, in 1906, published * the results of an elaborate and accurate set of experiments on steel, lap-welded, "wrought" tubes, carried out under his direction at the McKeesport works of the National Tube Company. The tests were made in two series, as follows:

"SERIES ONE. — This series of tests was made on tubes that were $8\frac{5}{8}$ inches outside diameter, for all the different commercial thicknesses of wall, and in lengths of $2\frac{1}{2}$, 5, 10, 15 and 20 feet between transverse joints tending to hold the tube to a circular form. The chief purpose of this series of tests was to furnish data for determining which of the existing formulas, if any, were applicable to modern lap-welded steel tubes, especially when used in comparatively long lengths, such as well casings, boiler tubes, and long plain flues.

"SERIES TWO. — This series of tests was made on single lengths of 20 feet between end connections, tending to hold the tube to a circular form. Seven sizes, from 3 to 10 inches outside diameter, and in all the commercial thicknesses obtainable, have been tested to date. The chief purpose of these tests was to obtain, for commercial tubes, the manner in which the collapsing pressure of a tube is related to both the diameter and the thickness of wall."

The tests were so numerous and the results so trustworthy as to prove (*a*) the inapplicability of existing published formulas, based on Fairbairn's experiments, to the conditions of modern practice; and (*b*) to establish the following facts and formulas:

(1) The length of the tube between such transverse joints as tend to maintain cylindrical form, if not less than about 6 diameters, has no practical effect on the intensity of collapsing pressure.

(2) The formulas for modern lap-welded Bessemer steel tubes:

(*A*) $p_c = 1000\left(1 - \sqrt{1 - 1600\frac{t^2}{D_0^2}}\right)$; for values of p_c less than 581, or of $\frac{t}{D_0}$ less than 0.023.

* *Transactions of Am. Soc. Mech. Engrs.*, Vol. 27 (1906), pp. 730–822.

(*B*) $p_c = 86{,}670 \dfrac{t}{D_0} - 1386$; for values of p_c and $\dfrac{t}{D_0}$ higher than given above.

p_c = intensity of collapsing pressure in pounds per square inch;

t = thickness of shell in inches;

D_0 = outside diameter in inches.

Example (*A*). Let $t = .206$ inch; and $D_0 = 10.027$ inches.

$$p_c = 1000\left(1 - \sqrt{1 - 1600\left(\frac{.206}{10.027}\right)^2}\right) = 430 \text{ pounds per square inch.}$$

Example (*B*). Let $t = .319$ inch; and $D_0 = 10.000$ inches.

$$p_c = 86{,}670 \frac{.319}{10.000} - 1386 = 1379 \text{ pounds per square inch.}$$

(3) As the apparent fiber stress at collapse varied from 7000 for the thinnest to 35,000 pounds per square inch for the thickest walls, for similar material, it appeared that the strength of a tube subjected to a fluid collapsing pressure is not dependent alone upon either the elastic limit or ultimate strength of the material.

40. Distorted tubes. A later set of experiments* was made by Stewart upon tubes distorted by previous testing, to determine a formula for tubes distinctly *not cylindrical;* the formula deduced by him is:

$$p_{c_1} = 0.0926 \frac{p_c - 47.55}{(M - 0.874)^{1.25}} + 47.55.$$

p_c = intensity of collapsing pressure of normally round tube, in pounds per square inch (§ 39, formula *A* or *B*);

p_{c_1} = intensity of collapsing pressure of a *distorted* tube, in pounds per square inch;

M = maximum outside diameter divided by minimum outside diameter at the place of greatest distortion.

Example. Let 430 pounds per square inch be the collapsing pressure for the normally round tube computed above (§ 39); if distorted so that maximum outside diameter = 10.8 inches, minimum outside diameter = 9.0 inches; then

* *Transactions of Am. Soc. Mech. Engrs.*, Vol. 29 (1907), pp. 123–130.

$$p_{c_1} = .0926 \frac{430 - 47.55}{\left(\frac{10.8}{9.0} - 0.874\right)^{1.25}} + 47.55 = 190 \text{ pounds per square inch.}$$

NOTE. "This formula represents exceedingly well the results of the experiments on the lap-welded Bessemer-steel tubes that were subjected to successive retests; and is strictly applicable, for the kind of distortion to which it applies, to lap-welded Bessemer-steel tubes for a range of thickness of wall from 0.15 to 0.20 inch for 10-inch tubes whose lengths are 20 feet between end connections tending to hold them to a circular form. The practical range of applicability is of course beyond the above narrow limits, but to just what extent is as yet, in the absence of more complete experiments, unknown."

41. In computing the probable collapsing pressure, one should bear in mind that the field of facts, except within the limits of these experiments, is practically unexplored. Outside of these, the engineer must choose one of the many existing formulas, and apply it with a very large factor of safety; which, however, may be reduced materially if the shell can be made and maintained truly cylindrical.

42. The hydraulic press. The hydraulic press, first put into use by Bramah, will serve to illustrate further the principles of fluid pressures. See figure 8.

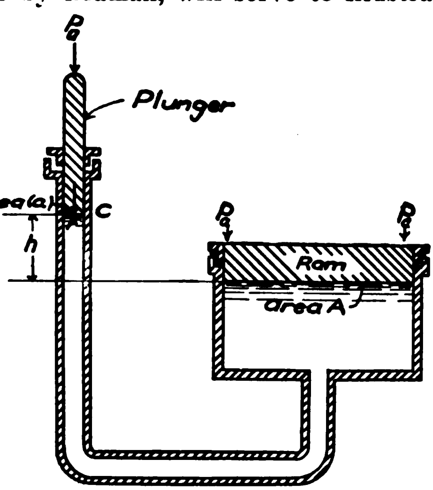


FIG. 8.

Let the plunger area $a = 1$ square inch.

Let the ram area $A = 1000$ square inches.

The force F on plunger at C in addition to the force due to atmospheric pressure $= 100$ pounds.

The intensity of pressure due to F at C is $\frac{100}{a} = 100$ pounds per square inch.

The intensity of pressure at C due to F and the atmosphere $= 100 + p_a$.

Since this is transmitted undiminished to the ram, the intensity

of pressure on the bottom of the ram corrected for difference in elevation $h = 100 + p_a + 1 \times .433$.

Since there is atmospheric pressure on top of the ram, the effective intensity of pressure on the bottom of the ram is 100 + .433 = 100.433 pounds per square inch.

Since the area $A = 1000$ square inches, the total vertical upward pressure on the ram = 100.433 × 1000 = 100,433 pounds.

43. The water piezometer. If a tube, open at the top, is inserted in a body of still water, the height of the column of water inside the tube will correspond to the intensity of pressure at the point where the tube is inserted. Such a tube used for measuring pressure is usually called a piezometer. See figure 9 *a*.

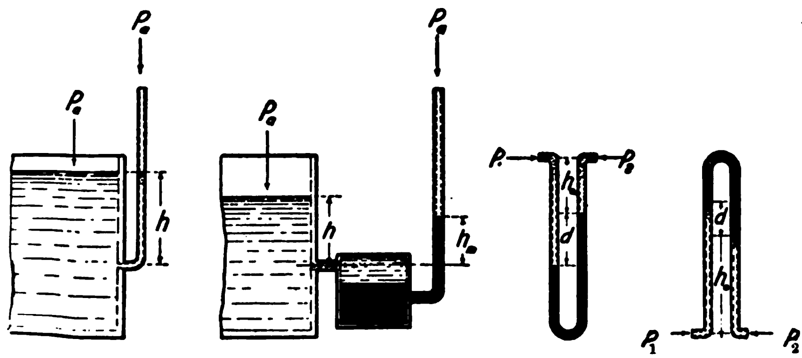


FIG. 9 *a*. Water Piezometer. FIG. 9 *b*. Mercury Gauge. FIG. 9 *c*. FIG. 9 *d*. Differential Gauges.

44. The mercury piezometer, or mercury pressure gauge. If a tube inserted in a body of still water is led to a closed reservoir containing mercury, into which a second tube is inserted, the mercury will stand in the second tube at a height (h_m) above the point where the first tube is inserted in the water, equal to h divided by the specific gravity of the mercury. See figure 9 *b*.

The height of mercury column $h_m = \dfrac{h}{13.5956} = .07355\, h$.

h_m = height of mercury column in feet.

h = height of equivalent water column in feet.

1 inch in height of mercury is equivalent to $\dfrac{1}{12 \times .07355}$ or 1.133 feet of water.

For the details of a mercury gauge see figure 73.

45. The differential gauge with a heavy liquid. If a U-tube (see figure 9 *c*), partly filled with a liquid heavier than water, have also water or other lighter liquid under different intensities of pressure in each leg, on top of the heavier liquid, the tops of the two columns of heavier liquid will have a difference in elevation or *deflection* (*d*) depending upon the difference in the pressure p_1 and p_2, and the relative densities of the two liquids.

If the liquids are quiet, the equation of equilibrium, using heads to represent pressures, may be written as follows:

$$s\left(\frac{p_1}{\gamma}+h_0+d\right)=s\left(\frac{p_2}{\gamma}+h_0\right)+s'd.$$

s = the specific gravity of the lighter liquid;
s' = the specific gravity of the heavier liquid.

Then $$\frac{p_1}{\gamma}-\frac{p_2}{\gamma}=d\left(\frac{s'-s}{s}\right).$$

γ here is the weight of a unit column of the lighter liquid.

If $\frac{s'-s}{s}=.25$, $\frac{p_1}{\gamma}-\frac{p_2}{\gamma}=0.25\,d$, and d of the lighter liquid is four times the actual difference in pressure heads.

If $\frac{s'-s}{s}=.10$, the scale is ten times exaggerated.

If $\frac{s'-s}{s}$ equals more than 1.0, the difference d is less than the actual difference in pressure heads and the scale is not enlarged but reduced. Such reductions in scale are frequently needed where the difference in pressures is great, in order to keep the size of the gauge within practical dimensions. Figure 35 shows such a gauge for use in a pitometer.

Example. Let the heavy liquid in the tube (figure 9 *c*) be carbon tetrachloride, having a specific gravity of 1.25; find the difference in pressure heads corresponding to a deflection of 2 feet, if the lighter liquid is water ($s=1.0$).

$$\frac{p_1}{\gamma}-\frac{p_2}{\gamma}=2.0\frac{1.25-1}{1}=0.5 \text{ foot of water.}$$

If the heavy liquid is mercury ($s'=13.6$) and $d=2$, the difference in pressure heads is,

$$\frac{p_1}{\gamma}-\frac{p_2}{\gamma}=2\frac{13.6-1}{1}=25.2 \text{ feet of water.}$$

46. The differential oil gauge. If a U-tube, partly filled with oil and inverted, have water or other liquid heavier than the oil under difference of pressure sustaining the oil in each leg (see figure 9 *d*), the difference (d) in the heights of the two water columns will be an exaggeration of the difference in the pressure heads due to p_1 and p_2.

If the liquids are quiet, equilibrium exists; and

$$s'\left(\frac{p_1}{\gamma} - h_0 - d\right) = s'\left(\frac{p_2}{\gamma} - h_0\right) - sd,$$

$$s'\left(\frac{p_1}{\gamma} - \frac{p_2}{\gamma}\right) = d\,(s' - s)$$

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} = d\frac{(s' - s)}{s'}.$$

γ here is the weight of a unit column of the heavier liquid.

If the heavier liquid is water ($s' = 1$), and the lighter is some oil ($s = 0.90$), and $d = 1.0$ feet,

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{d\,(1 - 0.90)}{1} = 0.1\,d = 0.10 \text{ foot.}$$

The nearer the specific gravity of the lighter liquid approaches that of the heavier liquid, the greater will the exaggeration in reading be. Williams, Hubbell, and Fenkell, who reinvented and established the use of the differential oil gauge, found that kerosene was a very satisfactory oil to use.

The specific gravity of both liquids, even if one is water, should be precisely determined for use in differential gauges.

Problems

1. A vessel similar to the one shown in figure 1 has the vertical tube filled with water to a height of 408 feet above the bottom of the cylinder. (*a*) Compute the absolute intensity of pressure in pounds per square inch on the inside along the bottom line. (*b*) Compute the total axial component pressure tending to blow off the ends of the horizontal cylinder.

2. A rectangular solid 10 feet by 10 feet by 10 feet is submerged in water. Its top surface is horizontal, and 30 feet below the water surface. (*a*) What is the total normal pressure on the top? (*b*) on the bottom?

3. A steel cylindrical tank with its axis vertical is 30 feet high and 10 feet in diameter, inside dimensions. It contains water 12 feet deep, the remaining space being filled with compressed air at 100 pounds per square inch as indicated by an ordinary steam gauge placed at the top.

(*a*) Compute the absolute intensity of pressure on the top and bottom of the inside of the tank.

(*b*) What are the total effective vertical component pressures on the top and bottom?

4. Compute the thickness of a cast-iron pipe 30 inches internal diameter under a head of 180 feet of water, using the N. E. W. W. Association formula and making the customary allowance for water hammer.

5. For the same head and water hammer allowance, as in problem 4, compute the necessary thickness of a 30-inch riveted steel pipe. Allowable stress 15,000 pounds per square inch, and joint efficiency 70 per cent.

6. What internal intensity of pressure should burst the pipe of problem 4, if the ultimate strength of the cast iron is 16,500 pounds per square inch in tension?

7. What internal intensity of pressure should burst the pipe of problem 5, if the ultimate strength of steel is 60,000 pounds per square inch in tension?

8. Compute by Fairbairn's formula the collapsing pressure for a steel shell ¼ inch thick, 20 inches external diameter, and 8 feet long with atmospheric pressure on the interior.

9. A lap-welded steel tube has outside diameter 9.625 inches, and thickness 0.344 inch. Under what intensity of external pressure should it collapse

10. A lap-welded steel tube has outside diameter 8 inches, thickness 0.16 inch. Under what intensity of external pressure should it collapse?

11. Given a 48-inch pipe with quarter bend, internal intensity of pressure 100 pounds per square inch. What is the resultant of the axial thrusts on the bend?

12. Water under a pressure of 20 pounds per square inch stands in the upper part of one leg of a differential gauge (as shown in figure 9 *c*) and at 10 pounds in the other leg. What will be the deflection of the heavier liquid (*a*) if its specific gravity is 13.6? (*b*) if its specific gravity is 1.40? (*c*) if its specific gravity is 1.95? What will happen if a liquid lighter than water instead of heavier is placed in this kind of a tube?

13. A liquid having a specific gravity of .95 is placed in the top of a differential oil gauge (figure 9 *d*); the oil is supported on water having a specific gravity of 1.005. The deflection is 0.4 foot. What is the difference in intensity of pressure in the water in the two legs in pounds per square inch?

14. A hydraulic jack with a 2-inch ram and ¾-inch plunger is to lift a weight of 1.5 tons. The leverage of the handle is 12 to 1. What force must be applied to the handle?

15. The plunger of a hydraulic press is 1 inch in diameter. The diameter of the ram is 7 inches. The leverage is 6 to 1. What weight could be lifted on the ram if the force acting at the end of the handle is 90 pounds?

CHAPTER III

FLUID PRESSURES OF VARYING INTENSITY

Plane Surfaces

47. A plane surface subjected to pressures of varying intensity. If an immersed plane surface is not parallel to the free surface of the liquid, the intensity of pressure, since it is proportional to the head at any point, will have a different value for every part of the surface.

The irregular plane surface $ABCD$, area A, immersed in a liquid and inclined at any angle θ with the free surface, will represent a general case from which may be deduced the magnitude of the total normal pressure on any plane surface, and of its components, its direction, and its point of application. See figure 10.

Let $a_1, a_2, \cdots a_n$ be elements of area of $ABCD$; and $h_1, h_2, \cdots h_n$ the heads upon these areas.

The intensity of pressure on $a_1 = h_1w = p_1$; on $a_2 = h_2w = p_2$; on $a_n = h_nw = p_n$.

The total normal pressure on $a_1 = h_1wa_1 = p_1a_1$; on $a_2 = h_2wa_2 = p_2a_2$; on $a_n = h_nwa_n = p_na_n$.

The total normal pressure (P) on the plane area $ABCD$ is the sum of the normal pressures on all its parts; that is,

$$P = \text{Limit } \Sigma(h_1wa_1 + h_2wa_2 + \cdots h_nwa_n).$$

Let the intersection of the plane of $ABCD$ with the free surface, which is the line $O-O$, be taken as a center of moments.

Then y is the shortest distance from $O-O$ to any element of area as indicated by subscripts;

and y_c is the shortest distance from $O-O$ to the center of gravity of $ABCD$;

y_p is the shortest distance from $O-O$ to the point of application of P.

From the figure,

$$h_1 = y_1 \sin\theta; \qquad h_2 = y_2 \sin\theta; \qquad h_n = y_n \sin\theta.$$

Substituting these values in the above equation for P,

$$P = \text{Limit } \Sigma\,(y_1a_1 + y_2a_2 + \cdots y_na_n) \sin\theta \times w.$$

The moment of an area is the area multiplied by the distance

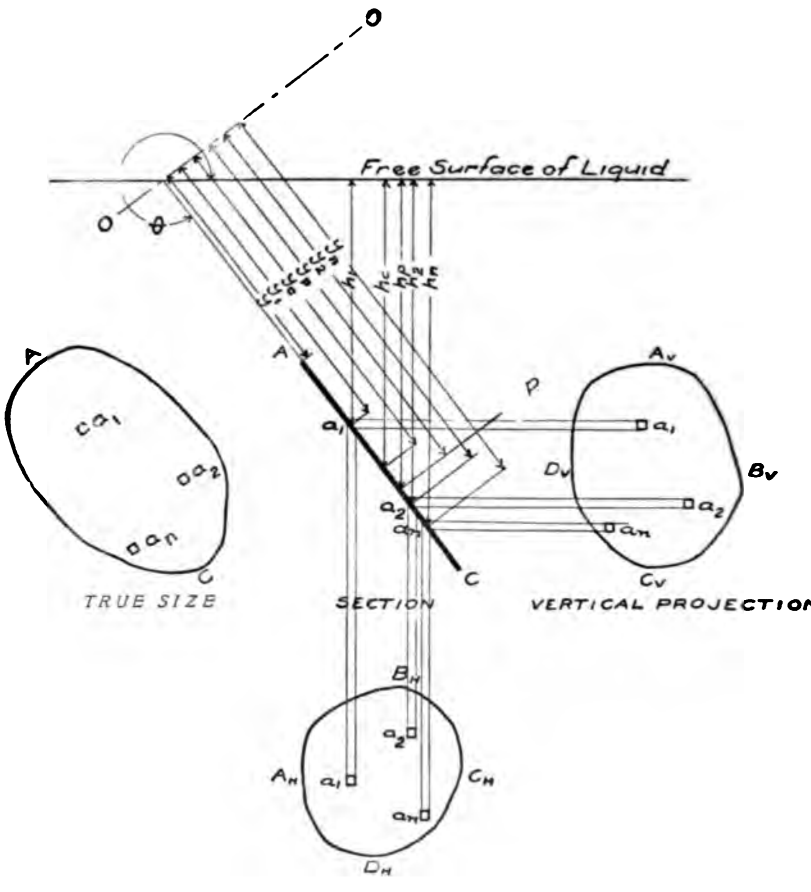


FIG. 10.

from its center of gravity to any center of moments, and equals the sum of the moments of all its parts about the same center. Expressed mathematically,

$$\int_{..}^{''}\int_{..}^{''} y\,\delta x\,\delta y\,^* = y_c\int_{..}^{''}\int_{..}^{''}\delta x\,\delta y = y_c A.$$

Therefore, Limit $\Sigma(y_1a_1 + y_2a_2 \cdots y_na_n) = y_cA$,
and $y_c \sin\theta = h_c$, the head on the center of gravity of $ABCD$.

Hence, **the magnitude of the total normal pressure,** (P), $= y_c \sin\theta \times w \times A = h_cwA$.

The direction of P is $(\theta - 90)$ measured from free surface.

The point of application of P. The distance from any center of moments to the point of application of a system of parallel forces equals the sum of the moments of all the forces divided by the sum of all the forces.

Using the same center of moments $O\text{–}O$, then

$(y \sin\theta aw)y =$ moment of the normal pressure on any infinitesimal area.

Hence, the sum of moments

$$= \text{Limit}\ \Sigma(y_1^2a_1 + y_2^2a_2 + \cdots y_n^2a_n)\sin\theta w,$$

and $$y_p = \frac{\text{Limit}\ \Sigma(y_1^2a_1 + y_2^2a_2 + \cdots y_n^2a_n)\sin\theta w}{\text{Limit}\ \Sigma(y_1a_1 + y_2a_2 + \cdots y_na_n)\sin\theta w} = \frac{\int\int y^2\,\delta x\,\delta y}{\int\int y\,\delta x\,\delta y}.$$

That is, $$y_p = \frac{\text{Moment of Inertia of } A}{\text{Moment of } A} = \frac{I'}{M} = \frac{I + Ay_c^2}{Ay_c}$$
$$= y_c + \frac{I}{Ay_c} = y_c + \frac{Ak_0^2}{Ay_c} = y_c + \frac{k_0^2}{y_c}.$$

Also $$y_cy_p = k_0^2 + y_c^2.$$

$I =$ moment of inertia of an area about an axis parallel to $O\text{–}O$ through its center of gravity $= Ak_0^2$.

$I' =$ moment of inertia of area about any other parallel axis.

$I + Ay^2 =$ general equation for finding I' about an axis distant y from the given axis of gravity.

$k_0 =$ radius of gyration about a parallel axis through the center of gravity.

* $\int_{..}^{''}\int_{..}^{''}\delta x\,\delta y$ is the expression for an element of area;

y is the distance from the center of moments to any area;

$\int_{..}^{''}\int_{..}^{''} y\,\delta x\,\delta y =$ sum of the moments of the elements of area.

Components of P. The horizontal pressure (H),

$$P\cos(\theta-90)=P\sin\theta=h_c w(A_V B_V C_V D_V).$$

The vertical pressure (V),

$$P\sin(\theta-90)=P\cos\theta=h_c w(A_H B_H C_H D_H).$$

The direction of H is here 180°. The direction of V is here 270°.

SUMMARY. The magnitude of **the total normal pressure on any immersed plane surface equals the intensity of pressure at its center of gravity multiplied by the area** of the surface.

Direction of the total normal pressure. Since the direction is fixed by the position of the surface, P is always perpendicular to the surface and toward it.

The point of application of the total normal pressure, or of its components, is through a point on the immersed surface at a distance measured parallel to the surface and distant $\frac{k_0^2}{y_c}$ below the center of gravity. When the pressure is of uniform intensity, $y_p=y_c$.

48. Formulas for regular areas. For regular areas, the precise values of P and y_p are most easily found by integrating between their proper limits the expressions:

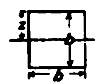
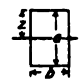
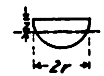
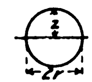
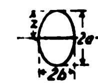
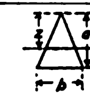
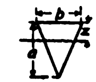
$$P=y_c\sin\theta wA=w\sin\theta\int\int y\,\delta x\,\delta y;\ \text{and}\ y_p=\frac{\int\int y^2\,\delta x\,\delta y}{\int\int y\,\delta x\,\delta y}.$$

49. Table VII gives the properties of the more common shapes thus computed. For other shapes the properties may be found in treatises on calculus and in various handbooks.

50. If an area is composed of one or more regular areas, the precise value of P and y_p for each area may be found as above, and the total normal pressure and its point of application computed by the ordinary methods of composition of forces, or more simply by the following rules.

51. Property of center of gravity. If $y_1, y_2, \cdots$ are the distances from any fixed axis to the centers of gravity of the partial areas

PROPERTIES OF COMMON REGULAR SHAPES; w = WEIGHT OF CUBIC FOOT OF LIQUID

Neutral Axis ($N.A.$)--------- Z = distance from extreme upper point of area to $N.A.$	 Square	 Rectangle	 Semicircle	 Circle	 Ellipse	 Triangle	 Triangle
Area (A)	b^2	ab	$\frac{\pi r^2}{2}$	πr^2	πab	$\frac{ab}{2}$	$\frac{ab}{2}$
Z	$\frac{b}{2}$	$\frac{a}{2}$	$.424\,r$	r	a	$\frac{2}{3}a$	$\frac{a}{3}$
I = Moment of inertia about $N.A.$	$\frac{b^4}{12}$	$\frac{ba^3}{12}$	$.109\,r^4$	$0.785\,r^4$	$0.785\,a^3b$	$\frac{ba^3}{36}$	$\frac{ba^3}{36}$
$\frac{I}{Ay_c}$	$\frac{b^2}{12\,y_c}$	$\frac{a^2}{12\,y_c}$	$\frac{r^2}{14\,y_c}$ (app.)	$\frac{r^2}{4\,y_c}$	$\frac{a^2}{4\,y_c}$	$\frac{a^2}{18\,y_c}$	$\frac{a^2}{18\,y_c}$
When $Z = y_c$, $\frac{I}{Ay_c}$	$\frac{b}{6}$	$\frac{a}{6}$	$\frac{r}{6}$ (app.)	$\frac{r}{4}$	$\frac{a}{4}$	$\frac{a}{12}$	$\frac{a}{6}$
When $Z = y_c$, y_p	$\frac{2}{3}b$	$\frac{2}{3}a$	$\frac{3}{16}\pi r$	$\frac{5}{4}r$	$\frac{5}{4}a$	$\frac{3a}{4}$	$\frac{a}{2}$
When $Z = y_c \sin\theta$ $P = ZAw$	$\frac{b^3}{2}w$	$\frac{a^2b}{2}w$	$\frac{2}{3}r^3w$	πr^3w	πa^2bw	$\frac{a^2b}{3}w$	$\frac{a^2b}{6}w$
Radius of gyration squared = k^2	$\frac{b^2}{12}$	$\frac{a^2}{12}$	$.696\,r^2$	$\frac{r^2}{4}$	$\frac{a^2}{4}$	$\frac{a^2}{18}$	$\frac{a^2}{18}$

A_1, A_2, ⋯, and y = the distance from the same axis to the center of gravity of the total area A, then

(I) $$Ay = A_1y_1 + A_2y_2 + \cdots.$$

(For, the sum of the moments in one direction equals the moment in the opposite direction.)

52. Properties of radius of gyration. If k_1, k_2, ⋯ are the radii of gyration of the partial areas A_1, A_2, ⋯ about any fixed axis, and k = the radius of gyration of the total area A about the same axis, then

(II) $$Ak^2 = A_1k_1^2 + A_2k_2^2 + \cdots.$$

(For, the "moment of inertia" of the total area equals the sum of the moments of inertia of the separate parts.)

If k = radius of gyration about any axis, and

k_0 = radius of gyration about a parallel axis through the center of gravity, then

(III) $$k^2 = k_0^2 + j^2,$$

where j = the distance between the axes.

53. Point of application, that is, center of pressure on a submerged area.

If y_p = distance to center of pressure,

and y_c = distance to center of gravity of the area,

then $$y_p \times y_c = k^2, \ = k_0^2 + y_c^2. \quad \therefore\ y_p = y_c + \frac{k_0^2}{y_c},$$

where k = radius of gyration about an axis in the surface.

If the y_p and y_c for the partial areas are known, the k^2 for each partial area can at once be found, and hence the k^2 for the total area, by (II).

Note that the y_p, y_c, and k will always stand in the following order of magnitude: $$y_p > k > y_c.$$

54. If an area is irregular, it may be divided into parallel strips, or small areas of such dimensions as the required precision demands, and the normal pressure on each subdivision treated as an individual force; and the total normal pressure approximately determined by the formula

$$P = w \sin\theta\Sigma(y_1a_1 + \cdots y_na_n).$$

EXAMPLES

55. Center of moments. A very convenient center of moments is the line of intersection of the immersed surface, or the surface produced, with the free surface of the liquid, which will be taken in the following examples, and designated *O–O*.

56. A vertical wall having its upper edge at or above the water surface. A wall with vertical faces, as shown in figure 11, forms the partition between two compartments of a reservoir. One is empty; the other has water 20 feet deep. Compute the total normal pressure on the face in contact with the water.

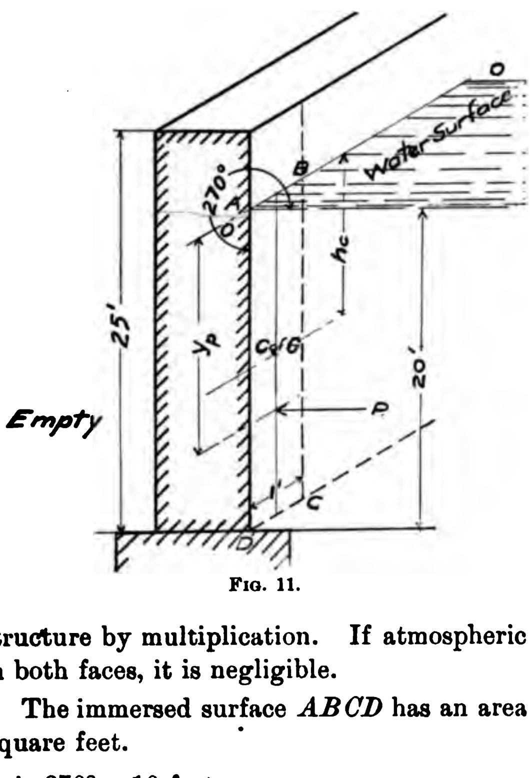


FIG. 11.

In computations for retaining walls, dams, and similar structures, it is customary to consider a section 1 foot long. If the structure is homogeneous and symmetrical, the computations for a one-foot strip may be applied to the entire structure by multiplication. If atmospheric pressure (p_a) exists on both faces, it is negligible.

The magnitude of *P*. The immersed surface *ABCD* has an area 20 feet × 1 foot = 20 square feet.

$$h_c = y_c \sin 270° = 10 \text{ feet.}$$

Therefore, $P = h_c wA = 10 \times 62.4 \times 1 \times 20 = 12{,}480$ pounds, or (from Table VII)

$$\frac{a^2b}{2}w = \frac{20 \times 20 \times 1}{2} \times 62.4 = 12{,}480 \text{ pounds.}$$

The direction of *P* is 180°.

The horizontal and vertical components of P.

$$H = P \cos 180° = 12{,}480 \text{ pounds.}$$

$$V = P \sin 180° = 0.$$

The point of application of P (y_p). Since the figure is symmetrical, the point of application, or center of pressure, is on a vertical bisecting the surface $ABCD$, and distant from O–O,

$$y_c + \frac{k_0^2}{y_c}; \text{ but } k_0^2 = \frac{a^2}{12}, \text{ and } y_c = \frac{a}{2}; \ \frac{k_0^2}{y_c} = \frac{a}{6}; \text{ (Table VII).}$$

Therefore, $y_p = 10 + \frac{20}{6} = \frac{2}{3} \times 20 = 13\frac{1}{3}$ feet below O–O.

57. A vertical wall having its upper edge below the water surface. A masonry dam, as shown in figure 12, has a vertical upstream face. The water is 30 feet deep on the upstream face, and there is no water pressure on the downstream face. Compute the total normal pressure on the upstream face.

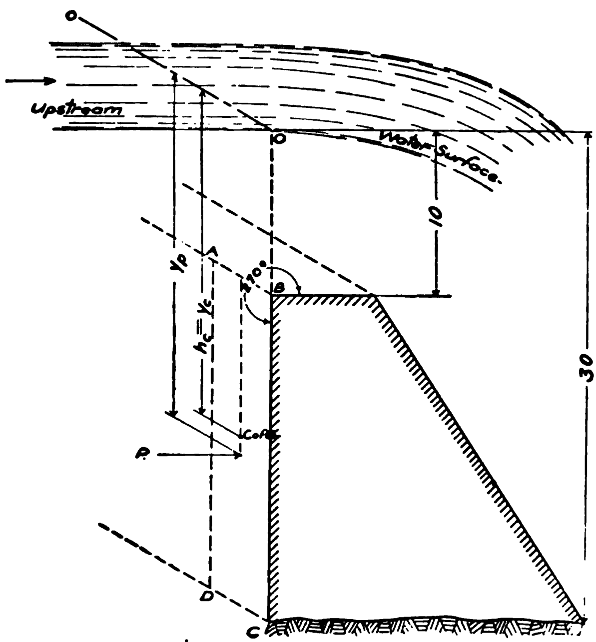


Fig. 12.

Consider a section 1 foot long.

Since atmospheric pressure (p_a) acts on both faces, it is negligible.

The magnitude of P. The immersed surface $ABCD$ has an area 20 feet × 1 foot = 20 square feet.

$$h_c = y_c \sin 270° = 20 \text{ feet.}$$

Therefore, $$P = h_c w A = 20 \times 62.4 \times 20 = 24{,}960 \text{ pounds.}$$

The direction of P is 0°.

The horizontal and vertical components of *P*.

$$H = P \cos 0° = 24{,}960 \text{ pounds.}$$

$$V = P \sin 0° = 0.$$

The point of application of P (y_p). The point of application is on a vertical bisecting the surface $ABCD$, distant from $O-O$,

$$y_p = y_c + \frac{k_0^2}{y_p}; \text{ but } \frac{k_0^2}{y_c} = \frac{a^2}{12\,y_c}; \text{ (Table VII).}$$

Therefore, $y_p = 20 + \dfrac{20 \times 20}{12 \times 20} = 20 + 1\frac{2}{3} = 21.67$ feet.

58. A vertical sluice gate covering a circular opening. Compute the total normal water pressure on a vertical gate covering

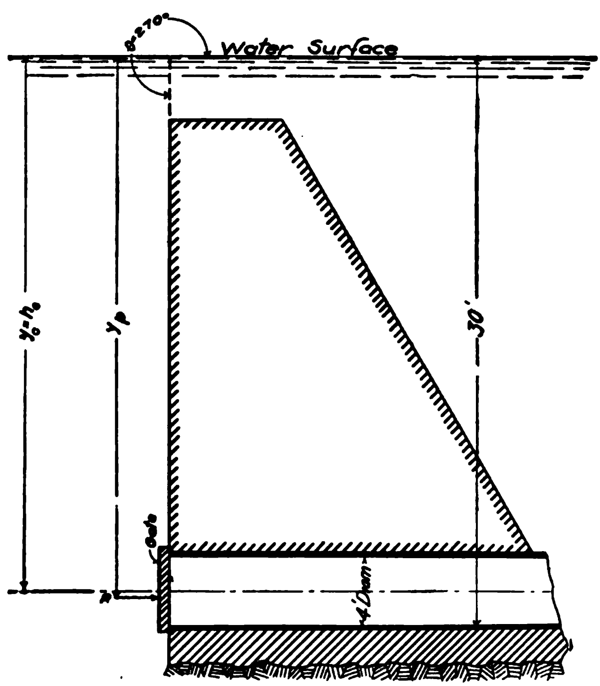


FIG. 13.

a 4-foot circular pipe, as shown in figure 13. Neglect atmospheric pressure (p_a).

The magnitude of P. The area $A = 12.57$ * **square feet.**

$$y_c = 30 - 2 = 28\text{; and } h_c = 28 \sin 270° = 28 \text{ feet.}$$

Therefore, $P = h_c wA = 28 \times 62.4 \times 12.57$ * $= 21{,}962$ **pounds.**

The direction of P is 0°.

The point of application of P is on a vertical through the center of gravity of the opening and below it a distance $\frac{k_0^2}{y_c}$. From Table VII,

$$\frac{k_0^2}{y_c} = \frac{r^2}{4\,y_c}.$$

Therefore, $$y_p = 28 + \frac{2 \times 2}{4 \times 28} = 28.036 \text{ feet.}$$

If by a very rapid closing of the gate, and withdrawal of the water, a complete vacuum should be produced in the pipe, the effective head would be increased by the head due to atmospheric pressure, and become $28 + 33.9 = 61.9$ feet.

P would then be 48,550 pounds, and

$$y_p = 33.9 + 28 + \frac{2 \times 2}{4 \times 61.9} - 33.9 = 28.016 \text{ feet.}$$

Under an assumption of *uniform* intensity of pressure, the point of application would be at the center of gravity, or $y_p = 28$, an error of about $\frac{1}{5}$ to $\frac{1}{10}$ of 1 per cent, and obviously negligible. The larger the area, and the nearer the water surface, the more y_p differs from y_c. Whether to consider the pressure on a gate as of uniform or of varying intensity is a question of the degree of precision required by the problem.

59. An inclined gate or valve covering a circular opening. Compute the total normal pressure on an inclined gate covering a 6-foot circular pipe, as shown in figure 14. Given $y_c = 10$ feet. Neglect (p_a).

The magnitude of P. The area of the opening A is an ellipse in which $a = \frac{r}{\sin 60°}$ and $b = r$.

$$\text{Therefore, } A = \frac{\pi r^2}{\sin 60°} = \pi r^2 \times 1.155 = 32.65 \text{ square feet.}$$

$$h_c = y_c \sin 240° = 10 \times .866 = 8.66 \text{ feet.}$$

* See Tables of Areas of Circles, LXVII and LXIX.

Therefore, $P = h_c wA = 10 \sin 60° \times 62.4 \times 32.65 = 17{,}640$ pounds.

The direction of P is 330°.

Horizontal and vertical components of P.

$$H = 17{,}640 \cos 330° = 17{,}640 \times .866 = 15{,}270 \text{ pounds.}$$
$$V = 17{,}640 \sin 330° = 17{,}640 \times .5 = 8820 \text{ pounds.}$$

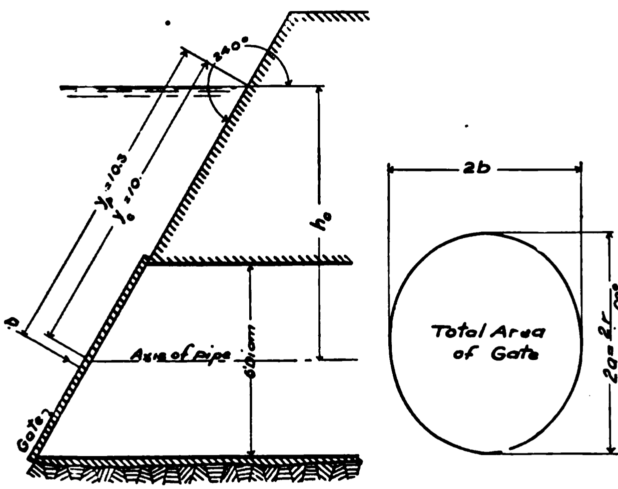


FIG. 14.

The point of application of P. The point of application of P is on a vertical through the center of gravity of the opening, and below it a distance $\frac{k_0^2}{y_c}$. From Table VII, $\frac{k_0^2}{y_c} = \frac{a^2}{4\,y_c}$.

Therefore,

$$y_p = 10 + \frac{(3 \times 1.155)^2}{4 \times 10} = 10 + \frac{12}{40} = 10.3 \text{ feet from } O\text{–}O.$$

If in computing y_p, uniform pressure were assumed, y_p would be 10 instead of 10.3, an appreciable error, but only 3 per cent.

60. A lock gate with sea water on only one side. Let figure 15 represent an elevation of a lock gate, with sea water on only one side. For sea water, take the weight of a cubic foot (w) as equal to 64 pounds.

The total normal pressure on $ABCD$, its direction, and its point of application are to be determined.

On account of its shape, the area $ABCD$ can not be readily treated singly; but inspection shows that it is composed of a rectangular area, two triangles, and a segment of a circle.

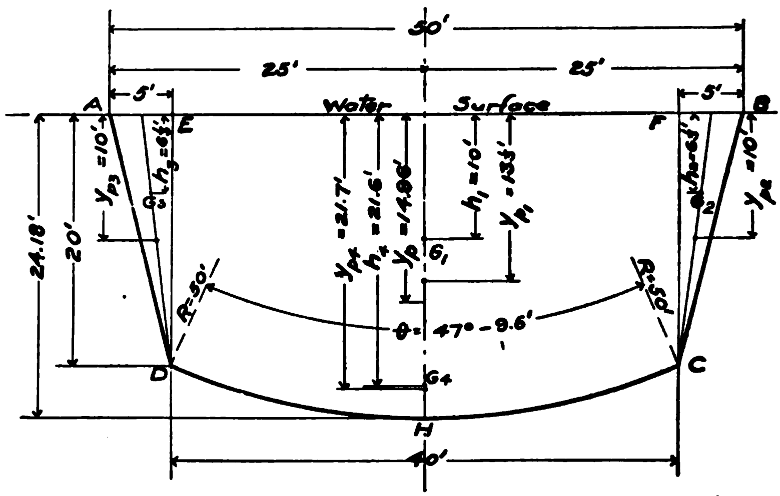


FIG. 15.

The total normal pressure on each area can be determined; and from these their resultant, the total normal pressure (P) on $ABCD$.

The area $ABCD =$ the sum of the areas $CDEF + BCF + ADE + CDH$.

Let the total normal pressure on $CDEF = P_1$; on $BCF = P_2$; on $ADE = P_3$; and on $CDH = P_4$. Let h_1, h_2, h_3, and h_4 be the heads on the centers of gravity of these four areas.

The magnitude of P_1. $P_1 = 10 \times 64 \times 40 \times 20 = 512{,}000$ pounds.

The direction of P_1 is 0°.

The point of application of P_1. $y_{p_1} = \frac{2}{3} \times 20 = 13\frac{1}{3}$ feet below the water surface.

The magnitude of P_2 and P_3. Since the triangles BCF and ADE are equal, $P_2 = P_3 = \frac{a^2 b}{6} w$ (Table VII) $= \frac{20^2 \times 5}{6} \times 64 = 21{,}333$ pounds; and $P_2 + P_3 = 42{,}667$ pounds.

The direction of P_2 and P_3 is 0°.

The points of application of P_2 and P_3. Since the triangles are similarly placed with respect to the water surface,

$$y_{p_2} = y_{p_3} = \frac{20}{2} \text{ (Table VII)} = 10 \text{ feet below the water surface.}$$

The magnitude of P_4. Area of segment $CDH =$

$$\tfrac{1}{2} \times 50 \times 50\left(\frac{\theta\pi}{180} - \sin\theta\right) = 112.4 \text{ square feet.}$$

$\theta =$ angle subtending the chord CD, $= 47°\ 9.6'$.

Center of gravity of $CDH = \dfrac{40^3}{12 \times 112.4} = 47.45$ feet from center of circle, or 21.6 feet below the water surface.

$$P_4 = 21.6 \times 64 \times 112.4 = 155{,}382 \text{ pounds.}$$

The direction of P_4 is 0°.

The point of application of P_4.

$$y_{p_4} = y_{c_4} + \frac{k_0^2}{y_{c_4}} = 21.7 \text{ (nearly) below the water surface.}$$

Inasmuch as the segment is well under water, no serious error would result if y_{p_4} were assumed equal to y_{c_4}.

The magnitude of the total normal pressure, P. Since the component forces are all parallel,

$$P = 512{,}000 + 42{,}667 + 155{,}382 = 710{,}049 \text{ pounds.}$$

The direction of P. Since the direction of P_1, P_2, P_3, and P_4 is 0°, the direction of P, their resultant, is 0°.

The point of application of P. Since the figure is symmetrical about its center line, the point of application is on the center line, distant y_p from the water surface.

$$y_p = \frac{\text{sum of moments of } P_1, P_2, P_3, \text{ and } P_4}{P_1 + P_2 + P_3 + P_4};$$

therefore, $y_p =$

$$\frac{512{,}000 \times 13\tfrac{1}{3} + 42{,}667 \times 10 + 155{,}382 \times 21.7}{710{,}049} = 14.96 \text{ below the}$$

water surface.

61. A solid with water on two opposite parallel surfaces.

The rectangular solid, as shown in figure 16, has two vertical faces, each 33 feet high and 8 feet wide; the water stands 30 feet

deep on one side and 10 feet deep on the other. Compute the total normal pressures on the opposite faces, and the resultant pressure on the wall.

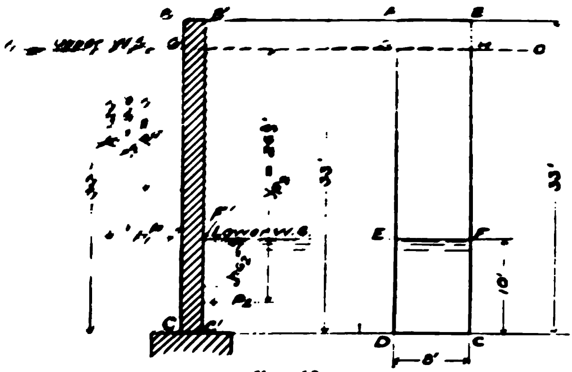


FIG. 16.

Let A_1 be the immersed surface $CDGH$; and let P_1 be the total normal pressure on A_1.

Let A_2 be the immersed surface $C'D'E'F'$; and let P_2 be the total normal pressure on A_2.

The total normal pressure on A_1 (P_1).

$$P_1 = h_{c_1} w A_1 = 15 \times 62.4 \times 30 \times 8 = 224{,}640 \text{ pounds.}$$

The direction of P_1 is 0°.

The point of application of P_1.

$$y_{p_1} = \tfrac{2}{3} \times 30 = 20 \text{ feet below water surface, } O\text{–}O.$$

The total normal pressure on A_2 (P_2).

$$P_2 = h_{c_2} w A_2 = 5 \times 62.4 \times 10 \times 8 = 24{,}960 \text{ pounds.}$$

The direction of P_2 is 180°.

The point of application of P_2.

$y_{p_2} = \frac{2}{3} \times 10 = 6\frac{2}{3}$ feet below the lower water surface; or 20 feet $+ 6\frac{2}{3}$ feet below the upper water surface, O–O, $= 26.67$ feet.

The resultant pressure (P).

Since P_1 and P_2 are parallel opposite forces,

$$P = P_1 - P_2 = 224{,}640 - 24{,}960 = 199{,}680 \text{ pounds.}$$

The direction of P is horizontal, and since P_1 is greater, the direction is 0°.

The point of application of P.

Taking moments about O–O,

$$y_p = \frac{224640 \times 20 - 24960 \times 26.67}{199680} = 19.17 \text{ feet below } O\text{–}O.$$

Note. In computing water pressures against dams, walls, and similar structures, the difference between the highest possible elevation to which the water can ever rise, and the lowest point of the dam, however deep in the ground, should in general be used. Moreover, the beginner should note that the difference between P_1 and P_2 is not identical with the pressure on $EFGH$. The total pressure on $CDEF = 25 \times 62.4 \times 10 \times 8 = 124{,}800$ pounds; while the pressure on $C'D'E'F' = 5 \times 62.4 \times 10 \times 8 = 24{,}960$ pounds, — one fifth as much; and they do not offset each other.

62. An irregular plane surface; method of approximation.

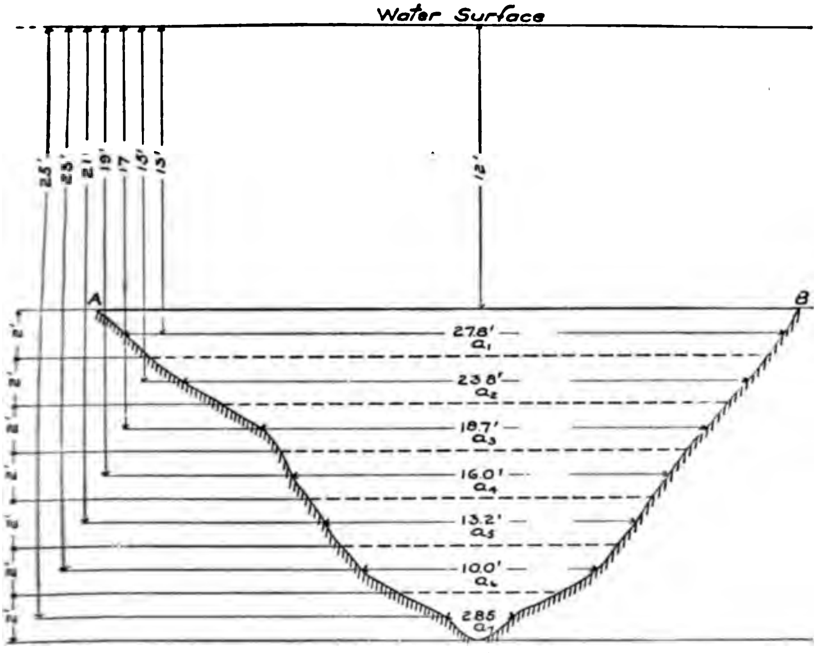


FIG. 17.

Figure 17 may represent an elevation of a cut-off wall of masonry or sheet piling, a coffer dam, or any irregular plane surface with water on only one side.

Divide the figure by horizontal lines 2 feet apart into areas $a_1 \cdots a_7$.

Assume that the center of gravity and the center of pressure of a_1, a_2, etc., are on horizontal lines midway between the upper and lower boundaries of each small area.

The computation to determine A, P, h_c, y_c, and y_p is put, for convenience, into tabular form, as follows:

1	2	3	4	5
Figure	a Areas sq. ft.	y Distance in feet from water surface to centers of gravity	ya Moment of areas	y^2a
a_1	55.6	13	725	9,420
a_2	47.6	15	714	10,710
a_3	37.4	17	636	10,810
a_4	32.0	19	608	11,550
a_5	26.4	21	554	11,640
a_6	20.0	23	460	10,580
a_7	5.7	25	142	3,550
	224.7		3839	68,260

$A = \Sigma(a_1 + \cdots a_7) = 224.7 =$ area of figure.

$y_c = \dfrac{\Sigma(y_1a_1 + \cdots y_7a_7)}{\Sigma(a_1 + \cdots a_7)} = \dfrac{3839}{224.7} = 17.0$ feet, distance from the water surface to the center of gravity.

$h_c = y_c \sin 270° = 17.0 =$ head on the center of gravity.

The magnitude of $P = \Sigma(y_1a_1 + \cdots y_7a_7) \sin \theta w = 3839 \times 62.4$
$= 239{,}550$ pounds.

The direction of P is 0°.

The point of application of P.

$$y_p = \frac{\Sigma(y_1^2a_1 + \cdots y_7^2a_7)\sin\theta w}{\Sigma(y_1a_1 + \cdots y_7a_7)\sin\theta w} = \frac{68260}{3839} = 17.8 \text{ feet.}$$

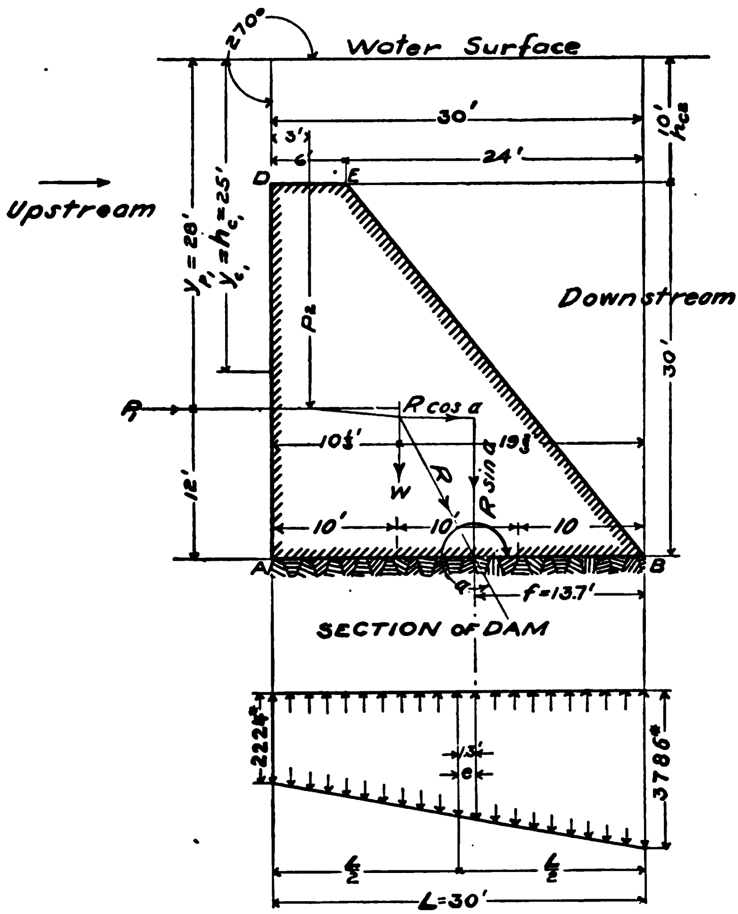
Similar, but more precise, approximate solution. By taking strips one foot wide instead of two feet, the following results would have been attained:

$P = 248290$ pounds; $y_c = h_c = 17.2$ feet; $y_p = 18.0$ feet.

The difference between the values of P in these two cases is about $3\frac{1}{2}$ %; and of h_c and y_p slightly over 1 %. If a higher degree of accuracy is needed, the area may be still more finely subdivided.

The horizontal coördinates of the center of gravity, and the center of pressure may be computed in a similar manner.

63. A masonry dam with water on one side. The dam shown in section in figure 18 is of solid masonry, which is assumed to weigh



Graphic Representation of Intensity of Pressure on Foundation

FIG. 18.

160 pounds per cubic foot. Consider a section one foot long. Although there may be water on the downstream side, its effect is usually neglected.

The stability of the structure is to be determined, under the action of the following forces:

The total normal pressure on the plane AD (P_1);

The total normal pressure on the plane DE (P_2);

The weight of one foot of the dam $ABDE$ (W); and

The resistance of the foundation.

The magnitude of $P_1 = y_{c_1} \sin 270° wA = 25 \times 62.4 \times 30 \times 1 =$ 46,800 pounds.

The direction of $P_1 = 270° + 90° = 360° = 0°$.

The point of application of P_1.

$$y_{p_1} = 25 + \frac{30 \times 30}{12 \times 25} \text{ from } O\text{–}O, = 28 \text{ feet.}$$

The magnitude of $P_2 = h_{c_2} \times w \times A = 10 \times 62.4 \times 6 \times 1 = 3744$ pounds.

The direction of $P_2 = 270°$.

The point of application of $P_2 = \frac{6}{2}$ from D, $= 3$ feet.

The magnitude of $W = \left(6 \times 30 + \frac{24 \times 30}{2}\right) \times 160 = 86{,}400$ pounds.

The direction of $W = 270°$.

The point of application of W is in a vertical through the center of gravity, which is distant from AD, $\frac{180 \times 3 + 360\,(6 + 8)}{540} =$ $10\frac{1}{3}$ feet.

Conditions of stability of a masonry dam. To insure stability and permanency, a dam should be built with suitable provision against:

I. Sliding bodily as a whole on its foundation; or in part along any plane in the structure.

II. Excessive stresses in the material in the structure, or excessive pressure on its foundation.

III. The percolation of water through, under, or around the dam.

I. **Sliding.** The forces tending to move the dam bodily down-stream along the plane AB are the following:

$$P_1 \cos 0° + P_2 \cos 270° + W \cos 270° =$$
$$P_1 \times 1 + P_2 \times 0 + W \times 0 = P_1 = \text{sliding force} = 46{,}800 \text{ pounds.}$$

If there is no cohesion between the base and the structure, then the only resistance to sliding is the friction, which equals the sum of the vertical components of all the forces multiplied by the coefficient of friction (μ), which for such cases may vary from 0.25 to 0.75. Take μ here $= .6$; then

The resistance to sliding

$$= \mu(P_1 \sin 0° + P_2 \sin 270° + W \sin 270°)$$
$$= 0.6(0 - 3744 - 86400) = 54{,}086 \text{ pounds.}$$

The factor of safety against sliding $= \dfrac{54086}{46800} = 1.15.$

When the frictional resistance is less than the sliding force (if for example $\mu = 0.4$), the structure would not be stable against sliding, if friction were the only resisting force.

If a masonry dam is built into a solid rock trench, as it should be, other resistances, such as the adhesion between the mortar and the stone, and in well-bonded rubble the shearing resistance of the masonry itself, will insure stability against sliding; and if so built, a dam stable in other respects will ordinarily be safe against sliding.

II. Stresses in the material or on the foundation. Stresses at any section, as, for example, in the plane AB, may be computed as follows:

Let R be the resultant of the forces P_1, P_2, and W; its direction be $\alpha°$; and f be the distance from B on AB at which R cuts AB.

The magnitude of R.

$R \cos \alpha =$ horizontal component of $R = P_1 = 46{,}800$ pounds.

$R \sin \alpha =$ vertical component of $R = P_2 + W = 90{,}144$ pounds.

$$R = \sqrt{90144^2 + 46800^2} = 101{,}500 \text{ pounds.}$$

The direction of R.

$$\frac{R \sin \alpha}{R \cos \alpha} = \tan \alpha = \frac{90144}{46800} = 1.926\text{; therefore, } \alpha = 360° - (62°\ 34')$$
$$= 297°\ 26'.$$

The point where the resultant (R) cuts the base. To find the point where R cuts AB, take moments of all forces about B, as a center.

$$R \sin \alpha \times f + R \cos \alpha \times 0 + P_1 \times 12 - P_2 \times 27 - W \times 19.67 = 0.$$

Therefore $f = \dfrac{P_1 \times 12 - P_2 \times 27 - W \times 19.67}{P_2 + W} = 13.7$ feet from B on AB.

If the point where R cuts AB coincides with the gravity axis of the area of the base, the intensity of pressure p_f on AB is uniform, and $p_f = \dfrac{R \sin \alpha}{L \times 1}$ pounds per square foot.

If the resultant (R) cuts the base on either side of the gravity axis, the pressure along AB will be of varying intensity. The toe nearer the point where R cuts the base will have a maximum intensity of pressure, and the other toe a minimum. The intensity of pressure at either toe A or B may be determined by the following expression:

$p_f = \dfrac{R \sin \alpha}{L}\left(1 + \dfrac{6\,e}{L}\right)$* the intensity of pressure in pounds per square foot at either toe.

e is the distance from the gravity axis of the base to the point where R cuts the base.

If e is measured downstream from the gravity axis, the maximum p_f is at the downstream toe; if measured upstream, the maximum is at the upper toe.

If $e = \dfrac{L}{6}$, which means that R cuts the base at the boundary of its middle third; $p_f = \dfrac{R \sin \alpha}{L}(1 \pm 1) = \dfrac{2\,R \sin \alpha}{L}$ at one toe, 0 at the other.

If e is greater than $\dfrac{L}{6}$, the minimum value of p_f changes to tension.

* This formula is by similarity taken over from the formula used to determine the extreme fiber stresses in a strut eccentrically loaded, viz.:

$S = \dfrac{R \sin \alpha}{a} \pm \dfrac{Mc}{I}$.

a = area of section; here, $a = L \times 1$.

c = any distance from neutral axis; here $c = \dfrac{L}{2}$.

I = moment of inertia of a; here $I = \dfrac{L^3}{12}$.

M = moment of external forces; here $M = R \sin \alpha \times e$.

S = intensity of stress at any point distant c from the neutral axis; and is tension whenever $\dfrac{R \sin \alpha}{a}$ is less than $\dfrac{Mc}{I}$.

Good practice does not sanction designing masonry (except reënforced concrete) to withstand tensile stresses; **the section should be shaped so that R shall not cut the base beyond the limits of its middle third**; if so proportioned, and if the other two conditions are satisfied, a dam may in general be considered stable.

p_f should obviously not exceed either the allowable crushing strength of the masonry or the allowable intensity of pressure on the foundation.*

In this case, $e = 1.3$ feet, measured toward B; then,

$$p_f \text{ at } B = \frac{R \sin \alpha}{L}\left(1 + \frac{6 \times 1.3}{30}\right) = \frac{90144}{30}(1. + .26) = 3786 \text{ pounds}$$

per square foot (compression).

$$p_f \text{ at } A = \frac{R \sin \alpha}{L}\left(1 - \frac{6 \times 1.3}{30}\right) = \frac{90144}{30}(1. - .26) = 2224 \text{ pounds}$$

per square foot (compression).

In figure 18, the lower diagram represents graphically the intensity of pressures along AB for this dam.

III. **Water percolating under or around the dam** will not only diminish its usefulness in storing water, but also reduce its stability.

If the sides and bottom of a dam are of earth, percolating water may wash out the foundations, and ultimately destroy the dam.

If the sides and bottom are of seamy rock, pressure may be transmitted through cracks from the upstream water, and cause an undesirable upward vertical pressure. This upward pressure is equal to the head on the wetted surface in the crack multiplied by w, multiplied by the area of wetted surface. Suppose, for example, as an extreme case, a crack extends along AB and nearly through to B; the upward vertical pressure would equal $40 \times 62.4 \times 30 = 74{,}880$ pounds; this is nearly equal to the total vertical downward force of 90,144 pounds, and its effect would practically nullify the frictional resistance.

The obvious safeguard is to prevent percolation as far as possible, by water-tight cut-off walls of masonry, concrete, or sheet

* See, for discussion of allowable pressures on foundations, Baker's *Treatise on Masonry Construction*; Corthell's *Allowable Pressures on Deep Foundations*; Wegmann's *Masonry Dams*; or other treatises and articles on dam construction.

piling placed as near the upstream face as practicable; but if some water must percolate, drains should be provided, to carry such water through without washing away either the dam or its foundations.

If a dam is also to serve as a spillway, or waste weir, and designed to discharge water over its top, it should be of sufficiently liberal dimensions to discharge the maximum flood flow, and to keep the water from overtopping the other parts of the structure. The stability of a dam is not infrequently determined by the capacity of a spillway.

This dam is of the type known as a *gravity dam*, because its weight is a most important factor in assuring its stability.

64. A framed dam. In contrast is shown, in figure 19, a dam which depends very little upon its weight for stability. Such

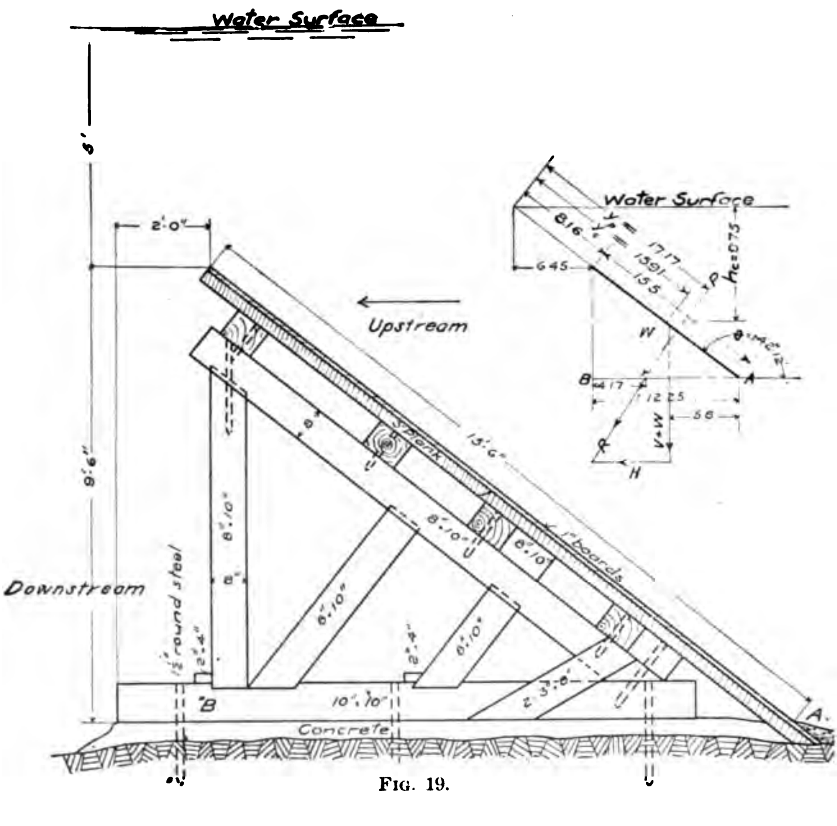


FIG. 19.

dams, which are now built in wood, steel, and reënforced concrete, must be designed as framed structures. Figure 19.

65. In designing large dams, the structure should not be treated as a whole, but part by part, beginning near the top. Successive horizontal sections may be assumed, the part above each section computed as if complete in itself, until by steps the whole structure is dimensioned and its stability determined.

Curved Surfaces

66. Curved surfaces subjected to fluid pressures of varying intensity. On a curved surface as on a plane the intensity of pressure varies as the distance from the free surface of the fluid; inasmuch as the normal forces on the different elements of area of the curved surface do not form a system of parallel forces, no single force can be said to be the total resultant force; but the total horizontal or vertical fluid pressure, or the total fluid pressure in any direction, may, however, be determined.

If the highest degree of precision is required, and the curved surface is regular, the magnitude, direction, and point of application of the components may be found by the integration of the proper differential equations. If the surface is not regular, and if the variation in intensities of pressure can not be neglected, the problem is usually solved by approximate methods. Two cases will illustrate approximate methods as applied to curved surfaces.

A deep cylindrical tank (axis vertical) filled with water. In a deep tank filled with water, there is a considerable variation in the intensity of fluid pressure. In determining the thickness of the shell, its surface may be divided for computation into sections, of which the dimensions will be more or less fixed by commercial considerations. The pressure may be assumed constant on each of these sections, and the necessary thickness of material t may be computed for successive sections by the formula for hoop tension.

Let figure 20 represent a deep cylindrical tank. Use the intensity of pressure at the point A for the cylindrical surface between A and B, and the pressure at the points, B, C, and D for each successive ring.

A metal shell. If the cylindrical tank of radius r (inches), shown in figure 20, be of steel or of any other homogeneous metal, and filled with water, the necessary thickness is

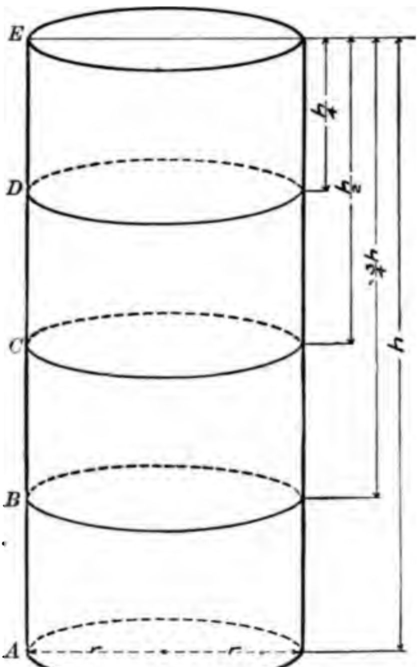


FIG. 20.

$$t \text{ at } A = \frac{pr}{S_a} = \frac{.433 \times h \times r}{S_a};$$

which is the required thickness of the circular plate of width AB.

$$t \text{ at } B = \frac{pr}{S_a} = \frac{.433 \times 3\,h \times r}{4\,S_a};$$

which is the required thickness of the circular plate of width BC.

In like manner, the thickness of each successive set of plates may be computed.

A wooden-stave tank. If the tank is of wooden staves hooped with metal, the requisite net cross section a of the material necessary to hold together the area defined by A and B may be determined as follows:

The force tending to rupture the cylinder AB (using the intensity of pressure at A) equals $.433\,h \times 2\,r \times AB$.

The resistance of the material in the hoops equals $2 \times a \times S$.

a = net cross section of material in the hoops in the section AB;

S = actual unit tensile stress in the material.

Since the resistance of the material of the hoops must be equal to the force tending to rupture the hoops,

$$0.433 \times h \times 2\,r \times AB = 2 \times a \times S.$$

Therefore, if a is given,

$$S = \frac{0.433 \times h \times r \times AB}{a} = \text{unit stress in tension in the hoops.}$$

If S_a, the allowable tensile stress, is given, the required cross-sectional area in the material in the hoops for section AB,

$a = \frac{.433 \times h \times AB \times r}{S_a} = \frac{p \times AB \times r}{S_a}$ = required area in square inches. This is merely another application of the computation of *hoop* tension (§ 29).

67. The horizontal and vertical fluid pressures on the inside of the curved surface of a tank; axis horizontal.

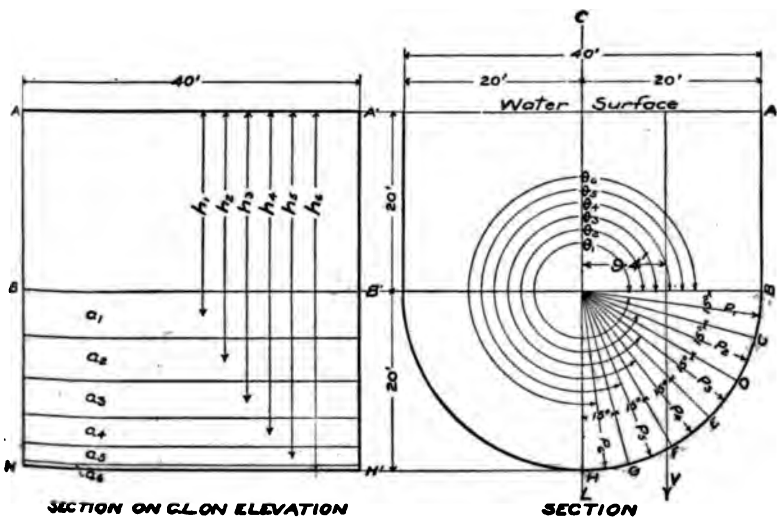


FIG. 21.

The determination by approximate methods of the magnitude and point of application of the total horizontal and vertical pressures on one quadrant, the area $BB'HH'$, of the tank shown in figure 21, is here given.

Divide the area $BB'HH'$ into six equal areas, designated a_1 to a_6; and assume each area to be a plane.

Assume that for a_1 the pressure is of uniform intensity, and is measured by h_1, the head on its horizontal center line. The point of application of P_1 is therefore h_1 feet below the water surface; also $h_1 = y_{p_1}$. In like manner, the total pressure on every one of the six areas may be determined.

The calculations are shown in tabular form, as follows:

(1) Figure	(2) Area sq. ft.	(3) Values of θ	(4) Values of h	(5) Values of ah	(6) $ah \sin \theta$	(7) $ah \cos \theta$	(8) $ah \sin \theta$ $20 \cos \theta$	(9) $ah^2 \cos \theta$
a_1	209.4	$352\frac{1}{2}°$	22.6	4730	620	4,680	12,280	105,800
a_2	209.4	$337\frac{1}{2}°$	27.7	5800	2,220	5,360	41,070	148,000
a_3	209.4	$322\frac{1}{2}°$	32.2	6750	4,110	5,350	65,400	172,100
a_4	209.4	$307\frac{1}{2}°$	35.9	7530	5,970	4,580	72,800	164,300
a_5	209.4	$292\frac{1}{2}°$	38.5	8060	7,450	3,100	57,300	119,100
a_6	209.4	$277\frac{1}{2}°$	39.8	8340	8,270	1,090	21,500	43,300
					28,640	24,160	270,350	752,600

$$H = \text{Limit } \Sigma w (ah_1 \cos \theta_1 + \cdots ah_6 \cos \theta_6) = 24{,}160 \times 62.4$$
$$= 1{,}507{,}600 \text{ pounds.}$$
$$V = \text{Limit } \Sigma w (ah_1 \sin \theta_1 + \cdots ah_6 \sin \theta_6) = 28{,}640 \times 62.4$$
$$= 1{,}787{,}100 \text{ pounds.}$$
$$y_p = \frac{\text{Limit } \Sigma w (ah_1^2 \cos \theta_1 + \cdots ah_6^2 \cos \theta_6)}{\text{Limit } \Sigma w (ah_1 \cos \theta_1 + \cdots ah_6 \cos \theta_6)} = \frac{752600}{24160} = 31.1 \text{ feet}$$

below the water surface.

The point of application of V is in a vertical line, of which the distance from the center line (C. L.) may be found as follows:

$$\frac{\text{Limit } \Sigma w (ah_1 \sin \theta_1 \, 20 \cos \theta_1 + \cdots ah_6 \sin \theta_6 \, 20 \cos \theta_6)}{\text{Limit } \Sigma w (ah_1 \sin \theta_1 + \cdots ah_6 \sin \theta_6)} = \frac{270350}{28640} = 9.4.$$

The other coördinates may be found in a similar manner.

If a higher degree of precision is required, smaller subdivisions should be made. In this case, the actual value of V is equal to half the weight of water in the tank; the value above computed differs from the weight by less than 1 %. Obviously, no higher degree of precision is needed. Any curved surface can be so treated for computation.

Problems

1. On one side of a sluice gate the water stands 8 feet above the bottom; on the other side, 4 feet; the gate is 6 feet wide. What is the total pressure against the grooves in which the gates slide? Where is the center of this pressure?

2. A flap valve, opening inwardly, automatically controls the water-supply in a small boiler. When the surface of the water is 3 feet above the center of the valve, and the steam gauge reads 20 pounds, at what height must the water stand in pipes outside, in order to cause flow?

3. A cast-iron sluice gate is placed at the bottom of a dam; its dimensions are 4 inches thick, by $1\frac{1}{2}$ feet wide, by 6 feet high; the water is 50 feet deep. If coefficient of friction of gate on its runners is 0.23, if gate is exposed to air on downstream side, what force will be required just to lift it? Assume the iron to weigh 480 pounds per cubic foot.

4. Compute the horizontal and vertical components of the normal pressure on an immersed rectangular area 6 feet by 8 feet, inclined 30° to the vertical, one of the longer edges being in the water surface.

5. Find the magnitude and point of application of the normal pressure on an immersed rectangle with sides 4 feet by 6 feet, with shorter side in the water surface.

6. Find the magnitude and point of application of the normal pressure on an immersed triangle of base 2 feet and altitude 3 feet, the plane being vertical, (*a*) with vertex in water surface and base horizontal, (*b*) with base in water surface.

7. Given a coffer dam of irregular outline as shown in figure 17, and assuming that the distance from the top of the irregular figure to the water surface is 24 feet, compute the total normal water pressure on the area ABC, and find the position of the center of pressure with reference to the water surface, and also to the vertical axis through A.

8. Figure 15 represents the outline of a caisson or gate forming the entrance to a lock. Assume that the sides are prolonged upwards until the water is 10 feet deeper than shown. When the water surface in the lock is flush with the top of the gate, find the total normal pressure on the face, and the point of application of the normal pressure. The lock is filled with sea water.

9. The partition wall between the compartments of a settling basin is 12 feet high. When on one side the water is within 1 foot of the top and the other side is empty, if masonry weighs 160 pounds per cubic foot, and 60 per cent of its total weight represents its ability to resist sliding, how thick must it be? Consider a strip 1 foot wide.

10. A small masonry dam 20 feet high, 4 feet thick at top, 10 feet thick at bottom, is symmetrical in cross section. The masonry weighs 140 pounds per cubic foot. When filled to top, what will be the resultant pressure on base, and how far from a perpendicular line through the center of gravity of the section will the resultant cut the base? Consider a strip one foot wide. Is this section stable?

11. In masonry dam of problem 10, if the upstream face is vertical, the dimensions being the same, and considering, as before, a section 1 foot wide, how far from the downstream toe will the resultant pressure cut the base?

12. A circular plate 4 feet in diameter is immersed so that the head on its center is 40 feet, its plane making an angle of 35° with the vertical. Compute the horizontal and vertical pressures on one side of it.

13. The upstream face of a dam has a slope of 60° with the horizontal. A penstock, 6 feet in diameter, covered by a gate is so set in the dam that the

center of the pipe is 20 feet below the surface of the water, measured along the face of the dam. Find the total normal pressure on the gate and its point of application.

14. What is the total normal pressure on the upstream face of the dam shown in figure 14? Consider a strip 1 foot wide.

15. A masonry dam is 6 feet thick at the top, 8 feet thick at the bottom, and 10 feet high. The water is 4 feet deep over the crest. Find the total normal pressure against the upstream face and its point of application. Consider a strip 1 foot wide. Is this section stable?

16. A masonry dam is 5 feet thick at the top, 40 feet thick at the bottom, and 40 feet high; upstream face vertical. Assume that the water stands 6 feet higher than the top, and that there is no water on the downstream face. Take the weight of masonry as 160 pounds per cubic foot and consider the pressure of water on the top of the dam. Determine (*a*) factor of safety against sliding if the coefficient of friction is 0.6; (*b*) distance from upstream toe to point where resultant cuts the base; (*c*) pressure in pounds per square foot at upstream and downstream toes.

17. A standpipe is 20 feet in internal diameter and 30 feet high; it is to be of riveted steel plates and to be entirely filled with water. The horizontal joints are to be 7.5 feet apart vertically; and all joints are to have an efficiency of 75 per cent. The allowable unit stress for steel in tension may be taken as 15,000 pounds per square inch. Compute the necessary thickness of each row of plates.

18. If the standpipe in problem 17 were to be made of wooden staves 2 inches thick, compute the required number of hoops made of $\frac{1}{2}$ inch diameter steel rods, and determine the spacing of the hoops.

CHAPTER IV

THE EQUILIBRIUM OF FLOATING SOLIDS

68. To secure complete equilibrium of a totally immersed or floating solid, **the resultant** horizontal and the resultant vertical components **of all forces** acting on it **must be zero; and it must have no tendency to rotate about any axis.**

Every face of a solid totally immersed in water, and every face below the water line of a floating solid,* is subjected to water pressure.

If the surface of the water in which the body is immersed is exposed to the atmosphere or compressed gases, or is under pressure due to mechanical forces, the intensity of pressure at any point on the solid is the sum of the intensity of pressure due to the head of water, and that due to forces acting on the water surface.

Whenever the intensity of pressure due to any force is transmitted with unchanged intensity to every face of the solid, the neglect of such a pressure will cause no error. In the following computations concerning the stability of bodies either immersed or floating in still water, the fluid pressures due to water alone will be considered, the atmospheric pressure being neglected.

69. Horizontal pressures. Let $ABCD$ in figure 22 a represent a vertical cross section of an immersed solid and let the solid be subdivided into elementary horizontal prisms. Let figure 22 b represent such a prism enlarged.

Since $a_{\prime}$, and $a_{\prime\prime}$, the elementary areas forming the ends of the prism in contact with the water, are equally distant (h feet) from the surface of the water, the intensity of pressure on $a_{\prime}$ and $a_{\prime\prime} = hw$; and the total normal pressure on $a_{\prime} = hwa_{\prime} = P_{\prime}$ and on $a_{\prime\prime} = hwa_{\prime\prime} = P_{\prime\prime}$.

The horizontal component of $P_{\prime} = H_{\prime} = hw$ multiplied by the

vertical projection of $a_{\prime}$; likewise the horizontal component of $P_{\prime\prime} = H_{\prime\prime} = hw$ multiplied by the vertical projection of $a_{\prime\prime}$.

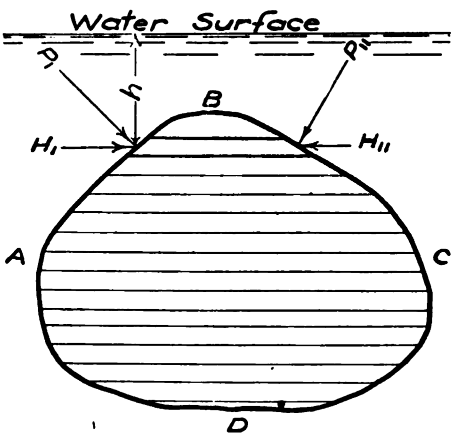


FIG. 22 *a*.

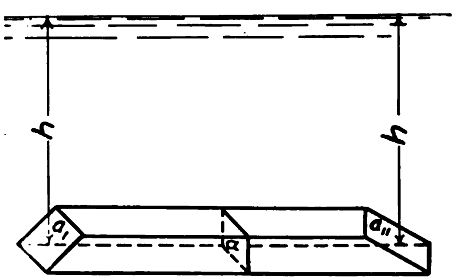


FIG. 22 *b*.

But since a, the cross-sectional area of the prism, is the vertical projection of both $a_{\prime}$ and $a_{\prime\prime}$, $H_{\prime} = hwa = H_{\prime\prime}$. Therefore the resultant horizontal component on the prism $= hwa - hwa = 0$.

On any other horizontal prism, the resultant will, for like reasons, be 0.

Since the sum of the volumes of all the parallel prisms is obviously the volume of the whole solid, the horizontal components parallel to the axis shown $(A - C)$, form, therefore, a system of parallel forces whose resultant is zero.

If, in like manner, the body be cut into elementary prisms parallel to a second axis also horizontal, and at right angles to the first considered, similar reasoning will show that the resultant horizontal component parallel to the second axis is also zero.

Since the sum of the components parallel to any two rectangular horizontal axes equals zero, there is no tendency to translation in a horizontal plane.

70. Vertical pressures. Consider the same solid cut into elementary vertical prisms, shown in figures 23 *a* and 23 *b*.

If a' and $a'' =$ elementary areas forming the ends of the prism, their horizontal projections $= a$, the horizontal cross-sectional area of the prism.

V', the vertical component of P', $= h'wa$.

V'', the vertical component of P'', $= h''wa$.

The resultant vertical fluid pressure on the prism $= V'' - V' = (h'' - h')wa$.

But $(h'' - h')a =$ volume of the prism, and also the volume of water displaced by it, and $(h'' - h')wa =$ weight of water displaced by the prism.

The magnitude of the resultant vertical pressure; buoyancy. Since the sum of the volumes of all the prisms equals the volume of the solid, the algebraic sum of all the vertical components,

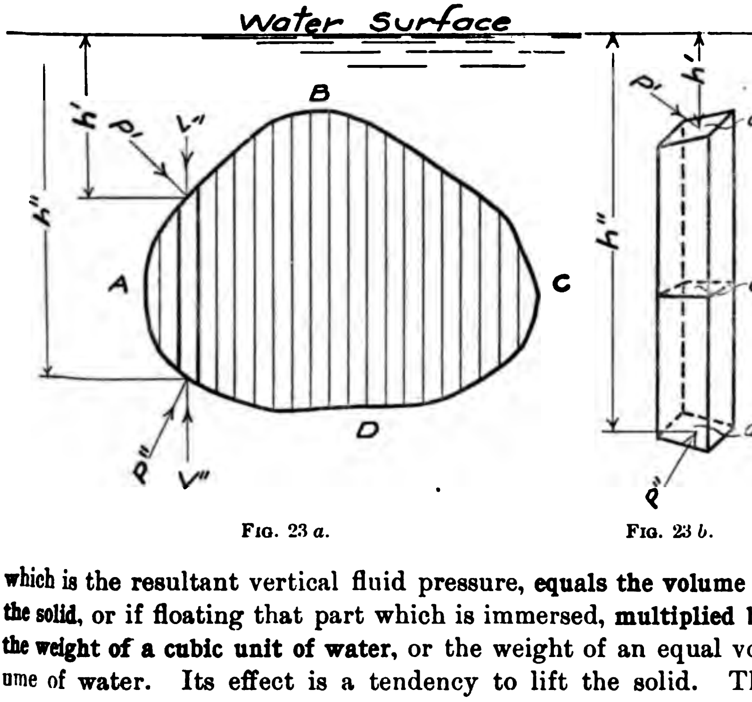


FIG. 23 *a*. FIG. 23 *b*.

which is the resultant vertical fluid pressure, **equals the volume of the solid,** or if floating that part which is immersed, **multiplied by the weight of a cubic unit of water,** or the weight of an equal volume of water. Its effect is a tendency to lift the solid. The **resultant vertical pressure** ($V'' - V'$) is therefore usually called **the buoyancy,** and will be designated B.

Direction of the resultant vertical pressure. Since the upward vertical fluid pressure is always greater than the downward vertical fluid pressure, the direction of B is 90°.

The point of application of B; center of buoyancy. The point of application of B is the center of gravity of the displaced water, called the center of buoyancy, and designated b.

If the body is wholly immersed and homogeneous, the center of gravity of the solid and the center of buoyancy are identical.

If it is hollow or of varying density, or not wholly immersed, the center of gravity of the solid and the center of buoyancy may not coincide.

71. Loss of weight. The weight (W) of the body is a downward vertical force; and the buoyancy is an upward vertical force W and B form, therefore, a set of parallel, opposite forces.

If B is less than W, a totally immersed body will be in equilibrium only when an additional vertical upward force equal to the difference between the weight of the body and the buoyancy is applied to it; that is, the body is supported by some external force. If the body is weighed while suspended in water, its apparent weight will be W minus B. Since W is its weight in air, its loss of weight is B.

If B is greater than W, an upward vertical force, B minus W will be exerted upon an immersed body; and to keep it immersed a vertical downward force equal in magnitude to B minus W must be applied to maintain equilibrium.

If unrestrained, this vertical upward pressure, B minus W, will cause the body to rise until it floats in equilibrium; which occurs when the weight of the displaced water, no longer of equal volume with the solid, equals W. In this position, B equals W and the apparent weight of the solid is zero. Its loss of weight is B.

The loss of weight of an immersed or floating body is merely the buoyant effect of the water, and equals the weight of the displaced water.

72. Depth of flotation; draught. The depth of flotation, or draught, of a floating body may be determined by computing: first, a volume of displaced water equal in weight to the body, including all loads, and the vertical resultant of all forces acting upon it; and then the depth of immersion, measured from the water surface to the lowest point of the solid, necessary to produce the required volume of displacement. The depth of flotation is constant only for a few shapes; usually, it varies as the body is tilted.

73. The stability of an immersed or floating body against overturning may be determined, as in any problem in statics, by taking moments of all the force acting upon the body, using any convenient axis of reference. For bodies of irregular shape the necessary computations, especially those required to determine the normal water pressures or their components, may be somewhat intricate. This work may be simplified in many instances by an investigation of the relative position of the center of gravity (or more precisely the point of application of all vertical forces) and the position of the center of buoyancy.

If the algebraic sum of the horizontal and vertical components of all forces equals zero, and if the center of gravity and center of buoyancy are in the same vertical line, the body will be in complete equilibrium.

74. **The buoyancy (B) of a floating body** is equal in magnitude and opposite in direction to the sum of all the vertical downward forces, including the weight of the body and all loads, and which for convenience will be called the weight (W).

The center of buoyancy will be designated b. The center of gravity or the point of application of all vertical forces will be designated G.

75. **The axis of flotation.** If a floating body is in complete equilibrium, a vertical through G and b (both computed for the position in which it is then floating) is called the axis of flotation.

If the axis of flotation divides the body into symmetrical parts, the body will float upright.

The centers of gravity and of buoyancy when the body floats upright are the original centers of gravity and of buoyancy, designated G_0 and b_0.

76. **The angle of heel; change of trim.** The angular displacement in any given direction of the axis of symmetry measured from the vertical is called the angle of heel, θ. The length of the water line multiplied by the sine of the angle of heel is the change of trim.

77. **The righting couple; the upsetting couple.** If a floating body is tilted, so that G_0 and b_0 are not in the same vertical line, W and B, which are equal in magnitude, parallel, but opposite in

direction, will form a couple the magnitude of which equals $Bc = Wc$; c being the perpendicular distance between the lines of action of B and W. The effect of this couple will be either to restore the body to an upright position or to cause further tilting—

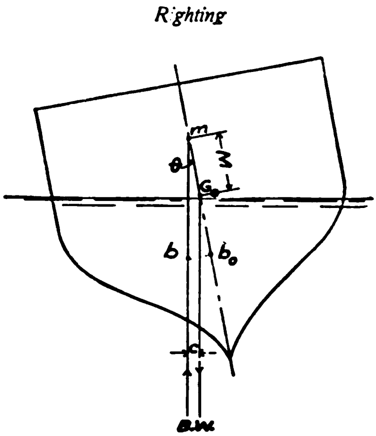


FIG. 24 *a*.

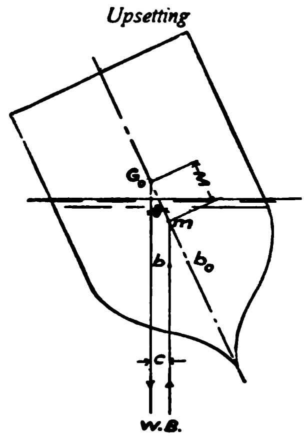


FIG. 24 *b*.

If, upon the removal of the couple which caused tilting, the **couple *Bc* tends to restore** the body to its original position of equilibrium, ***Bc* is a righting couple**; and the body is stable (see figure 24 *a*).

If, on the contrary, **the couple *Bc* tends to rotate** the body still farther from the original position of equilibrium, ***Bc* is an upsetting couple**; and the body is unstable (see figure 24 *b*).

78. The metacenter. The point where the vertical through the center of buoyancy cuts an inclined axis of symmetry is called the metacenter (m).

79. The metacentric height. The distance measured on the axis of symmetry from the metacenter, for the position in which the body is floating, to the original center of gravity is called the metacentric height (M). For only a few cases is it constant; usually it varies with every change in position. The longitudinal and transverse metacentric heights are more frequently considered, but other values may be required.

Determination of the metacentric height (M). If a floating body is in equilibrium with its axis inclined, the couple (designated Fa) causing it to heel must be equal to the righting couple Bc.

Since $c = M \sin \theta$, θ being the angle of heel,

$$Fa = B \times M \sin \theta; \text{ and } M = \frac{Fa}{B \sin \theta}.$$

For a vessel already constructed, M may be computed from the inclination caused by shifting loads (included in the weight W).

Example. A weight of 100 tons (included in the displacement) is moved transversely 10 feet on a vessel of 10,000 tons displacement, causing it to heel through an angle of 10 minutes of arc. Compute the transverse metacentric height. $F = 100$; $a = 10$; $B = 10{,}000$.

Therefore $M = \dfrac{100 \times 10}{10000 \times \sin 10'} = 34.5$ feet.

80. The proper metacentric height (M_0). The limiting minimum value of the metacentric height which occurs when the axis of symmetry is vertical is called the proper metacentric height.

81. Stability shown by the position of the metacenter. Reference to figures 24 *a* and 24 *b* will show that, with the axis of symmetry inclined, if the metacenter (m) is above G, Bc will be a righting couple; if m is below G, Bc will be an upsetting couple.

If, when the axis of symmetry is vertical, m is below the center of gravity while the body may be in equilibrium, a slight push may produce complete upsetting.

82. Application of principles of flotation. Many complicated problems arise which require the application of the principles just considered, which are for the most part connected with the work of naval architects. The question of the stability of booms, rafts, dredges, gas holders, and similar structures of simple design belong, however, to the work of other engineers. The following example will illustrate the computations for determining the stability of flotation.

Example

83. Consider the transverse stability of a scow 20 feet wide, 12 feet deep, and 50 feet long, symmetrically constructed and loaded, having an average weight including all loads of 41.6 pounds per cubic foot, and floating in still water.

Let $ABCE$ in figures 25 *a* and 25 *b* represent the cross section of the scow: (*a*) with axis of symmetry vertical; (*b*) with this axis tilted transversely.

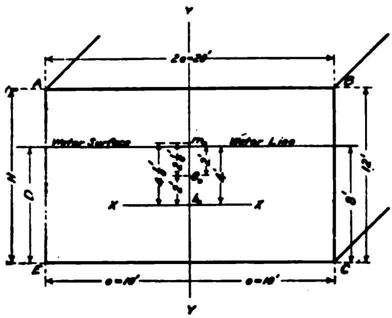


FIG. 25 *a*.

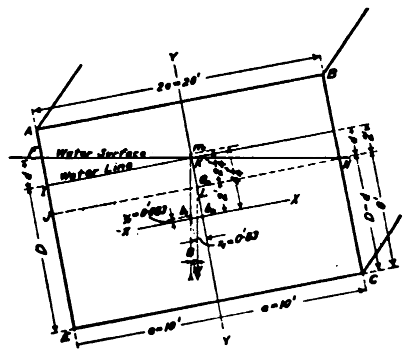


FIG. 25 *b*.

Let D = depth of flotation in feet when axis of symmetry is vertical; draught.

d = the distance in feet that the water line is raised above the water surface in tilting the vessel, measured on the face.

θ = angle of heel corresponding to d.

$2e$ = breadth of the vessel.

H = height of the vessel.

Stability with the axis of symmetry vertical. The center of gravity is seen to be on the axis of symmetry $Y-Y$, midway between the top and bottom of the scow; or 2 feet below the water surface; this is the original center of gravity, G_0. See figure 25 *a*.

Since $\quad 12 \times 20 \times 41.6 \times 50 = D \times 20 \times 62.4 \times 50$
$\quad = \text{weight of water displaced};$

Therefore, $$D = \frac{41.6}{62.4} \times 12 = 8 \text{ feet.}$$

The center of buoyancy is on the axis of symmetry 4 feet below the water surface; this is the original center of buoyancy, b_0.

Since the sum of the horizontal components and the sum of the vertical components of all forces are both zero (§§ 69 and 70), and the center of gravity and the center of buoyancy are in the same vertical line, the scow is in a position of complete equilibrium.

Stability with the axis of symmetry inclined. Consider the effect of tilting the scow as shown in figure 25 *b*.

If **the loading remains constant**, the amount of water displaced must be constant for any angle of heel. Therefore, **the volume of the wedge of water displaced on one side of the axis of symmetry equals the volume of the wedge released on the opposite side.**

In this position, the sum of the horizontal components and the sum of the vertical components of all forces are both zero (§§ 69 and 70).

It is necessary to find the relative positions of new center of buoyancy b_1 and the original center of gravity G_0, and thus to determine whether the body is stable against overturning. This may be done by any convenient method with reference to any set of axes.

Let the axis of symmetry be the $Y\text{-}Y$ axis, and a perpendicular to $Y\text{-}Y$ through b_0 be the $X\text{-}X$ axis.

To find the center of buoyancy. The center of buoyancy (b_1) in the tilted position shown coincides with the center of gravity of the area $CEFH$, which is made up of four figures, rectangle $CEJH$, triangles HKL and FKI, and rectangle $LJIK$.

Let x_1 and y_1 be coördinates of b_1.

Taking moments about the axis $Y\text{-}Y$,

$$2\,Dex_1 = ((D-d)2\,e\times 0) + \left(\frac{de}{2}\times\frac{e}{3}\right) - \left(\frac{de}{2}\times\frac{2\,e}{3}\right) - \left(de\times\frac{e}{2}\right);$$

$$x_1 = -\frac{2\,de^2}{6\,De} = \frac{de}{3\,D}, \text{ measured to the left from } Y\text{-}Y; \tag{1}$$

Taking moments about the axis $X\text{-}X$,

$$2\,Dey_1 = -\left[(D-d)2\,e\times\frac{d}{2}\right] + \frac{de}{2}\left(\frac{D}{2}+\frac{d}{3}\right) + \frac{de}{2}\left(\frac{D}{2}-\frac{2\,d}{3}\right) + de\left(\frac{D}{2}-\frac{d}{2}\right);$$

$$y_1 = +\frac{d^2e}{6\,De} = \frac{d^2}{6\,D}, \text{ measured upward from } X\text{-}X. \tag{2}$$

To find the angle of heel (θ_1) for this position:

$$\tan\theta_1 = \frac{d}{e};\quad \cot\theta_1 = \frac{e}{d}. \tag{3}$$

To find the location of the metacenter (m_1) for this position: by definition, m_1 is on the axis $Y-Y$ and distant from b_0, b_0m_1;

$$b_0m_1 = x_1 \cot \theta_1 + y_1 = \frac{de}{3D} \times \frac{e}{d} + \frac{d^2}{6D} = \frac{e^2}{3D} + \frac{d^2}{6D}. \qquad (4)$$

To find the metacentric height (M_1) for this position: by definition the metacentric height is the distance

$$M_1 = G_0m_1 = b_0m_1 - b_0G_0. \qquad (5)$$

To find the proper metacentric height (M_0). The equation (4) for finding the position of the metacenter $b_0m_1 = \frac{e^2}{3D} + \frac{d^2}{6D}$ is a general one for a vessel of this shape, provided the top is not immersed; when $d = 0$, $b_0m_0 = \frac{e^2}{3D}$; and

$$M_0 = G_0m_0 = \frac{e^2}{3D} - b_0G_0. \qquad (6)$$

The locus of the center of buoyancy. As the vessel is heeled through a series of inclined positions, the path of the center of buoyancy is a parabola, of which $\frac{e^2}{3D}$ is a subnormal, and therefore a constant.

The magnitude of the couple *Bc*.

$$c = M_1 \sin \theta_1 = (b_0m_1 - b_0G_0) \sin \theta_1;$$

and

$$Bc = W(b_0m_1 - b_0G_0) \sin \theta. \qquad (7)$$

The scow is in equilibrium. Because m_1 in the tilted position of the scow is clearly in such a location that Bc is a righting couple, and because the horizontal and vertical forces are in equilibrium, the scow is in complete equilibrium. Successive positions may be investigated in a similar manner.

Numerical results.

From (1), $x_1 = \frac{de}{3D} = \frac{2 \times 10}{3 \times 8} = 0.83$ feet.

From (2), $y_1 = \frac{d^2}{6D} = \frac{2^2}{6 \times 8} = 0.083$ feet.

From (3), $\tan \theta_1 = \frac{d}{e} = .2$; $\cot \theta_1 = 5.0$; $\sin \theta_1 = .196$.

From (4) and (5), $M_1 = \frac{e^2}{3D} + \frac{d^2}{6D} - b_0G_0 = \frac{10^2}{3 \times 8} + \frac{2^2}{6 \times 8} - 2 =$ 2.25 feet.

From (4) and (6), $M_0 = \frac{e^2}{3D} - b_0 G_0 = \frac{10^2}{3 \times 8} - 2 = 2.17$ feet.

From (7), $Bc = 20 \times 12 \times 50 \times 41.6 \times 2.25 \times .196 = 220{,}150$ foot pounds.

In order to hold the scow in the tilted position, a couple $F_1 a_1 = 220{,}150$ foot pounds must be exerted without altering the displacement.

For example, a load $F_1 = 22{,}015$ pounds already on board on the end of derrick boom, if swung athwart the scow a distance $a_1 = 10$ feet, would just tilt the scow into the position shown.

84. **The formulas** (1) to (7), derived to compute x, y, θ, M, and Bc with reference to a transverse plane, **are general formulas for any rectangular outline** with reference to any plane, provided the deck is not submerged.

Other forms of solids may be computed by similar methods, although in some cases more intricate. Every case resolves itself into finding the position of the actual center of buoyancy relative to the original center of gravity (chosen for convenience) by the ordinary methods of mechanics. Moving a weight already on board changes the actual center of gravity, and if the new position of the center of gravity were determined it would be found, when equilibrium is established, to be on a vertical through the corresponding position of the actual center of buoyancy.

85. Formulas for regular solids.

Certain regular solids frequently used in engineering practice are shown below, together with the formulas for computing x, y, θ, $b_0 m$, and $b_0 m_0$.

d = the distance by which the water line is raised out of water.

D = the draught when the axis of symmetry is vertical.

r_0 = outside radius, and r_1 = inside radius of a cylinder or sphere.

b_0 = center of buoyancy when axis of symmetry is vertical.

G_0 = center of gravity when axis of symmetry is vertical.

b = center of buoyancy when axis of symmetry is in any inclined position.

m = metacenter when axis of symmetry is in any inclined position.

A homogeneous sphere; or a homogeneous cylinder with its axis of symmetry horizontal. For the position shown in figure 26,

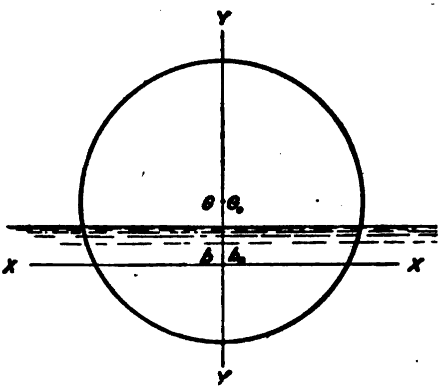


FIG. 26.

$x=0$; $y=0$; $\theta=0$; $b_0m = b_0G_0 = 0$; and $M=0$.

Such solids are therefore stable against transverse rotation in any position of tilting. For a sphere the conditions of stability in any plane are identical with a cylinder in a transverse plane.

The longitudinal stability of a cylinder may be determined by the application of principles already explained, perhaps most easily by cutting it into vertical sections parallel to its axis and using an approximate method; the elementary cross sections in this case may be considered rectangular and the elementary solids may be considered parallelopipeds.

A cylindrical, flat-bottomed vessel floating on end See figure 27.

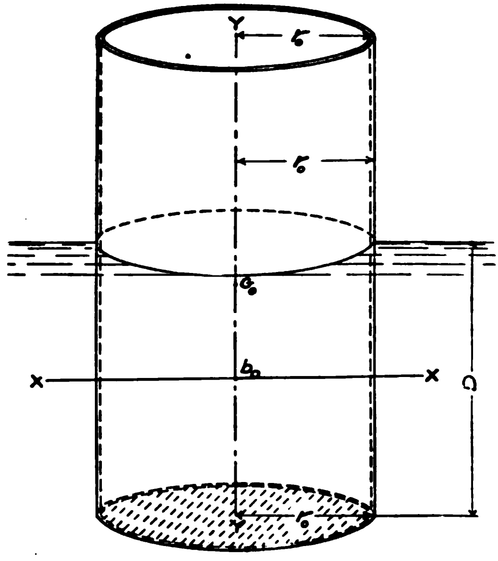


FIG. 27.

$$x = \frac{dr_0}{4D};$$

$$y = \frac{d^2}{8r_0};$$

$$\tan\theta = \frac{d}{r_0};$$

$$b_0m = \frac{r_0^2}{4D} + \frac{d^2}{8D};$$

$$b_0m_0 = \frac{r_0^2}{4D}.$$

A cylindrical caisson; open at top and bottom. See figure 28.

$$x = \frac{d}{4\,r_0 D}(r_0^2 + r_1^2)\,;$$

$$y = \frac{d^2}{8\,D r_0}(r_0^2 + r_1^2)\,;$$

$$\tan\theta = \frac{d}{r_0}\,;$$

$$b_0 m = \frac{1}{4\,D}(r_0^2 + r_1^2)\left(1 + \frac{d^2}{r_0}\right);$$

$$b_0 m_0 = \frac{1}{4\,D}(r_0 + r_1^2).$$

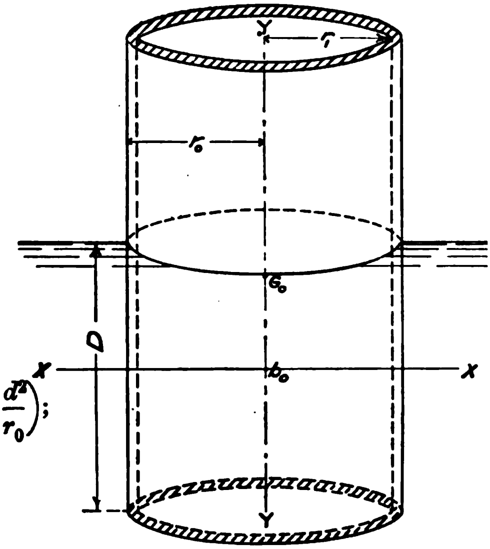


FIG. 28.

A cylinder with closed top and no bottom, with thin walls. In a cylinder with closed top and no bottom, disregarding the volume of walls, the center of buoyancy (b) always maintains the same position, viz. on the axis, midway between the two water surfaces inside and outside; and unless the center of gravity is below the center of buoyancy, the cylinder will not stand vertical unless supported. For example, a gas holder. See figure 29.

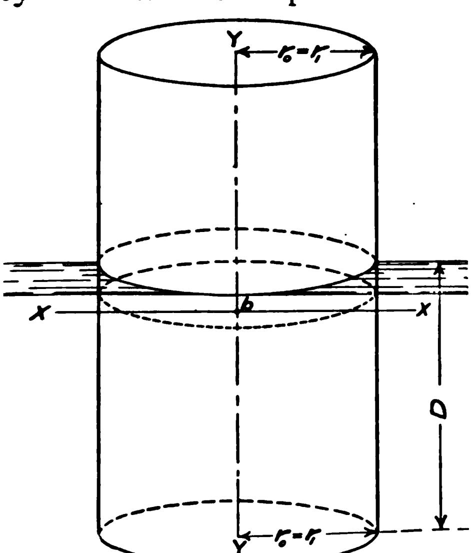


FIG. 29.

86. Approximate method. To determine the stability of a complicated shape, plot to scale the successive cross

CHAPTER V

FUNDAMENTAL PRINCIPLES OF HYDROMECHANICS

87. The fundamental laws of mechanics form the basis of hydraulic formulas. Such of the laws of mechanics * as are fundamental to hydraulics, briefly stated in the following paragraphs, will in their proper connection, when necessary, be further elaborated.

88. The velocity of falling bodies. The velocity (V) acquired in falling from rest through a height (h) under the action of gravity alone,

$$V = \sqrt{2gh} = 8.02\sqrt{h} \text{ (feet per second).}$$

(See Table LXI.)

If the body has an initial vertical downward or upward velocity (V_0) feet per second,

$$V = \sqrt{2gh} \pm V_0 \text{ (feet per second).}$$

89. The space traversed by falling bodies. A body falling from rest under the action of gravity alone will in t seconds fall a vertical distance,

$$h = \tfrac{1}{2} gt^2 \text{ (feet).}$$

A body moving with an initial vertical downward or upward velocity (V_0) will in t seconds travel a vertical distance,

$$h = \tfrac{1}{2} gt^2 \pm V_0 t \text{ (feet).}$$

90. The path of a body when the initial velocity is not vertical. If the initial velocity V_0 is not vertical, but has a direction $\theta°$, the vertical distance traversed,

$$h = \tfrac{1}{2} gt^2 \pm V_0 t \sin\theta \text{ (feet);}$$

and the horizontal distance,

$$x = V_0 t \cos\theta \text{ (feet).}$$

* It is assumed that the student is familiar with the laws of mechanics; the formulas are here stated merely for convenience of reference.

7. A ship of 5000 tons displacement is upright and symmetrical when a weight $P = 40$ tons is amidships. Moving P across the deck to one side a distance of 10 feet caused the bob of a pendulum 15 feet long to move through 10 inches. What is the metacentric height?

8. A dredge scow, with its bucket extended and empty, floats on an even keel. When the bucket has a load of 5 tons at a distance of 25 feet from center line of scow, a pendulum 12 feet long swings 4 inches. Find the metacentric height. Weight of scow and load = 1,000 tons.

9. Given a scow 36 feet wide, 15 feet high, and 60 feet long. The draught is 9 feet. If its water line is tilted 6 feet above the water surface, find for this position:—

(*a*) Location of its center of buoyancy.
(*b*) Metacentric height.
(*c*) Righting couple.

96. Energy of pressure; potential energy. Since a change in the intensity of pressure from p pounds per square foot to 0 is capable of producing in a fluid a velocity, $V=\sqrt{2g\frac{p}{\gamma}}$, in this case $\gamma = w$, then the potential energy of pressure,

$$E = W\frac{p}{w} \text{ (foot pounds).}$$

97. The force required to produce acceleration or retardation. Acceleration or retardation is the rate of change of velocity. A constant force, F pounds, which, acting on a body weighing W pounds, will cause it to change its velocity dv feet per second in an interval of time dt, may be found as follows:

$$F=\frac{W}{g}\frac{dv}{dt}=\frac{W}{g}\left(\frac{V_1-V_0}{t}\right); \text{ if } t = \text{one second, } F=\frac{W}{g}(V_1-V_0).$$

V_0 = the component of the original velocity parallel to the line of action of F.

V_1 = the component of the final velocity parallel to the line of action of F.

98. Centrifugal force. The force required to restrain a body of weight, W, from moving out of a circular path of radius, ρ, is called the centrifugal force, F_c.

The centrifugal retardation, $\frac{dv}{dt}$, is equal to $\frac{V^2}{\rho}$. Therefore

$$F_c=\frac{WV^2}{g\rho} \text{ (foot pounds).}$$

99. The principle of work. A body can be transferred into a new position, or changed in form or size, or undergo a change in velocity, only by overcoming resistances which oppose the change. This process is called doing work. The amount of work done is equal to the energy expended, irrespective of whether it is usefully applied, and is measured by the total resistance; it is equal to the force exerted, F pounds, multiplied by the space, S feet, through which resistance is overcome. This may be expressed:

$$\text{Work done} = \text{energy exerted} = FS \text{ (foot pounds).}$$

The change of velocity from V_0 to V_1 feet per second, or equivalent changes in elevation or pressure, in a distance S, requires a

force F, or an expenditure of energy of FS foot pounds, equal to the change in energy of the weight W. Therefore,

$$FS = W\frac{(V_1^2 - V_0^2)}{2g} = W(h_1 - h_0) = W\left(\frac{p_1 - p_0}{\gamma}\right).$$

100. Application of the principle of work to falling water. If the initial velocity, V_0, $= 0$, and $V_1 =$ the final velocity, $S = h$, the distance through which water falls, and $F =$ the force of gravity $= W$; then

$$FS = Wh = \frac{WV_1^2}{2g}.$$

Even though the time required to fall through the distance, or to change the velocity or pressure, may require several seconds, if W pounds of water are changed every second in elevation, velocity, or pressure, the formula represents the work done every second.

101. Horse Power. The rate of expenditure of energy is usually stated primarily in foot pounds per second, or in horse power (HP); one horse power equals 550 foot pounds per second.

Therefore, $$\frac{FS}{550} = HP.$$

102. Conservation of energy. When there are no frictional resistances, the energy expended in doing work is not lost, but merely transformed; energy exerted in a body changing its motion or position is represented by an exact equivalent amount of energy in some form in the body itself. Hence, if the body does not receive or part with energy, the total energy is a constant, expressed as follows:

Total energy = kinetic energy + potential energy = constant.

This law appears in hydraulics as Torricelli's and Bernoulli's theorems.

103. Torricelli's theorem. In 1643, Torricelli enunciated the theorem that "*the velocity of a fluid passing through an orifice in the side of a reservoir, is the same as that which is acquired by a body falling freely in vacuo from a vertical height, measured from the surface of the fluid in the reservoir to the center of the orifice.*"

104. Bernoulli's theorem. In 1738, Bernoulli showed that the law of the conservation of energy is applicable to the flow of fluids. Bernoulli's theorem may be stated thus:

At every section of a continuous and steady stream of frictionless fluid, the total energy is constant; that whatever energy is lost as pressure is gained as velocity. Therefore, in terms of head,

Total energy = velocity head + pressure head + head due to position = constant: or, in every stream section,

$$H_T = h_v + h_p + h_e = \frac{V^2}{2g} + \frac{p}{\gamma} + h_e.$$

H_T is the total head.

h_e is the vertical distance from any reference plane to the point at which p is measured.

Note that p may be more or less than atmospheric pressure.

Consider a stream of frictionless fluid having a steady volume of flow of W pounds per second. Let A and B represent any two consecutive points in the stream.

At A the velocity is V_1 feet per second; the pressure head, h_{p_1} feet; and the elevation of A, h_{e_1} feet. At B, V_2 is the velocity, h_{p_2} is the pressure head, and h_{e_2} is the elevation.

At section A the total energy $= W\left(\frac{V_1^2}{2g} + h_{p_1} + h_{e_1}\right) = WH_T$;

At section B the total energy $= W\left(\frac{V_2^2}{2g} + h_{p_2} + h_{e_2}\right) = WH_T.$

Therefore, $$\frac{V_1^2}{2g} + h_{p_1} + h_{e_1} = \frac{V_2^2}{2g} + h_{p_2} + h_{e_2} = H_T.$$

This formula, properly modified to include the effect of frictional resistances, is the basis of all empirical formulas for the flow of water.

105. Frictional resistances modifying theoretic formulas. To the movement of water are opposed frictional resistances, so called, of which neither the character nor the laws governing their action have been precisely defined. Many experiments, some highly trustworthy, others less reliable, have been made for the purpose of determining the laws governing fluid friction. So far, these investigations have made available a large mass of invaluable information, but have produced laws of only limited

applicability. From these researches, however, hydraulic coefficients covering a wide range of conditions have been derived. Methods of reducing these resistances have also been learned, as well from practice as from experimental research. Theoretic formulas expressing the fundamental principles of hydromechanics are the basis of practical formulas, serving not only to determine the character of problems, but also the limiting value of results. Practical formulas are, therefore, theoretic formulas modified by numerical coefficients derived from practice and research, — or, in other words, empirical formulas.

Problems

1. The water tank of a locomotive has its intake 9 feet above the water in the feed trough placed in the middle of the track. What is the least speed in miles per hour which will project water through a scoop into the tank (*a*) neglecting frictional resistances; (*b*) assuming that frictional resistances absorb enough energy to raise water 7 feet?

2. Compute for heads of 15, 40, and 150 feet (*a*) the equivalent intensity of pressure in pounds per square inch; (*b*) in inches of mercury; (*c*) the velocity due to the head; (*d*) the discharge in cubic feet per second through an opening having an area of 1 square inch; (*e*) also the equivalent discharge in gallons per minute.

3. Compute the head due to velocity for velocities of 2, 6, 8, and 20 feet per second.

4. Show by a sketch and assumed values of your own the application of Bernoulli's theorem to the flow of a frictionless fluid in a closed channel (pipe under pressure).

5. A jet of water flows from a nozzle with a velocity of 30 feet per second, at an angle of 50 degrees. Assuming no air resistance, determine its path.

6. If 100 pounds of water per second discharge from a nozzle at a velocity of 40 feet per second, what gross horse power is available?

7. If 2000 pounds of water per second are reduced in pressure in flowing through a turbine of from 40 pounds per square inch above atmospheric to 10 pounds per square inch below atmospheric pressure, compute the energy expended and the gross horse power.

8. A mass of water weighing 100 pounds is flowing with a velocity of 20 feet per second through a curved channel bent to a radius of 30 feet. Compute the force in pounds exerted on the water to keep it in its curved path.

9. What force must be exerted to change the velocity of 1000 pounds of water from 20 feet per second to 5 feet per second in one second?

10. A stream of water, 3000 cubic feet per second, falling through 40 feet, will make available how much gross horse power?

CHAPTER VI

THE FLOW OF WATER — METHODS OF MEASUREMENT

106. Flow of water. Flow means the general translatory movement of a body of water, and is caused by inequalities of elevation or pressure in its different parts.

107. A stream of water is a body of water spouting through the air or flowing through a channel cut out of or made up of solid materials.

108. A channel is the bed of a stream, that is, any natural or artificial receptacle containing a flowing body of water.

109. Free discharge. A stream if surrounded on all sides by gas or vapor is said to have free discharge, or free deviation; and the stream is usually called a jet. The discharge from an orifice into air or the open end of a pipe gives the condition of free discharge, or free deviation.

110. Submerged discharge. If a stream which is flowing into another body of water has its entire cross section below the water surface of the latter, the flow is said to be wholly submerged; if only part of its cross section is below the water surface, the flow is partly submerged.

111. Flow in open channels. If the upper surface of the flowing water is free (exposed to atmospheric pressure only) and the other surfaces are in contact with a solid which forms the stream bed, the intensity of pressure at every point in the stream will be measured by the vertical distance of the point below the free water surface, and the condition of flow in an open channel exists.

112. Flow in closed channels, or flow under pressure. If the flowing water has no free surface, and if the intensity of pressure

at every point in the stream is in excess of that produced by the mere depth of water, the flow is under pressure, and the condition of flow in a closed channel exists (see A to E, figure 31).

113. **The volume of flow; discharge.** The volume of flow, commonly called the discharge, is the quantity of water passing through any stream cross section in a unit of time, usually a second. The discharge equals the cross-sectional area of the stream multiplied by the mean velocity of the water in the section.

A cubic foot per second, often called a second foot, is the standard unit of measurement of stream flow in American practice; other units are, however, frequently used (see Chapter I).

Let Q = the discharge in cubic feet per second.

A = the cross-sectional area of the stream in square feet.

V = mean velocity of the water in feet per second in the section under consideration.

Then $$Q = AV. \tag{1}$$

From (1) $$V = \frac{Q}{A}. \tag{2}$$

And also from (1) $$A = \frac{Q}{V}. \tag{3}$$

The relations expressed by formulas (1), (2), and (3) hold for any system of units.

114. **The cross-sectional area.** The cross-sectional area (A) is the actual area of any stream cross section taken at right angles to the axis of the stream, or to the general translatory direction of flow at the point where the discharge is to be measured.

The area should be computed by the ordinary rules of mensuration from dimensions determined by direct calibration.

Distinction must be made between the cross-sectional area of a channel itself, and the cross-sectional area of the stream flowing in the channel; the latter is the cross-sectional area (A) required, and may be less than the former.

115. **Mean velocity of flow.** In any stream cross section, the velocity of flow will be different at different points. In a smooth pipe flowing full, for instance, the velocity is greatest at or near

the center, and least along the sides. The mean translator velocity (V), parallel to the axis of the channel, is the velocit required in determining the volume of flow.

The velocities at different points in the cross section of a strea may be determined directly or indirectly by suitable instruments and from these velocities the mean velocity may be compute Or the mean velocity may be directly deduced from the laws motion modified by coefficients determined by experiment.

116. Steady flow. For any period of time during which t area of a stream cross section, the pressure, and the direction a the mean velocity of flow in that section are all constant, the fl is said to be steady; that is, the discharge is constant for succe sive equal intervals of time. If the discharge varies with th time, it is unsteady. Absolutely steady flow is never obtained it is merely more or less closely approximated. Conditions o steady flow, although often lasting but a short time, are usuall assumed in computations of discharge. If all the condition shown in figure 31 are constant, the flow is steady.

117. Uniform flow. For all successive cross sections of a strea in which the mean velocities of flow are equal, the flow is said t be uniform, that is, the discharge is constant and the successiv cross-sectional areas are equal. See B to C, and D to E, in figu 31. If the velocities or the areas change from place to place, eve though the discharge is steady from time to time, the flow is sai to be variable. The channel shown on figure 31 represents as whole a case of variable flow.

The distinction between uniform and variable flow is importan since the resistances to uniform flow are much less than the resis ances to variable flow; and coefficients for uniform flow, unle properly corrected, should not be applied to cases of variab flow.

118. Principle of continuity of flow. If the flow of a strea is steady, whether uniform or variable, the area of any strea cross section multiplied by the mean velocity of flow in th section will equal the discharge. In figure 31 let A_1, A_2, A_3, etc be successive areas, and V_1, V_2, V_3, etc., corresponding velocitie

Then $$Q = A_1 V_1; \; = A_2 V_2; \; = A_3 V_3; \text{ etc.} \tag{4}$$

$$V_1 = \frac{Q}{A_1}; \; V_2 = \frac{Q}{A_2}; \; V_3 = \frac{Q}{A_3}; \text{ etc.} \tag{5}$$

$$A_1 = \frac{Q}{V_1}; \; A_2 = \frac{Q}{V_2}; \; A_3 = \frac{Q}{V_3}; \text{ etc.} \tag{6}$$

119. The wetted perimeter. The wetted perimeter (*w.p.*) is that part of the boundary line of the cross section of a channel which is in contact with the flowing water. In a river, the wetted perimeter is the length of the line outlining the bottom and sides; in a circular pipe of diameter (D) flowing full, it is the circumference of a circle (πD); if flowing half full, it is the half circumference $\frac{\pi D}{2}$.

120. The mean hydraulic radius. The mean hydraulic radius (R) is the cross-sectional area (A) of a stream divided by its wetted perimeter; therefore,

$$\text{The mean hydraulic radius, } R = \frac{A\text{, the cross-sectional area}}{w.p.\text{, the wetted perimeter}}.$$

121. The head lost in flow. The flow of water sets up resistances, in the overcoming of which energy is irrecoverably expended. For a given stream the sum total of energy thus expended increases with the distance traveled, as both theory and ample experience prove. The difference in the sum totals of energy between two stream sections is, therefore, the energy irrecoverably expended, or "lost," by the stream between these two places. If the flow is steady, this loss of energy may be expressed by the difference between the sum totals of the heads due to velocity, pressure, and elevation existing at any instant in two successive sections; this difference in heads is called **the lost head**.

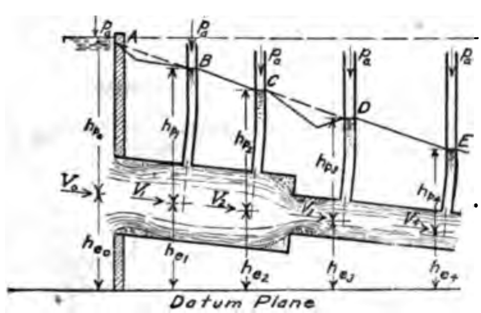


FIG. 31.

Lost head will in general be designated h_λ.

Consider the following case of steady flow of a stream through a series of variously formed channels. See figure 31.

Let

$V_0, V_1, \cdots =$ mean velocities of flow at successive sections.

$h_{p_0}, h_{p_1}, \cdots =$ pressure head at successive sections, $= \frac{p_0}{\gamma}, \frac{p_1}{\gamma}, \ldots$.

$h_{e_0}, h_{e_1}, \cdots =$ elevations of center of channel above datum plane at successive sections.

$p_a =$ atmospheric pressure, and the head due to $p_a = \frac{p_a}{\gamma}$.

$Q =$ a steady volume of flow.

$W = wQ =$ weight of volume of flow.

Consider the head lost between sections A and B which may represent any two consecutive sections.

By Bernoulli's theorem,

$$wQ\left(\frac{V_0^2}{2g} + h_{p_0} + h_{e_0} + \frac{p_a}{\gamma}\right) = wQ\left(\frac{V_1^2}{2g} + h_{p_1} + h_{e_1} + h_\lambda + \frac{p_a}{\gamma}\right). \tag{7}$$

Therefore, the head lost between A and B

$$h_\lambda = \frac{V_0^2 - V_1^2}{2g} + h_{p_0} + h_{e_0} - h_{p_1} - h_{e_1}. \tag{8}$$

122. The hydraulic grade line. If, on the profile of a channel, vertical lines be drawn to represent graphically the pressure head at every section, a line drawn to connect the upper ends of these vertical lines is called the hydraulic grade line. The hydraulic grade line of a river, or other open channel, is a longitudinal profile of the water surface.

The line $ABCDE$ in figure 31 is the hydraulic grade line of the channel shown, during flow; if the flow is entirely stopped by an obstruction, the horizontal line through A becomes the hydraulic grade line. The hydraulic grade line as usually shown represents only heads due to elevation and intensity of pressure; to show the total head at each section it should include also the velocity head.

123. The slope of the hydraulic grade line. Since the total loss of head increases with the length of a channel, the hydraulic grade line which represents graphically the pressure head at every sec-

tion will have a general downward inclination in the direction of flow.

For uniform flow the grade line has a constant slope which may be measured by the sine (S) of the angle of inclination.

$$S = \frac{h_\lambda}{L}. \tag{9}$$

h_λ = head lost in a length of channel L.

For variable flow the grade line will be broken and the inclination varies from section to section; but at places where the velocity is suddenly increased (see C to D, figure 31) the grade line will take a sudden drop and then a sudden rise. This sudden drop is not all a loss of energy, but means that momentarily pressure is being converted into velocity; and the rise means that by diminution of velocity the pressure is being restored.

124. Conditions affecting the resistance to flow. As the proportion of the total head to be used in overcoming resistance, producing velocity, or remaining in the stream depends upon the velocity of flow and the resistances to flow, it is important to know what factors affect resistances to flow.

Resistance is independent of the pressure exerted by the water on the wetted perimeter.

Resistance is approximately proportional to the total area of rubbing surface; that is, the length of the channel multiplied by the wetted perimeter.

Resistance increases with increased roughness of channel; as some power of the mean velocity, 1 or higher; with an increase in the amount of suspended matter in the water; with abrupt changes in the cross-sectional area of the channel; at bends; or at junctions with other channels.

Resistance decreases with a rise in temperature.

125. Irregular motion of flowing water. In flowing water, the surface in open channels, and the pressure in closed channels, oscillates; perhaps more at the sides than near the center. Except under very low velocities and favorable conditions, the particles of water do not move in parallel stream lines, but cross and recross each other in many directions; making the stream motion a combination of translatory and eddy motion. The eddy motion

is greatest along the perimeter, and increases with increased roughness of the perimeter. Very precise observations, following at very close intervals of time, should therefore show fluctuations in velocity, without appreciable changes in the discharge. These fluctuations, or *pulsations*, sometimes appear in gaugings. If slight, they are not considered, since their effect is practically eliminated in the period of time required to make one measurement, and they are not usually detectable by the methods of measurement ordinarily used; but very turbulent water will interfere with accurate measurement of discharge. The effect of turbulent motion is shown in increased resistance to flow.

126. The critical velocity. Turbulent eddying motion exists in nearly all cases in practical hydraulic problems, and the resistance to flow varies in proportion to some power of the mean velocity between 1.7 and 2.0 or more. Certain investigations, however, have shown that at very low velocities the motion of the water is in parallel stream lines, that is, without the disturbance due to eddying motion; and the resistance to flow varies nearly directly as the mean velocity of flow. The velocity at which turbulent eddying motion begins or ceases is called the critical velocity.

Reynolds * made experiments to determine the point of critical velocity, and found that there were two critical values for any pipe or tube; "one at which steady motion changed into eddies, the other at which eddies changed into steady motion." The former change was found to occur at velocities considerably higher than the latter; and the two critical points are, therefore, called "*the higher critical velocity*" and "*the lower critical velocity.*"

The higher critical velocity. Reynolds found, by experiments with streams of colored water in straight glass tubes, that when the velocity was increased by small increments, the velocity at which stream line, or non-sinuous flow, changed into eddy motion may be determined by the following equations:

$$v_c = \frac{1}{43.79}\frac{P}{D} \text{ (metres per second); or}$$

$$v_c = .2458\frac{P}{D} \text{ (feet per second).} \qquad (10)$$

* Osborne Reynolds, *Phil. Trans. of the Roy. Soc.*, 1883, pp. 935 et seq.

D = the diameter of the pipe in metres, or feet;

$P = (1 + .0336\ T + .000221\ T^2)^{-1}$ is the temperature correction;

T = temperature of the water, degrees Centigrade.

The lower critical velocity. To determine the point at which eddies ceased, when the velocity is decreased instead of increased, Reynolds studied the relation between the loss of head and the mean velocity of flow in straight pipes. The results were shown graphically by plotting the logarithms of the loss of head against the logarithms of the mean velocity of flow, a method which was devised by Reynolds. Curves so constructed are called "*logarithmic homologues.*" (See Table XLIX for a more detailed explanation.)

Every homologue will (if the experiments plotted include sufficiently low velocities) have two branches, both straight lines; the lower having a slope of 1 to 1; the upper having a slope of n to 1 (n being greater than 1). The intersection of these two branches may, for practical purposes, be assumed to be the point of critical velocity; although there is, in fact, a short portion of the curve near the intersection where the curve does not follow a clearly definable law. From his own experiments Reynolds thus determined a formula for the lower critical velocity, and verified his results by plotting in a similar manner Poiseuille's and Darcy's experiments. Reynolds's formula for the lower critical velocity is:

$$v_c = \frac{1}{278}\frac{P}{D}\ \text{(metres)};\quad \text{or}\quad v_c = 0.0387\,\frac{P}{D}\ \text{(feet)}. \qquad (11)$$

Experiments by Barnes and Coker* show values for the higher critical velocity fully double those of Reynolds, and for the lower critical velocity as little as half as much as Reynolds.

All these experiments showed that disturbances in the supply tank, or jarring of the pipes, made a marked change in the point of critical velocity. For practical conditions the point of critical velocity can not be very precisely determined; and except for small pipes is usually too low to be considered.

* *Proc. of the Roy Soc.*, Vol. 74, pp. 341 to 356.

The resistance to flow for velocities under the critical velocity for capillary tubes and small pipes may be approximately computed by the following formula.

Hazen's formula * is:

$$V = cSD_i^2\left(\frac{t+10}{60}\right). \tag{12}$$

S = the slope of the hydraulic grade line.

V = the mean velocity of flow in feet per second.

D_i = the diameter in inches.

t = the temperature of the water, degrees Fahrenheit.

c = a factor; from Saph & Schoder's experiments on brass pipes Hazen determined c to be from 462 to 584; Williams and Hazen use a value of 475 in their hydraulic tables.†

127. Hazen's formula for the flow of water through sand. The velocity of water when flowing through a bed of sand is usually below the critical velocity. Hazen ‡ proposed as the result of experiment the following formula for the flow of water through closely packed sand with the pores completely filled with water, and in the entire absence of clogging.

$$v = cd^2\frac{h}{l}\left(\frac{t+10}{60}\right). \tag{13}$$

v = the velocity of the water in metres daily (24 hours), in a solid column of the same area as that of the sand.

c = a factor.

d = the effective size of sand grain in millimetres; the size such that 10 per cent by weight of the particles in a given sand are smaller, and 90 per cent larger than itself.

h = the loss of head.

l = the thickness of sand layer through which the water passes.

t = temperature of the water, degrees Fahrenheit.

h and l must be in the same units of measure.

* Allen Hazen, *Trans. Am. Soc. C. E.*, Vol. 51, pp. 316 to 319.

† Williams and Hazen, *Hydraulic Tables*, p. 15.

‡ Allen Hazen, *Filtration of Public Water Supplies*, p. 22.

The value of c may be as high as 1200 for very uniform and perfectly clean sand; and as low as 400 for very closely packed sands containing a good deal of alumina or iron, especially if they are not quite clean.

Example. Compute the volume of flow in millions of gallons daily (24 hours), through a bed of new clean sand one acre in area, 5 feet thick, having an effective size of grain of 0.3 mm. If the loss of head is 0.5 feet, and the temperature 50° F.,

$$v = 1000 \times .3^2 \times \frac{.5}{5.0}\left(\frac{50 + 10}{60}\right) = 9 \text{ meters.}$$

Volume of flow in 24 hours $= 9. \times 3.281 \times 43{,}560 \times 7.48 = 9.63$ million gallons.

METHODS OF MEASUREMENT

128. **Kinds of methods.** The following methods have gradually proved to be the most successful for measuring the discharge of moving water under the conditions which obtain in rivers, canals, pipes, and other carriers.

Measurements by —

Volume.	Orifices.	Current meters.
Weight.	Nozzles.	Slope of the hydraulic grade line.
Pitot tubes.	Weirs.	Water wheels.
Venturi meters.	Floats.	House meters.

The order in which these methods are tabulated merely indicates the order in which they are treated.

Numerous other contrivances for measuring water have been devised, chiefly for the purpose of eliminating certain steps in the process of recording and computing the measurements; but as far as principles are concerned they may all be fairly included in the methods already mentioned.

The only real criterion. Strictly speaking, measurements of water by volume or by weight are the only real standards; because the other methods of measurement depend upon the use of coefficients, directly or indirectly derived by comparison with volumetric or weight measurements.

129. **Volumetric measurement of discharge.** The determination of the rate of flow, by observing the time required to fill a tank

or basin to a known capacity, is not only the most obvious, but at the same time, the most precise method of measuring discharge; and should produce results free from most of the irregularities and errors unavoidable in other methods.

The apparatus required comprises: a water-tight tank or basin of which the volumetric capacity at different elevations of the water surface may be accurately calculated from the inside dimensions, and which is arranged to receive and discharge the water; suitable valves and gauges attached to the channel of flow by which a nearly constant discharge may be maintained during an experiment; some contrivance by which the flow may be quickly diverted into and away from the basin without measurable spilling; gauges for observing the elevations of the water surface; a stop watch; the instruments for calibrating the tank; and the necessary notebooks for record.

Procedure. (1) Measurements of the inside dimensions of the tank or basin, and a computation of its capacity at successive elevations of the water surface, which are usually read from a gauge. (2) Determinations of the accuracy of the settings of the gauges. (3) Arrangement of the valves to obtain the desired rate of flow. (4) Observations of the elevation of the water surface at least at the beginning and end of each experiment. (6) Determinations of the leakage, if any, at different elevations of the water surface. (7) A computation of the rate of flow from the volume deposited in the tank during the time covered by each experiment, that is,

$$Q = \frac{\text{volume in cubic feet}}{\text{time in seconds}}, \text{ cubic feet per second.}$$

Application. For relatively small discharges of a few gallons or a half cubic foot per second or less, tanks may be easily arranged for measurement; and tanks of large dimensions, generally in laboratories, have been constructed for just this purpose, some capable of measuring several cubic feet per second. In several instances, where the volumetric method has been made the standard form of measurement, masonry locks or channels have been used, the existence of which has made possible the extreme accuracy of the weir measurements of Henri Bazin, James B. Francis, and Fteley-Stearns. Water supply reservoirs are often available for

measuring moderate discharges if the reservoir is of regular dimensions, and if the leakage can be measured. Ordinarily, however, the expense and difficulty of constructing large water-tight tanks is so great that the volumetric method is confined to small discharges, and used in calibrating other measuring devices in laboratories. One very common use is for testing water meters to be used to measure domestic consumption.

130. **Measurement of discharge by weight of water.** The determination of the rate of flow by observing the time required to fill a tank set upon weighing scales can be made with the same degree of precision as by the volumetric method; and it is sometimes feasible where the volumetric method can not be used.

The apparatus required comprises: one or more water-tight tanks which, when full, are within the capacity of the weighing scales, usually in duplicate, so that one may be filling while the other is emptying; suitable valves for regulating the flow; a stop watch; a thermometer; and the necessary notebooks for record.

Procedure. (1) Calibration of the weighing scales and other instruments used. (2) Observation of the time required to fill the tanks. (3) Weighing the water. (4) A reduction of the weight to volume, with temperature corrections when necessary. (5) A computation of the discharge from the volume and time.

If $W =$ the total weight during an experiment, $w =$ the weight of a cubic foot of water, and $t =$ time of an experiment in seconds;

Volume in cubic feet $= \frac{W}{w}$, and the discharge $(Q) = \frac{W}{w \times t} =$ cubic feet per second.

Application. This method, like the volumetric method, is applicable to relatively small discharges of a few gallons, rarely more than a cubic foot per second, but is just as precise, certainly in laboratory experiments for standardizing other methods of measurement. This method or the volumetric should be used wherever possible.

131. **Pitot tubes.** The Pitot tube is a bent tube, usually of brass, drawn to a small open end, and placed in the water to measure the head or pressure due to the kinetic energy of flow; for practical use it must be combined with some kind of a pie-

zometer. When properly constructed and calibrated, the Pitot tube may be used to measure the discharge of open or closed channels with a high degree of precision; and this method has made possible measurements of water never attempted before. The subject of water waste prevention in water mains has been studied successfully by means of some form of Pitot tube. See Chapter VII.

132. The Venturi meter. The Venturi meter, the principle of which is based upon Bernoulli's theorem, is an instrument used for determining the rate of flow in pipes under pressure, by observing the difference in pressure at two successive sections of a special casting put into the pipe line, one section being of the same diameter as the pipe and the other of less, the entire apparatus resembling two nozzles with the small ends together. The Venturi meter, if rated accurately, and free from unusual disturbing conditions, should measure water probably with an error not exceeding 3 per cent. This form of meter is used chiefly to measure the discharge in relatively large pipes, where the slight loss of head necessary for purposes of measurement makes this meter the only method of measurement possible. See Chapter VIII.

133. Orifices. Orifices are openings of regular form inserted in the sides or bottoms of channels or basins. They are among the oldest water measuring devices, and in principle are based upon Torricelli's theorem. With the cross-sectional area of the opening known, the measurement is made by obtaining during a suitable period of time the exact difference in elevation between the center of the orifice and the water surface above it; from this difference the velocity may be computed, and from the area and velocity the discharge. The actual cross-sectional area of the jet is very materially modified by the shape of the opening, by the character of the edge presented to the water, and by the head. For these reasons the coefficients of discharge are rarely unity, and vary in wide ranges with changes in the conditions; but have been very well established by a large number of experiments for so-called standard, sharp-edged orifices of small dimensions, one square foot or less in area. With such small orifices and other conditions of measurement good, a very high degree of precision may be ob-

tained; and the error should not be over one or two per cent. High precision requires thorough knowledge of all the experiments on which the coefficients are based and exact application to specific instances. For large openings, the coefficients available can be rarely used with confidence, owing to modifying conditions, the effect of which are not well known. See Chapters IX and X.

134. Nozzles. The nozzle, which is a converging cone attached to a pipe, is one of the simplest forms of measuring device, requiring only a reading of the pressure at its base to determine the discharge; and standard nozzles, like those of John R. Freeman, are usually constructed to give very nearly the theoretical quantity due to the head. They are well adapted to measuring the flow under pressure under a variety of conditions, at a hydrant or blow-off or the end of any pipe; and may be easily transported, and quickly set up. They may be made and calibrated to measure the discharge of pipes of moderate sizes, perhaps 24 inch or less, with a high degree of precision; and are peculiarly well fitted for measuring the discharge of pumps and fire apparatus. See Chapter XI.

135. Weirs. The weir is a vertical rectangular, trapezoidal or V-shaped overfall notch in a dam or bulkhead, used to measure the flow in open channels; and from long usage is perhaps the best known and most thoroughly trusted standard device for measuring water. This very fact makes great precaution and thorough study of conditions in each case the more imperative. The construction of a good weir is relatively simple; but a precise determination of the discharge requires a thorough knowledge of the effect of the velocity of approach, and of the coefficients of discharge. The number and range of experiments, upon which the different coefficients of discharge are based, are so limited, especially as to head, that the conditions of experiment must be reproduced with absolute fidelity to prevent erroneous results. On the other hand, where the conditions for measurement are good, a very high degree of accuracy may be obtained, which should give results well within two per cent of the truth. See Chapter XII.

136. Float measurements. Surface and subsurface floats in various combinations, and running definitely determined distances

in an observed time, may be used to determine the velocity of flow between two or more consecutive cross sections of an open stream. Float measurements are limited to rivers, power and other canals, and open channels in general. The number of published results of float measurements is very limited, and the precision obtainable varies with many factors. See Chapter XIII.

137. Current meters. The current meter is a small water wheel or propeller, revolving with the current, to which is geared some form of revolution counter, by which the number of revolutions of the wheel can be registered, and which may be placed at desired points in the cross section of an open channel. The meter must be calibrated or rated, in order to determine the relation between the number of revolutions of the wheel and the velocity of the water in the same unit of time. The current meter, if properly constructed, kept in rating, and skillfully handled, will measure the velocities of a running stream with a high degree of precision; and it can be used with good results where the cross sections of the stream are too irregular to allow the use of any other instrument. See Chapter XIV.

138. Measurements by cross sections and slopes. A very fair estimate of the discharge in an open or closed channel may be made if successive cross sections are uniform or nearly uniform and the difference in elevation or head between the two sections is known. This method is merely the reverse process of designing channels, and depends upon the selection of coefficients based upon experiments made under conditions which must necessarily differ to some extent from those of any case under consideration; and an estimate which requires many unverified assumptions is necessarily open to criticism. It is very difficult to fix the degree of precision possible in an estimate of this kind, but the estimate may well be more than ten per cent in error, even where the conditions are pretty well known, and yet be near enough to make this method extremely useful if due allowance can be made in advance to cover the probable error. See Chapters XV and XVI.

139. Water wheels. Water wheels of the ordinary reaction type, by means of measured guide and bucket openings, or the nozzles of impulse wheels, when calibrated and checked by occa-

sional measurements by other methods, not only make excellent water meters, but in many water power plants furnish the only regular measure feasible. See Chapter XVIII.

140. House meters. There are many commercial meters for measuring the use of water by water consumers, which depend for their accuracy upon an occasional test or rating by the weight or volumetric methods. These meters are usually of three types: (1) meters fitted with disks or vanes rotating in closed channels; (2) meters of the positive displacement or piston type; (3) meters like the Venturi meters, which depend upon observations of the pressure at two successive sections of different cross-sectional area.

Instruments. In connection with the various methods of measurement mentioned in this chapter, engineering instruments of many kinds, both common and uncommon, are used for making the observations. Such instruments as are especially pertinent to hydraulics will be described in this book in connection with the kind of work on which they are used.

Problems

1. Area of a sand filter is 1½ acres. Depth of sand, 1¼ metres. Effective diameter, 0.2 millimetre. Temperature, 50° F. $c = 800$. Head, 0.41 feet. Compute the flow in millions of gallons per day.

2. Calculate the head necessary to filter 6 million gallons per day through two successive layers of sand each 2 feet thick, the lower having an effective diameter of 0.35 millimetre, the upper of 0.20 millimetre. Temperature, 50° F. $c = 800$. Area = 4 acres.

3. Compute the higher and lower critical velocities for straight smooth pipes of ½, 1, 1½, and 2 inches diameter, with water at 0° C. and 20° C. Compute also the slopes corresponding to these velocities.

CHAPTER VII

THE PITOT TUBE; PIEZOMETERS; THE PITOMETER

141. The Pitot tube. If a tube of glass or smooth metal, op at both ends and bent through 90°, is held in running water wi one leg horizontal, parallel to the stream lines and opposed to t current, and the other leg vertical and open at the top (positi A, figure 32), the impact of the moving water will cause a colu of water to stand in the vertical leg at some height h above t surrounding water surface. This height h depends upon, a should theoretically be proportional to the square of, the veloc (V) at the point O; the height of the water column $Y+h$ is the fore a measure of the pressure head plus the velocity head at t point O.

Experiments* have proved that in tubes having small poi so shaped as not to disturb the flow, with openings of cyl drical or converging or diverging cone form, h very nearly equ $\frac{V^2}{2g}$. Therefore with such forms $V = (2gh)^{\frac{1}{2}}$. If h can be me ured, the velocity at any point may be calculated.

The tube A is called an *impact tube;* tubes B and C and D intended to measure pressure head ("Y") but do not.

In 1780 Pitot † was the first to use such a device to determ the velocity of flow; hence the name. His apparatus consis of a bent cylindrical tube with one leg horizontal and its ori opposed to the current, and a straight vertical cylindrical tu set with its bottom opening at the same level as the orifice of bent tube (tubes A and D, figure 32). h being the difference

* Darcy, *Ann. des Ponts et Chaussées*, 1858, 1st semestre; Bazin, *Ann. Ponts et Chaussées*, 1890, 1st semestre; White, *Journal Asso. of Eng'g. Soci*, 1901; Gregory, *Trans. Am. Soc. M. E.*, December, 1903; Williams, Hubbell, Fenkell; *Trans. Am. Soc. C. E.*, 1902.

† *Mémoires de l'Académie*, November, 1732.

level in the two vertical tubes, Pitot assumed that V was equal to $(2gh)^{\frac{1}{2}}$.

This apparatus did not give a true measure of the velocity head h because the vertical tube is not a true piezometer and did

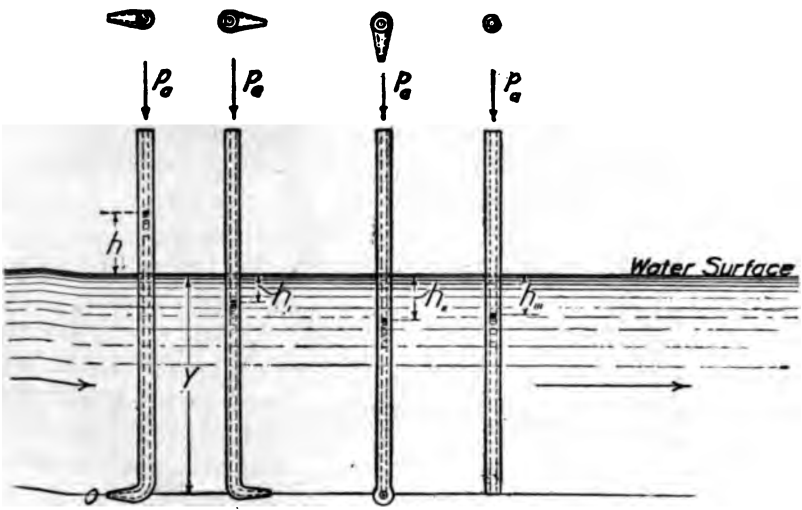


FIG. 32.

not accurately measure the pressure head "Y."* Pitot's tube was in fact little more than a scientific toy until about 1850, when Darcy made practical Pitot tubes which he and Bazin used in their experiments upon the flow of water.

142. Darcy's experiments. Darcy determined by experiment that the chief three difficulties in the way of the practical use of Pitot's tube were † as follows:

(1) The oscillations of the water surfaces in the tubes and the disturbances in the stream due to the tubes make an accurate measurement of the head "h" difficult. If the velocity is feeble, any differences in elevation may entirely disappear.

To reduce oscillations and disturbances as much as possible, Darcy made the orifices much smaller than the tubes, the diameter of the orifices being .0015 metre and of the tubes .01 metre.

* See Darcy, *Ann. des Ponts et Chaussées*, 1858, 1st semestre, p. 353.

† *Ann. des Ponts et Chaussées*, 1858, 1st semestre, pp. 351–359.

(2) An observer looking at the tubes from above easily measure with his eye the differences of water le cially if they are slight.

To facilitate reading, Darcy designed an apparatus in figure 33.* Two vertical glass tubes, one connecte impact tube, the other to the press are set parallel to each other; at G unite in one; and at this point is At H and I are two valves both on Between the tubes is placed a graduat To operate, suction is applied above rarefies the air in both vertical legs; closed, and simultaneously H and I. are then lifted out of the water and re

(3) A tube projecting into a strea direction does not give a true measu pressure head ure 32). He termine the between the difference in levels and the a coefficient fo must be deter calibration.

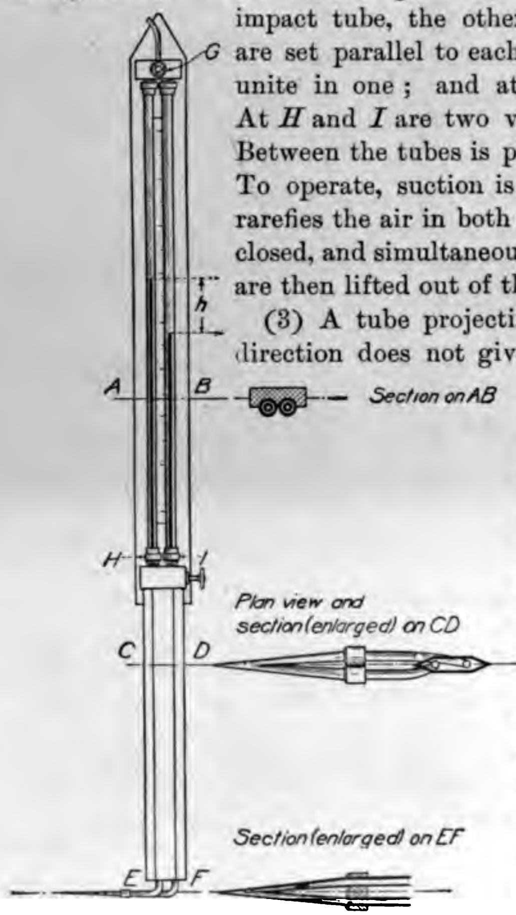


FIG. 33. — Darcy-Bazin Pitot Tubes.

Figure 33 a one of Darcy a set of tubes. recognizing th ties of measur sure head manner, deter experiment th setting a bent different positions in succession, in the center of pipe different diameters as follows:

(1) Against the current (Position A in figure 32

* Darcy and Bazin, *Recherches Hydrauliques*, Part I, plate i

† *Mouvement de l'Eau dans les Tuyaux*, 1857, pp. 220–224.

(2) In the direction of the current (Position *B*).

(3) At right angles to the current (Position *C*).

The water levels in the tubes thus set were compared with a piezometer set flush with the side of the pipe, and at the elevation of the pipe center. From twenty-one experiments on each setting, with velocities ranging from 1.4 to 16.3 feet per second, he found the following mean differences in level in terms of his computed center velocity V.

$$h = 1.07\frac{V^2}{2g};\quad h_{,} = .43\frac{V^2}{2g};\quad h_{,,} = .68\frac{V^2}{2g};\ \text{(see figure 32)}$$

h being in excess of the pipe pressure; and $h_{,}$ and $h_{,,}$ being less.

143. Pitot-Darcy tubes. At the request of Darcy, Baumgarten had constructed and rated (in 1855) three different arrangements of tubes* described below, and determined their coefficients for use with Darcy's formula:

$$V = K(2gh)^{\frac{1}{2}}.$$

h = difference in feet in water levels in the two tubes composing the apparatus.

K = the coefficient of velocity, *i.e.* the ratio of the actual velocity, to the velocity computed as equal to $(2gh)^{\frac{1}{2}}$.

The tubes were rated first in running water with floats; second by moving the tubes in still water at a known velocity; with the following results:

Description of Apparatus	Value of K	Method of Rating
1. The impact tube was directed against the current, and the other at right angles.	.848 .797	With the aid of floats. In still water.
2. Both tubes were directed against the current, but one was closed at the end and pierced laterally by an orifice 1 millimetre in diameter.	.975 .864	With the aid of floats. In still water.
3. The impact tube was directed against the current, and the other in the direction of the flow, opening downstream.	.998 .991	With the aid of floats. In still water.

* Darcy and Bazin, *Recherches Hydrauliques*, pp. 302–303.

With several other tubes, which Darcy himself constructed, obtained a coefficient $K = 1$ when the pressure orifice did n project into the stream, but was simply an opening pierced in copper tube.

144. Bazin's tube. Bazin, in his weir experiments,* used Pitot-Darcy tube having an impact orifice normal to the curren and a lateral orifice opening without any projection in a surfa by which the stream flowed tangentially. The tubes were ins

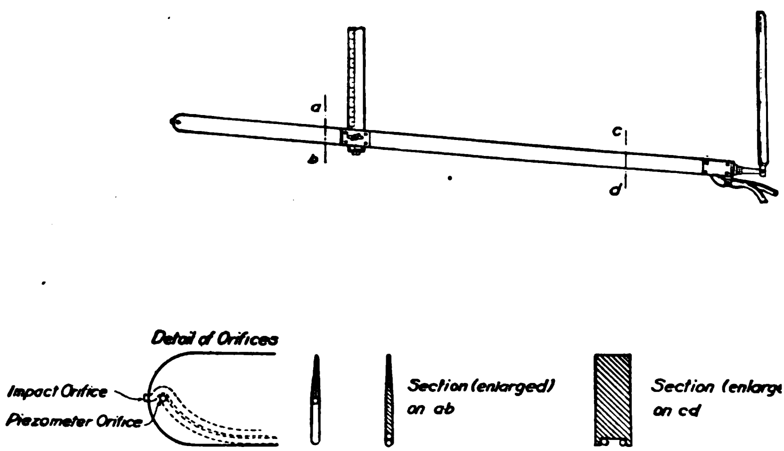


FIG. 34. — Bazin's Pitot Tube.

on a thin board beveled to a sharp top edge, which Bazin stat was similar to Darcy's apparatus. It had a coefficient K equ to 1. See figure 34.

145. Mills's experiments upon piezometers. Hiram F. Mills published in 1878 the results of some six thousand observatio made with extraordinary accuracy to determine the proper for of piezometer orifice. With twenty-two openings varied in sha and direction and a range of velocities from 0.6 to 8.9 feet p second, he found that with **an orifice whose edges are in the pla of the side of the channel and passage normal thereto, the piezomet column** will stand neither above nor below the surface of the strea but **will indicate the true height of the water surface in an op**

* *Ann. des Ponts et Chaussées*, 1890, 1st semestre, p. 54.

† Chief Engr. Essex Co., Lawrence. *Trans. Am. Academy of Science*, 1878.

channel or the pressure in a closed channel; but if the passage inclines either upstream or downstream, or the edges of the orifice project beyond the plane of the side, the true height of the surface or the true pressure will not be indicated.

146. Conclusions from various experiments. From the experiments mentioned above, and from later American experiments of White,* Gregory,† Williams,‡ and others, it is concluded:

(1) That with almost any form of small circular impact orifice and a pressure orifice at right angles to and flush with a surface parallel to the stream flow, the differences in level in two vertical tubes connected to these orifices is very nearly the head due to velocity.

(2) That the coefficient K of a given tube is constant for all velocities.

(3) That relatively slight inaccuracies in pointing the piezometer orifice may result in considerable errors; for this reason there is, under certain conditions of use, difficulty or uncertainty in measuring the pressure head. A bent tube with its orifice pointing directly downstream (position B, figure 32) may often be used to advantage instead of a true piezometer; because, although the height of the column of water which will rise in such a tube will be less than the true pressure head (Y), it will have a regular relation to the velocity head; and because the apparatus is not very sensitive to deviations in pointing. Such a tube combined with an impact tube is called a pitometer; the coefficient K of a pitometer is considerably less than 1.

(4) That in measuring velocities in a jet of water, only an impact tube is required; and its cofficient may be taken as 1.

(5) That, especially in measuring low velocities, some form of differential gauge is required in order to magnify the differences of water levels in the two tubes so that they can be read (§ 45).

(6) That all forms of tubes must be rated to determine the coefficient K, and to test the accuracy of construction, and periodically rerated to detect effects of wear.

147. Rating Pitot tubes. Rating in still water. Tubes have been successfully rated by moving them in still water at a known

* *Journal Asso. of Engr. Soc.*, August, 1901.
† *Trans. Am. Soc. M. E.*, December, 1903. ‡ *Trans. Am. Soc. C. E.*, 1902.

speed, the velocity V, at which the tube is moved through the water in feet per second, being assumed to have an effect in producing head (h) equivalent to an equal velocity of running water. Therefore $K = \dfrac{V}{(2gh)^{\frac{1}{2}}}$.

Rating in running water. Present practice, however, favors a rating in moving water under conditions as nearly as possible like those in which the measurement of flow is to be made. Readings of head h are taken at the center of subdivisions of equal area so spaced as to give a complete traverse of the channel, and from these the values of velocities due to the heads. As K is independent of velocity, the uncorrected mean velocity (V_m) may be taken as the arithmetical mean of all these velocities.

The actual discharge Q is measured during the period of observation by volume, weight, or by another calibrated measuring device.

The theoretic discharge (Q_t) equals the mean velocity (V_m) multiplied by the cross-sectional area (A) of the stream or jet.

$$Q_t = V_m A.$$

Therefore the actual discharge,

$$Q = KV_m A; \text{ and } K = \frac{Q}{V_m A}.$$

148. Determination of discharge. The discharge is determined as follows: (1) all the apparatus is tested to determine its accuracy; (2) calibrations are made to determine the area of the cross section of the stream, which should be subdivided into parts of equal or convenient area, in pipes most readily by circles; (3) the apparatus should be set at the desired point in the stream (for closed channels insertion is made through a stuffing box set in a corporation cock set in the pipe); (4) the tubes are connected to the gauges either by stiff rubber or metal pipes; (5) the difference h between the water level in the two tubes is observed; (6) the velocity is determined for each observation as equal to $(2gh)^{\frac{1}{2}}$, and the mean velocity V_m in the stream is taken as the arithmetical mean of all the computations; (7) the discharge is computed by multiplying the cross-sectional area of the stream by mean velocity V_m, corrected by K. Therefore,

$$Q = KV_m A.$$

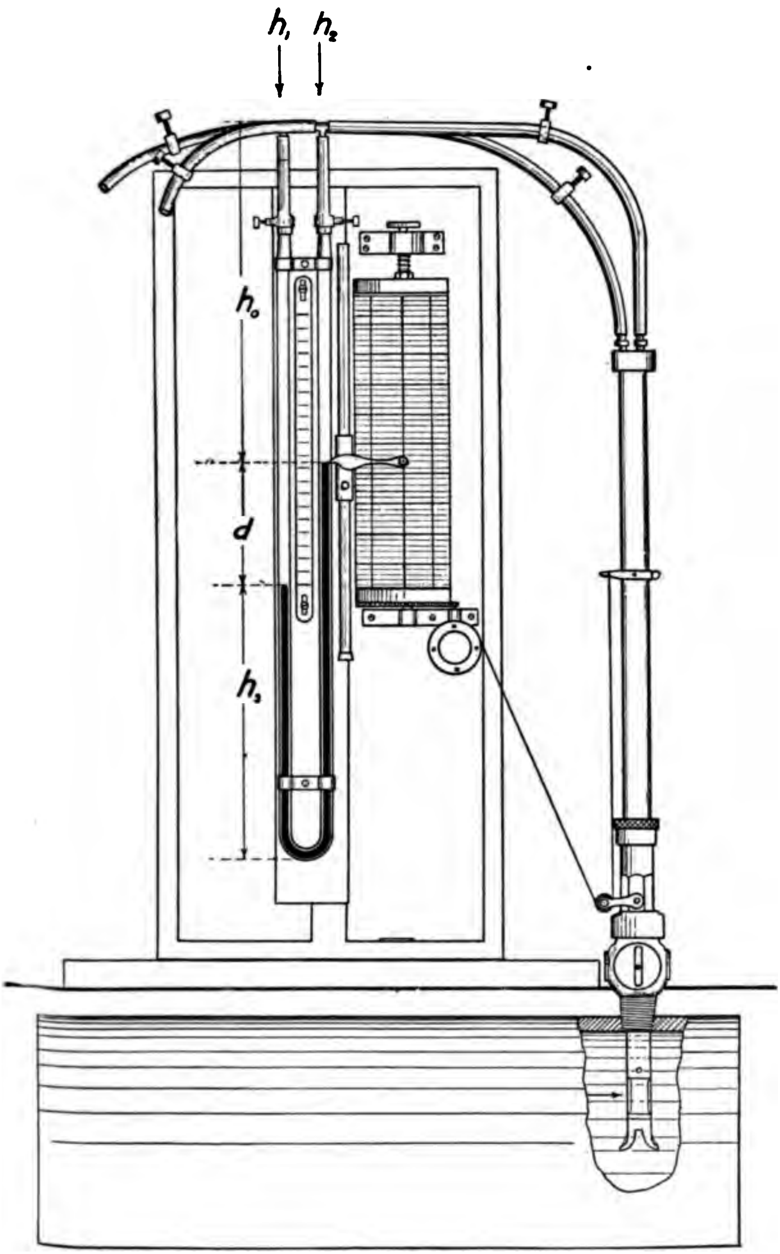


Fig. 35. — Portable Pitometer with Differential Gauge and Recording Apparatus.

149. The pitometer. The most recent form of the Pitot tube is called the pitometer.* See figure 35. It consists of a rod meter, and a differential gauge with suitable connections to which may be added either a revolving drum with paper for manually recording observations, or a photographic apparatus for automatic continuous records.

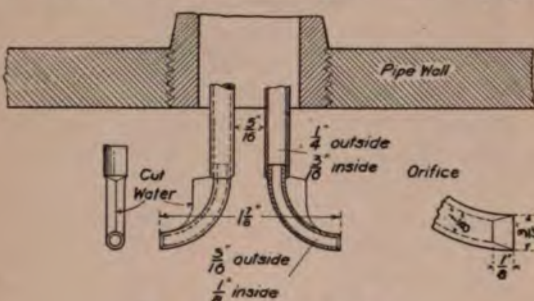


Fig. 36. — Pitometer Orifices.

The rod meter consists of a brass sheath of flat oval cross section containing two $\frac{1}{4}$-inch brass tubes, each terminating in a curved phosphor-bronze orifice with a cutwater. See figure 36. When in use one orifice points against the current, the other with the current, both tubes parallel with the current. For insertion the tubes are set in collars which allow them to be rotated through 180°.

The differential gauge used for measuring the difference in pressure in the two tubes is a glass U-tube half filled with a mixture usually of carbon tetrachloride and gasoline having a specific gravity usually of 1.25 to 1.50 and colored dark red. When in use water fills the remaining spaces. Thus the difference in pressure in the two tubes is magnified according to the specific gravity of the red liquid. Velocities as low as .5 foot per second may be measured. (See differential gauges, Chapter II, § 45.)

The coefficient (K) of this instrument has been established as a constant equal to .84.

The formula for velocity therefore becomes:

$$V = K[2g(s' - 1)d]^{\frac{1}{2}}.$$

s' = the specific gravity of the heavier liquid.

If used under varying temperatures, the specific gravities of the tetrachloride as well as of the water will change, and for high precision must be corrected.

* Developed by John A. Cole and Edward S. Cole, and owned by the Pitometer Co., New York.

If $K = .84$, and $s' = 1.25$,

$$V = .84\left(\frac{2gd}{4}\right)^{\frac{1}{2}} = 3.368\ (d)^{\frac{1}{2}}.$$

d = deflection of the tetrachloride, or the difference in elevation in feet between the tops of the two columns of the tetrachloride.

The photopitometer consists of a portable box in which a drum carrying Velox paper revolves before a fine vertical slit just in front of which is locked one leg of the U-tube. The rays of light are partly intercepted; and as the liquid rises and falls a line or band of shades is recorded on the sensitive film. The record of the pressure head in the pipe is also superimposed on the film by means of a movable finger operated by a gauge having an independent piezometer connection with the pipe; or the height of the heavy liquid in both tubes may be recorded. By scaling the ordinates the velocity may be computed at any time; and furthermore, the chart gives a graphic record of all changes in rate, which may at times be more useful than the absolute determination of discharge.

150. Gauging flow in a pipe by a pitometer. The traverse. As the pitometer measures only deflections due to the velocities at the point where the orifices are placed, either a number of point measurements must be made, or a ratio established between the velocity at the point measured, generally the center of the pipe, and the mean velocity of the entire cross section. With a constant rate of flow the velocity will vary from point to point in the stream, gradually increasing from the walls of the pipe toward the center. The mean velocity may be obtained as the arithmetical mean of observations made at the center of rings of equal value; but for convenience it is customary to divide the area into a center circle and three or four rings of dimensions which experience has shown to be convenient. These occasionally have to be varied if the traverse shows a distorted curve. Table VIII shows the usual subdivision for a traverse, and the corresponding areas.

The velocity curve. The deflections may be
during observations on a paper stre

TABLE VIII

TRAVERSE TABLE FOR PITOMETER GAUGINGS, SHOWING INNER DIAMETER OF EACH RING FOR ALL SIZES OF MAINS IN INCHES; AND GIVING THE AREA OF EACH RING AND CENTER CIRCLE IN SQUARE FEET

DIAMETER OF PIPE	AREA OF PIPE		RING *A*	RING *B*	RING *C*	RING *D*	CENTER CIRCLE *E*
Inches	Square Feet		Inches	Inches	Inches	Inches	Inches
48	12.566	Diam. . .	44	36	28	16	16
			Square Feet	Square Feet	Square Feet	Sq. Feet	Square Feet
		Area . . .	2.007	3.490	2.793	2.880	1.396
42	9.621	Diam. . .	38	32	24	14	14
		Area . . .	1.745	2.291	2.443	2.073	1.069
36	7.069	Diam. . .	34	28	20	10	10
		Area . . .	0.764	2.029	2.094	1.637	0.545
30	4.909	Diam. . .	28	24	18	12	12
		Area . . .	0.633	1.134	1.375	0.982	0.785
24	3.142	Diam. . .	22	18	14	8	8
		Area . . .	0.502	0.873	0.698	0.720	0.349
20	2.182	Diam. . .	18	14	10	6	6
		Area . . .	0.415	0.698	0.524	0.349	0.196
18	1.767	Diam. . .	17	15	11	7	7
		Area . . .	0.191	0.349	0.567	0.393	0.267
16	1.396	Diam. . .	15	13	10	6	6
		Area . . .	0.169	0.305	0.377	0.349	0.196
14	1.069	Diam. . .	13	11	9	6	6
		Area . . .	0.147	0.262	0.218	0.246	0.196
12	0.785	Diam. . .	11	9	7	5	5
		Area . . .	0.125	0.218	0.175	0.131	0.136
10	0.545	Diam. . .	9	8	6	4	4
		Area . . .	0.103	0.093	0.153	0.109	0.087
8	0.349	Diam. . .	7	6	5	3	3
		Area . . .	0.082	0.071	0.060	0.087	0.049
6	0.196	Diam. . .	$5\frac{1}{2}$	$4\frac{1}{2}$	$3\frac{1}{2}$	2	2
		Area . . .	0.031	0.055	0.043	0.045	0.022
4	0.0873	Diam. . .	$3\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{1}{2}$		$1\frac{1}{2}$
		Area . . .	0.0205	0.0327	0.0218		0.0123

wards on cross-section paper and a mean curve drawn through these points. From the deflections, the velocities may be computed and a velocity curve plotted, and from this, the mean velocity scaled for any desired ring.

The "pipe coefficient." The mean velocity for the whole pipe section is the total of these ring volumes divided by the area of the pipe as shown on the diagram, figure 37. The mean velocity divided by the center velocity, as read from the curve, is the pipe coefficient, or "decimal." This coefficient must be obtained either with a constant center velocity * or from many previous observations on the same pipe; once determined for a given pipe and given conditions of lining, it may be taken as practically constant for all velocities.

Discharge. The mean velocity of each ring in feet per second multiplied by the area of the ring in square feet gives the ring volume in cubic feet per second. The total discharge is the sum of the ring volumes of flow.

Example. The following diagram (figure 37) and computations illustrate the usual method of computing discharge.

DEFLECTION AND VELOCITY CURVES FROM 24-INCH SUPPLY MAIN

RING	RING AREAS SQ. FT.	RING VELOCITY FT. PER. SEC.	RING VOLUMES OF FLOW CU. FT. PER. SEC.
A	.502	1.57	.788
B	.873	1.87	1.632
C	.698	2.05	1.431
D	.720	2.18	1.570
E	.349	2.24	.781
	3.142		6.202 Total discharge of pipe.

$$\frac{\text{Discharge of Pipe}}{\text{Area of Pipe}} = \frac{6.202}{3.142} = 1.97 \text{ Mean Velocity.}$$

$$\frac{\text{Mean Velocity}}{\text{Center Velocity}} = \frac{V}{V_c} = \frac{1.97}{2.24} = .879 = \text{Pipe Coefficient.}$$

* In determining the coefficient an auxiliary pitometer is frequently placed in tandem to indicate the center velocity, while the traverse is being made.

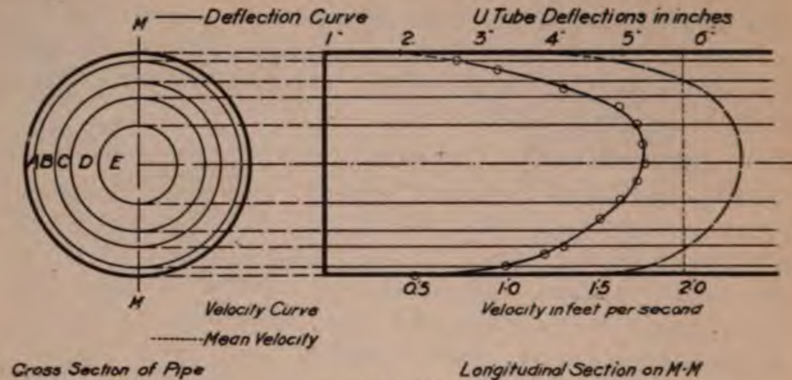


Fig. 37.

151. Deflection and velocity curves for several pipes taken from observations are shown in figure 38; all the pipes here shown had been in service, some for many years at the time of measurement. Figure 112, Chapter XV, also shows a set of velocity curves.

152. Continuous measurements of discharge. Having determined the "pipe coefficient," the tubes may be fixed with the orifices at the center of the pipe, and the observed center velocity, at any rate of flow, may be reduced to mean velocity by this pipe coefficient.

153. Application. The pitometer may be used to measure the velocity and hence the volume of flow either in open or closed channels without interfering with the use of the channel, with relatively small expense for installation and operation. It has proved its value in detecting waste or misuse of water in street mains and other channels and where changes in rates of flow are significant, and in measuring the slip of pumping engines. In using these meters in old pipes, ample allowance due to reductions of area due to incrustations must be made; and the certain change in the distribution of velocities due to incrustations and roughness must be provided for. Where possible, a number of point measurements of the velocity will produce a higher degree of accuracy than the single observation at the center and the use of the pipe coefficient.

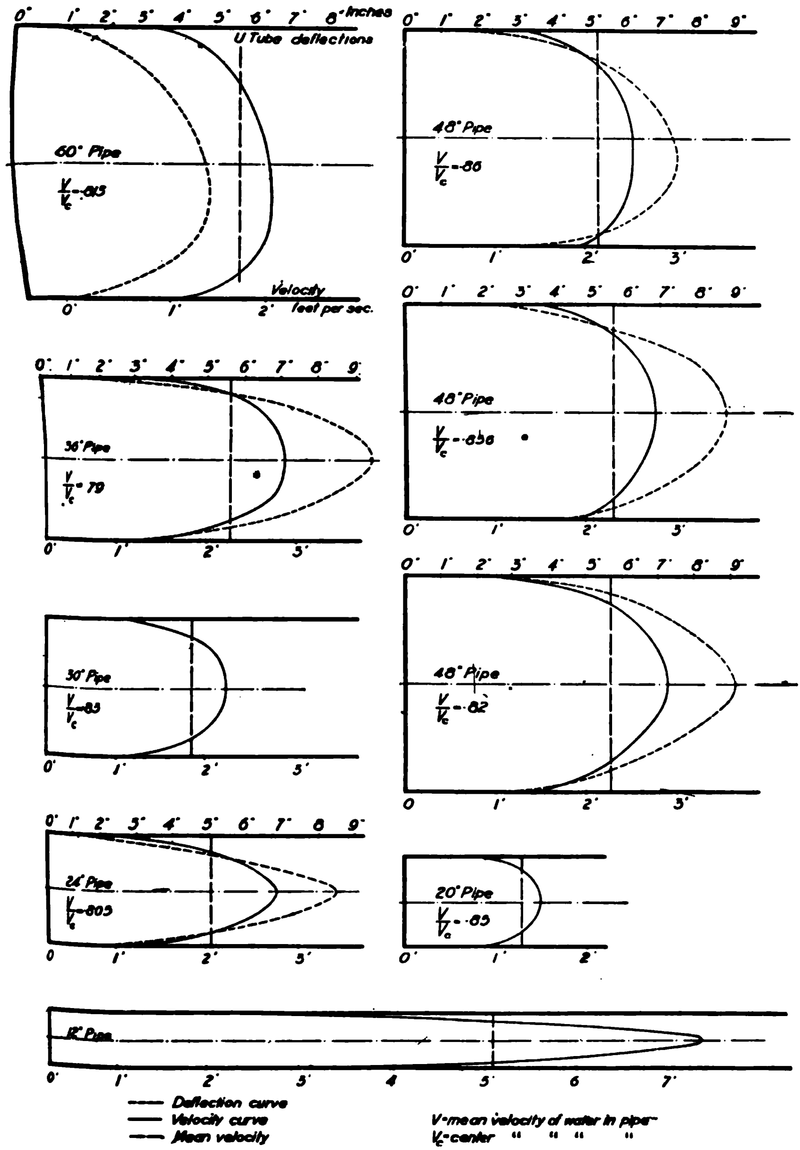


FIG. 38.—Diagrams of Actual Pitometer Traverses in Old Pipes.

I

Problems

1. If the specific gravity of carbon tetrachloride in a differential gauge is 1.25, compute for a pitometer the theoretic velocities in feet per second corresponding to the following deflections in inches:

.5 1.0 4.0 8.5

Also for same deflections when specific gravity is 1.5.

2. The cross section of a 42-inch pipe is divided by concentric rings into 5 equal areas. A Pitot tube, inserted at the center of each section in succession gave the following differences between the heights of water in the impact and pressure tubes, starting from the top and going down.

1.236″, 1.885″, 2.32″, 2.57″, 2.83″, 2.59″, 2.11″, 1.79″, 0.24″.

The coefficient of the pitometer is .84. Find the mean velocity, plot the velocity curve to scale, and determine the pipe coefficient and discharge.

3. A pitometer is set with its orifices at the center of a pipe. The water level in the impact tube is at elevation 123.7 feet, in the pressure tube at 123 feet, both measured from some datum plane. Calculate the mean velocity of flow in the pipe. K is .84 and the pipe coefficient is .85.

4. A pitometer is set with its orifices at the center of a 16-inch pipe. Compute the discharge in gallons per minute for each of the following deflections of the differential gauge containing carbon tetrachloride, (a) when its specific gravity is 1.25, (b) when its specific gravity is 1.50. Deflections, 2, 9, and 16 inches. Pipe coefficient is .86.

5. Traverses of a 10-inch pipe made with a pitometer at two different times gave the following deflections in inches of tetrachloride (if specific gravity is 1.50):

	A	B	C	D	E	D	C	B	A
(a)	0.8	1.19	1.55	1.74	1.90	1.84	1.67	1.41	1.10
(b)	2.64	3.92	5.10	5.57	5.93	5.89	5.40	4.50	3.70

The points of observation were midway between concentric rings spaced as in Table VIII, beginning at the top of the pipe.

Draw deflection and velocity curves, compute the discharge, and determine the pipe coefficient for both sets of observations. K is .84.

CHAPTER VIII

VENTURI METER

154. The Venturi meter. The Venturi meter is a practical application of Bernoulli's theorem to the measurement of water flowing in pipes under pressure. Two truncated hollow cones are inserted, as shown in figure 39, in a line of pipe having the same internal diameter as the base of the cones.

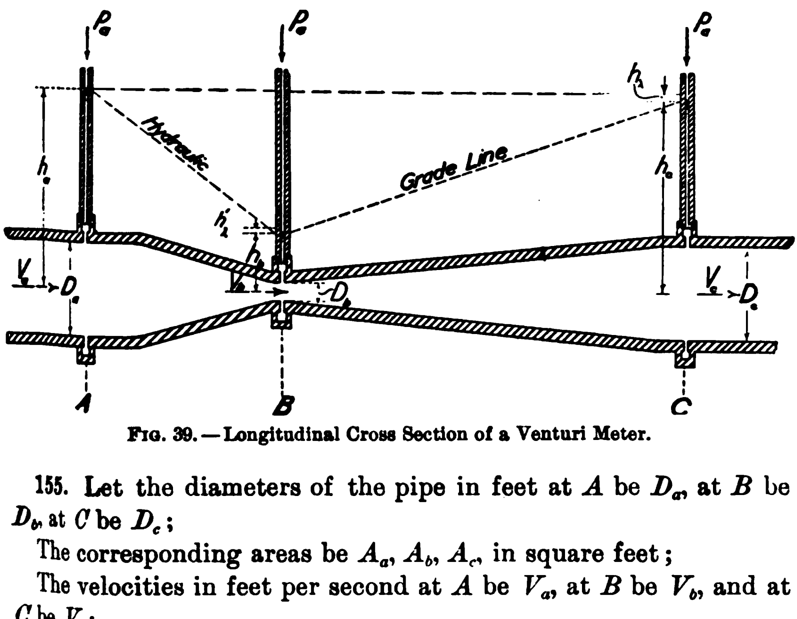


FIG. 39.—Longitudinal Cross Section of a Venturi Meter.

155. Let the diameters of the pipe in feet at A be D_a, at B be D_b, at C be D_c;

The corresponding areas be A_a, A_b, A_c, in square feet;

The velocities in feet per second at A be V_a, at B be V_b, and at C be V_c;

The pressure heads in feet measured from the center of the pipe at A be h_a, at B be h_b, and at C be h_c.

The upstream section at A is called the "inlet," the section at B is called the "throat." The section at C is the same area as at A. The pipe is filled with water flowing in the direction of the arrows, that is, from A to C.

Since the same quantity of water is passing through both inlet and the throat, and since the area at B is smaller, the velo at B is greater than at A; but as they have a constant relation any single meter, depending upon the ratio of the two are simultaneous observation of the pressure heads at the inlet throat provides a method of determining the velocity at the th (and also at the inlet), and from the velocity and areas the charge.

The formula for the discharge through a Venturi meter is rived as follows:

Let Q = actual discharge in cubic feet per second;

h_λ' = the lost head between A and B in fluid friction, changes in velocity.

By Bernoulli's theorem, the meter being set level,

$$\frac{p_a}{\gamma}+h_a+\frac{V_a^2}{2g}=\frac{p_a}{\gamma}+h_b+\frac{V_b^2}{2g}+h_\lambda'.$$

But $$Q=A_aV_a=A_bV_b=A_cV_c.$$

Then $$A_a^2V_a^2=A_b^2V_b^2;\text{ and }V_a^2=\frac{A_b^2V_b^2}{A_a^2}=\frac{D_b^4V_b^2}{D_a^4}.$$

Substitute the value of V_a^2 from (3) in (1) and transpose;

then $$\frac{V_b^2}{2g}\left(\frac{D_a^4-D_b^4}{D_a^4}\right)=h_a-h_b-h_\lambda'.$$

From (4) $$V_b=\frac{D_a^2}{(D_a^4-D_b^4)^{\frac{1}{2}}}[2g(h_a-h_b-h_\lambda')]^{\frac{1}{2}}.$$

And $$Q=A_bV_b=\frac{\pi D_b^2D_a^2}{4(D_a^4-D_b^4)^{\frac{1}{2}}}[2g(h_a-h_b-h_\lambda')]^{\frac{1}{2}}.$$

Since in the ordinary use of the meter h_λ' can not be measu its effect may be and usually is included by the use of a coeffic of discharge C.

Let $$h_a-h_b=H.$$

Then $$Q=C\frac{\pi D_b^2D_a^2}{4(D_a^4-D_b^4)^{\frac{1}{2}}}(2gH)^{\frac{1}{2}}.$$

NOTE. If h_b is less than atmospheric pressure, a not infrequent occurre

$$h_a-(-h_b)=h_a+h_b.$$

Formula (7) may be simplified for use with meters which have a given ratio of throat and inlet diameters.

Let this ratio $$R = \frac{D_a}{D_b};$$

and let $$K = \frac{\pi R^2}{4}\left(\frac{2g}{R^4 - 1}\right)^{\frac{1}{2}}. \tag{8}$$

Then $$Q = CKD_b^2H^{\frac{1}{2}}. \tag{9}$$

Values of R	$\frac{3}{1}$	$\frac{2\frac{1}{2}}{1}$	$\frac{2}{1}$
Values of K	6.338	6.381	6.505

156. The value of *C*. The coefficient C varies with the velocity at the throat, the ratio of the diameters at inlet and outlet, and the actual dimensions of the meter. As meters are ordinarily constructed, the coefficient C is between .97 and 1.0.

In Table IX (page 118) are given experimental values of C.

The total loss of head in a Venturi meter, that is, the lost head between A and C, is relatively small as compared with other types of water-meters.

In Table IX are given experimental values of the lost head for corresponding velocities.

157. Invented by Clemens Herschel. In 1886–1888 * Clemens Herschel invented this meter, established its scientific and commercial value, and named it after an Italian hydraulician who had discovered its principle.

When first used the pressures were observed by piezometers or manometers; and the meter could therefore be operated only when continued observations were possible, and now is frequently so used, especially in laboratory work; in 1892–1894 the engineers of the makers † of the meter devised a calculating machine or register driven by weights to indicate the total flow on a direct reading dial, and record the rate of flow on a chart. Figure 40 shows the general appearance of the outside of a meter tube; the distance A–B is usually about $\frac{1}{3}$ B–C. The tube is usually made

* *Trans. Am. Soc. C. E.* 1887, Vol. 17, p. 228 *et seq.*

† Builders Iron Foundry, Providence, R.I.

TABLE IX

Experimental Values of *C* for Certain Venturi Meters. Experimental Determinations of Lost Head (h_λ) in Certain Venti Meters

Velocities at throat in feet per second:

	5	10	15	20	25	30	35	40	45	50	55
Experiments of Coker & Strickland, *Trans. Can. S. C. E.*, 1902: Area of the throat, .000787 square feet. Ratio of throat area to inlet area, 1 to 18.3.											
* C			1.09	.102	.990	.982	.975	.968	.960	.955	.954
* h_λ.		.29	.89	1.5	2.1	2.8	3.5	4.4	5.5	7.1	12.3
Experiments of Marx, Wing, and Hoskins, *Trans. Am. Soc. C. E.*, 1898: Ogden Venturi on 54-inch pipe. Area of the throat, 3.547 square feet. Ratio of throat area to inlet area, 1 to 4.48.											
C furnished by makers of instruments.											
† h_λ	.05	.23	.53	.94							
Experiments of Clemens Herschel, *Trans. Am. Soc. C. E.*, 1887: Venturi on a 12-inch pipe. Area of the throat, 0.08684 square feet. Ratio of throat area to inlet area, 1 to 9 (practically).											
‡ C	.934	.982	.988	.989	.991	.992	.993	.995	.997	.999	
‡ h_λ	.06	.27	.57	.98	1.47	2.11	2.88	3.80	5.06	8.20	
Venturi on a 108-inch pipe. Area of the throat, 7.074 square feet. Ratio of throat area to inlet area, 1 to 8 (practically).											
‡ C	.985	.980	.975	.969	.966	.959	.954				
‡ h_λ	.04	.19	.39	.70	1.08	1.53	2.13				
Venturi on a 48-inch pipe. Area of the throat not stated. Ratio of throat area to inlet area, about 1 to 4.5.											
§ C	.998	.996	.991	.984	.980						
§ h_λ	.04	.16	.34	.62	1.00						

* These values have been interpolated from curve plotted from results of a experiments.

† These values the mean of losses shown by two meters, interpolated from cu plotted from results of above experiments.

‡ Values of C and h_λ taken from diagram accompanying above paper.

§ These values taken from curve: Plate III, *115 Experiments*, Herschel, 1 p. 48.

of cast iron with bronze-lined throat. Meters have been built of concrete with metal linings. Pressure-chambers (piezometer rings) surround the upstream end and throat, and pressure pipes lead therefrom to the manometer or register.

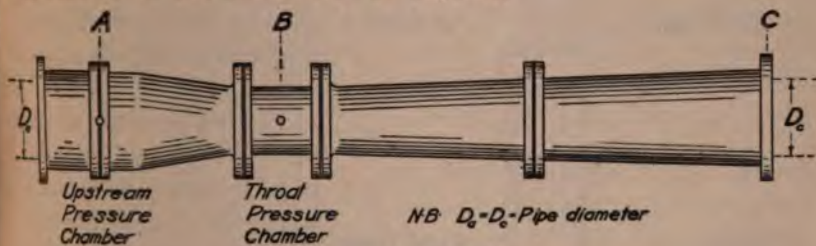


FIG. 40. — Venturi Meter.

158. Ratio of inlet and throat. In Herschel's first meter the ratio of inlet and throat diameters was 3 to 1, the areas being consequently 9 to 1. With this ratio the pressure at the throat is likely to be less than atmospheric; thus producing a positive column of water at the inlet, and a suction at the throat. This required certain additional corrections in computation, and a perfectly air-tight connection from the throat; matters easily managed in laboratory work but somewhat troublesome in practical installations. These meters should be, and are usually, designed to make both h_a and h_b greater than atmospheric pressure. This generally requires a smaller ratio of diameters than 3 to 1, depending, however, upon the diameter of the pipe, the pressure in the pipe, and the rate of flow.

159. Setting up and reading the Venturi meter manometer. A pressure pipe not less than $\frac{3}{4}$ inch inside diameter is run from valve A to the "upstream" or inlet pressure chamber of meter tube, also a similar pipe from valve B to the throat pressure chamber. These pipes should be preferably of lead, brass, or lead-lined iron or tin-lined iron, and should be connected at the sides of the pressure chambers, not at the top or bottom. A valve or corporation cock should be placed in each pipe close to the meter tube, so that if an accident occurs to the pipe the water can be shut off. The pipes should be laid on a slight grade, which may be either upward or downward, but there must be no summits where air can collect and affect the pressures. It is very necessary that there be unbroken

columns of water from the meter tube to the mercury in the ma nometer. These pressure pipes should be as short and direct a possible. Figure 41 show a manometer. Figure 4 shows a meter with regi ter * and recording attac ment.

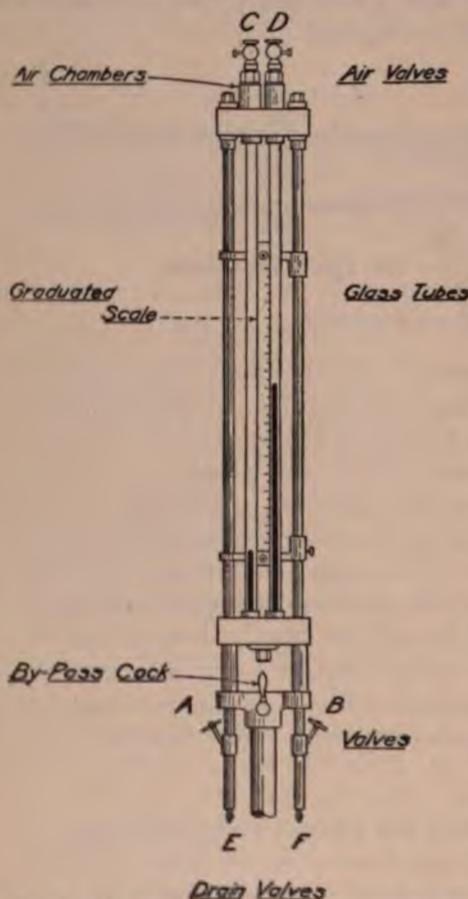


FIG. 41. — Mercury Manometer, or Pressure Gauge.

The manometer shou be filled about half fu of mercury by unscrewin one of the air chambers a inserting a small funnel.

The air valves C and are for the purposes drawing off any air th might collect in the a chambers. The by-pa cock is for the purpose equalizing the mercu levels and thus determi ing whether there is a air in either pressure pip It may also be opened cause a circulation water through the tw pressure pipes for the pu pose of cleaning them At other times this by-pas should be kept closed.

The drain valves E an F are for the purpose draining off any mercury that may accidentally be blown ov by an excessive pressure in one of the pressure pipes.

The glass tubes are ⅝ inch outside diameter and 36 inches lon set in rubber gaskets.

Determination of difference of head *H* by the manometer. Th sliding graduated scale is set with its zero at the top of the low

* For a description of the register and explanation of its working see *Journ Franklin Institute*, Vol. 147, p. 118.

column of mercury and the reading gives the difference in pressure in inches of mercury. This difference multiplied by the constant ratio 1.133 for reducing inches of mercury to feet of water gives H in feet of water, from which, by formula (7), the rate of discharge at the instant of observation may be computed.

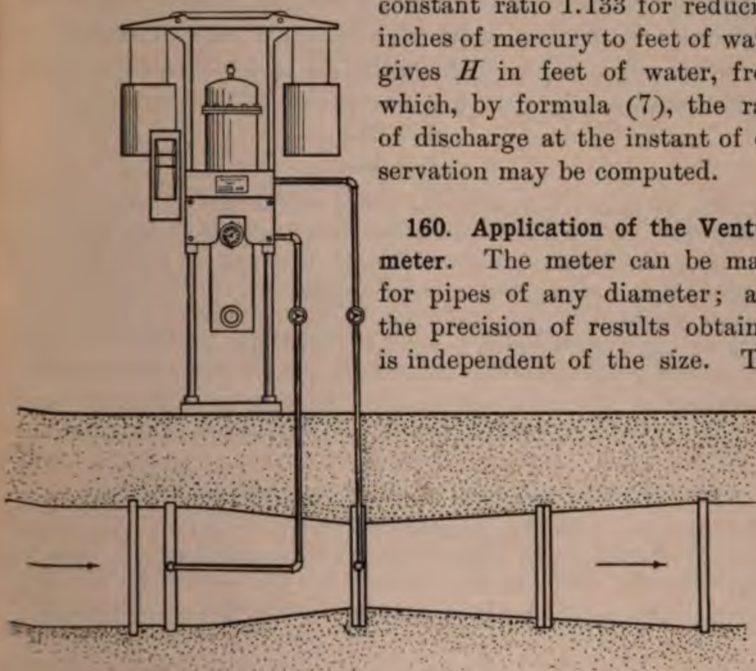


FIG. 42. — A Venturi Meter connected with a Continuous Register.

160. Application of the Venturi meter. The meter can be made for pipes of any diameter; and the precision of results obtained is independent of the size. The makers list the following sizes, designated by the inside diameter D_a in inches, as shown in figure 40: 2, 3, 4, 5, 6, and by 2 inches increment up to 60.

The capacity in U. S. gallons per 24 hours varies from 4000 for the 2-inch to 117,000,000 for the 60-inch. The 2-inch meter tube weighs about 50 pounds, and the 60-inch about 58,000 pounds. Smaller sizes than 2-inch are in use, and much larger than 60-inch up to 108 or 120 inches. The common sizes are 6 to 60 inches.

Experience has shown that, with the register, continuous measurements have been made with an error not exceeding 2 per cent. In laboratory experiments with mercury columns to measure the head at inlet and throat, rates of flow may be determined with an error frequently not exceeding 1 per cent. The total head lost between

the points A and C (figure 39) is so little that the insertion of a meter in the mains of a water supply system is rarely objectionable; and it may be used in high pressure power plants. It is frequently utilized in water and sewage filters to regulate the rate of filtration. Its chief use is in street mains, in measuring liquids heavier or lighter than water, and in filter control.

Problems

1. The following results were obtained by experiments on a Venturi meter; diameter of inlet = 3.92 inches, diameter of throat = 1.32 inches. (*a*) Run of 15 minutes: 52,042 pounds of water discharged; pressure at inlet, + 55 pounds per square inch; and at throat − 10.3 pounds per square inch. (*b*) Run of ten minutes: weight of water = 22,057 pounds; inlet pressure = + 14 pounds per square inch, and throat pressure = − 12.86 pounds per square inch. Gauge at inlet in both experiments was 16 inches above the center line of the pipe. Determine C.

2. In a Venturi water meter, of which the coefficient of discharge is .96, the larger diameter is 2½ feet and the smaller 1 foot, the heads are + 132 feet and + 11 feet. What will be the discharge in gallons per minute?

3. The ratio of the areas of the throat and inlet areas of a Venturi water meter are as 1 to 9. The larger pipe is 18 inches in diameter. What is its smaller diameter, and its discharge in cubic feet per second, if the coefficient of discharge is .98, and the head on larger end is 66 feet, on smaller end is − 15 feet?

4. The water level in the piezometer tube at the inlet (6 feet diameter) of a Venturi meter stands 49 feet above the center of the pipe. In the throat (2 feet diameter) the piezometer shows a negative reading of 15 inches of mercury. If the actual equals the theoretic discharge, how many gallons per day will be delivered?

5. The discharge through a Venturi meter is 810 gallons per minute; diameter of pipe is 12 inches. Ratio of diameters, 2 to 1. Pressure head at entrance is 21.4 feet. Find velocity at inlet and at throat, and pressure head at throat. C is .99.

6. Diameter of larger pipe in a Venturi meter is 3 feet; smaller, 1 foot. Heads are 110 feet and − 3 feet. Discharge is 65.7 cubic feet per second. What is coefficient of discharge?

CHAPTER IX

ORIFICES

Orifices in a Thin Wall

161. An orifice is an opening; in hydraulics an opening of some regular geometric form used in measuring the flow of water. The use of the term "orifice" is commonly limited to openings pierced in relatively thin, flat, stiff walls of metal, wood or masonry; but actually includes mouthpieces, short tubes, nozzles, gates, sluices, and other openings, even those in which the distance from inlet to outlet is considerable.

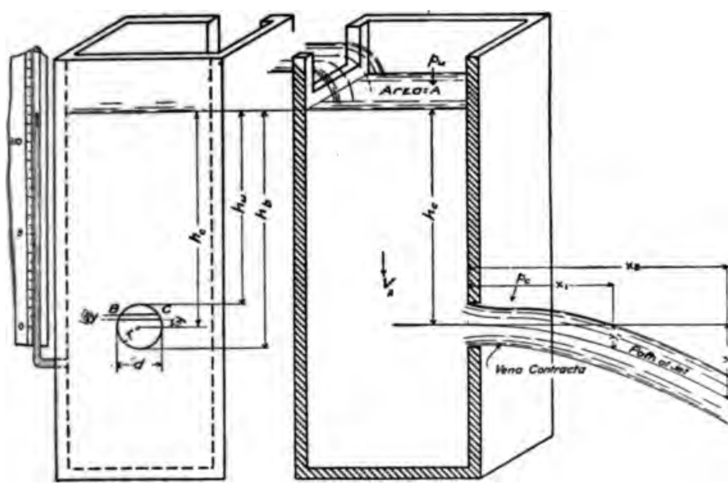


Fig. 43. — A Vertical Circular Orifice.

A vertical orifice is one set in a vertical plane. Figures 43 and 44 represent a circular and a rectangular vertical orifice.

A horizontal orifice is one set in a horizontal plane. Figure 48 represents a horizontal orifice.

162. Nomenclature. The following nomenclature will be used in discussing orifices of all kinds.

L = breadth in feet of a rectangular orifice.

O = height in feet of a rectangular orifice.

d = the diameter of a circular orifice in feet.

a = area of an orifice in square feet.

A = area of the channel of approach in square feet.

Q = the actual discharge in cubic feet per second.

Q_t = the theoretic discharge in cubic feet per second.

V = the actual mean velocity in feet per second through an orifice, that is, at the "vena contracta."

V_t = theoretic mean velocity in feet per second through an orifice.

V_A = the mean velocity in feet per second of the water flowing through the tank or channel of approach to the opening, called the velocity of approach.

p_u = the intensity of pressure on the upper surface of the water, due to external forces.

p_c = the intensity of pressure on the center of the orifice, due to external forces.

h_c = difference in elevation in feet between the water surface and the center of the orifice.

h_u = difference in elevation in feet between the water surface and the upper edge of the orifice.

h_b = difference in elevation in feet between the water surface and the bottom edge of the orifice.

$H_c = h_c + \left(\frac{p_u - p_c}{\gamma}\right)$ = head in feet on center corrected for difference in pressure on water surface and on the orifice.

W = the volume of discharge through the opening in pounds of water per second.

g = 32.16, the acceleration due to gravity in feet per second per second.

C = coefficient of discharge = $C_v C_c$.

C_v = coefficient of velocity.

C_c = coefficient of contraction.

163. Theory of the orifice. The determination of the volume of flow through an orifice is based on Torricelli's theorem, viz. that the velocity of water passing through an orifice is the same as that which would be acquired by a body falling freely in vacuo from the surface of the water to the center of the orifice; hence the theoretic velocity should be

$$V_t = (2gh_c)^{\frac{1}{2}}, \qquad (1)$$

and the theoretic discharge should be

$$Q_t = aV_t = a(2gh_c)^{\frac{1}{2}}. \qquad (2)$$

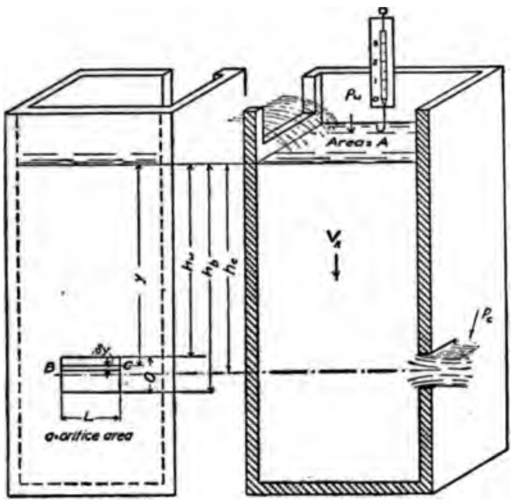


FIG. 44. — A Vertical Rectangular Orifice.

The actual mean velocity is slightly less than given by equation (1) and the actual discharge somewhat less than given by equation (2) for reasons that will be stated.

164. Difference in pressure on water surface and orifice; velocity of approach. The velocity and the discharge may be further modified by a difference between the intensity of pressure on the water surface above the orifice, and the intensity of pressure on the opening itself; and by the velocity of approach.

The effect of difference in pressure and of velocity of approach may be shown by Bernoulli's theorem,

$$W\left(\frac{V_t^2}{2g}\right) = W\left(\frac{V_A^2}{2g} + \frac{p_u - p_c}{\gamma} + h_c\right), \qquad (3)$$

$$\frac{V_t^2}{2g} = \frac{V_A^2}{2g} + \frac{p_u - p_c}{\gamma} + h_c,$$

Theoretic velocity, $$V_t = \left[V_A^2 + 2g\left(\frac{p_u - p_c}{\gamma} + h_c\right)\right]^{\frac{1}{2}}, \qquad (4)$$

Actual velocity, $$V = C_v\left[V_A^2 + 2g\left(\frac{p_u - p_c}{\gamma} + h_c\right)\right]^{\frac{1}{2}} \qquad (5)$$

and if $$H_c = h_c + \frac{p_u - p_c}{\gamma},$$

$$V = C_v(V_A^2 + 2gH_c)^{\frac{1}{2}}. \quad (6)$$

The effect of differences in pressure is the same as adding algebraically to h_c the algebraic difference between p_u and p_c reduced to equivalent feet of water.

The effect of velocity of approach. The kinetic energy, $\frac{V_A^2}{2g}$, due to the velocity of approach, will increase the actual velocity from the orifice by adding to the head. Whether this increase is significant is wholly a question of its magnitude relative to the degree of precision warranted by the other factors of the problem.

Since the velocity is not uniform in all parts of the section, the kinetic energy, in terms of the mean velocity, should be expressed as $n\frac{V_A^2}{2g}$.

$$Q = AV_A = C_c aV; \text{ therefore } V_A = \frac{C_c aV}{A},$$

Hence $n\frac{V_A^2}{2g} = n\frac{C_c^2 a^2 V^2}{A^2 2g}$, and by substituting this value in (6)

formula (6) becomes $V = C_v\left(n\frac{C_c^2 a^2 V^2}{A^2} + 2gH_c\right)^{\frac{1}{2}}$, (7)

The actual velocity $V = C_v(2gH_c)^{\frac{1}{2}}\left[\frac{1}{1 - C_v^2 C_c^2 n\frac{a^2}{A^2}}\right]^{\frac{1}{2}}$, (8)

Including all necessary factors, then,

the actual discharge $Q = Ca(2gH_c)^{\frac{1}{2}}\left[\frac{1}{1 - C^2 n\frac{a^2}{A^2}}\right]^{\frac{1}{2}}$. (9)

Neglecting differences of pressure and velocity of approach, formula (9) becomes $Q = Ca(2gh_c)^{\frac{1}{2}}$. (10)

The coefficient C should be selected for a head equal to $\left(H_c + n\frac{V_A^2}{2g}\right)$.

In Table X is shown the theoretic effect of the velocity of approach.

TABLE X

$C_v = .98$ and $n = 1.0$

$\frac{A}{C_c a}$	$\frac{1}{\left(1 - C_v^2 \frac{n C_c^2 a^2}{A^2}\right)^{\frac{1}{2}}}$	$\frac{A}{C_c a}$	$\frac{1}{\left(1 - C_v^2 \frac{n C_c^2 a^2}{A^2}\right)^{\frac{1}{2}}}$
1.33	1.479	5	1.020
1.5	1.321	10	1.0048
2.0	1.147	15	1.0022
2.5	1.087	20	1.0012
3.0	1.058	25	1.0008

It appears from this table that when $\frac{A}{C_c a} = 10$, the error in ignoring the velocity of approach is but .5 per cent; while on the other hand, as the ratio $\frac{A}{C_c a}$ decreases, the effect of velocity of approach becomes more significant, until if $\frac{A}{C_c a} = 1.33$, the increase in discharge would appear to be about 50 per cent. Such a large correction can not be considered anything but a mathematical extension of a formula, and has no reliability. With such high velocity of approach, ordinary coefficients do not apply, and few special ones are available. Indeed under such conditions the opening should probably not be classified as an orifice, but as a sluice, or an open channel with varying cross section, for which a rating at different depths should be made by actual measurements.

Example. What is the discharge through a vertical orifice .4 foot square set in a channel 2.4 feet in width, where the water is 2.4 feet deep, the bottom edge of the orifice being 1.0 foot above the bottom of the channel? The orifice is discharging into a condenser in which there is a vacuum of 27 inches of mercury.

Let $C_v = .98$; $C_c = .62$; and $n = 1$. Then

By (9) $Q = .98 \times .62 \times .4^2$

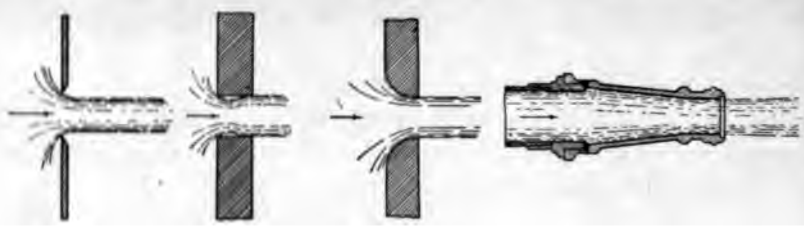
$$\times 8.02\left[\left(1.2 + \frac{14.7 - 1.47}{.433}\right)\left\{\frac{1}{1 - .98^2 \times .62^2\left(\frac{.4}{2.4}\right)^4}\right\}\right]^{\frac{1}{2}} = 4.39.$$

of jet. The jet of water issuing from an orifice us... contracts to an area smaller than the opening itself; again enlarges more or less irregularly; and does not maintain the cross-sectional shape of the opening itself except for circular orifices.

The path of the jet is that of a trajectory. (See figure 43.)

Vena contracta. The section of minimum area, called the "vena contracta," is usually distant from the inner plane of the orifice about .50 to .80 of the least dimension of the opening. In determining velocity and discharge, this is the critical section; because many experiments have demonstrated that the velocity at this point is nearly equal to $(2gh_c)^{\frac{1}{2}}$; and **the actual discharge through the orifice equals very nearly the area at the "vena contracta" ultiplied by $(2gh_c)^{\frac{1}{2}}$.**

The contraction is probably due to the inability of the water to hange instantly from rest, or the very low velocity of approach, relatively high velocity of discharge; and to the viscosity fluid; for this reason the contraction is greatest in the case a sharp-edged orifice in a thin wall, where the change in veity in passing from the channel to the orifice is greatest; and



A *B* *C* *D*

FIG. 45.

least in the case of a nozzle or mouthpiece, where the changes are purposely made most gradual. See figure 45.

In well-designed orifices, friction and eddy disturbances have little effect; and experience has shown that the actual loss of energy is slight.

In figure 45, *A* is an orifice in a thin wall; *B* is a sharp-edged orifice in a thick wall; *C* is a bell mouth, or rounded orifice; *D* is a nozzle.

166. The coefficient of contraction (C_c) is the ratio of the area of the jet at the "vena contracta" and the area of the orifice; that is,

$$C_c = \frac{\text{area at vena contracta}}{\text{area of orifice}}, \text{ also} = \frac{C}{C_v}.$$

Its mean value is about .62 for orifices in a thin wall.

The coefficient of contraction (C_c) may be found by direct measurements of the diameter of the jet at the contracted section,

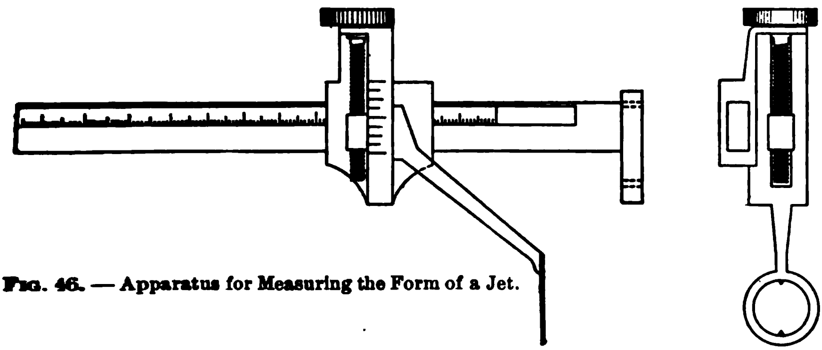


FIG. 46. — Apparatus for Measuring the Form of a Jet.

by such an apparatus as shown in figure 46, or it may be computed from experimental determinations of C and C_v.

167. The coefficient of velocity is the ratio of the actual mean velocity of the jet at the vena contracta and the velocity due to the whole head on the orifice; that is,

$$C_v = \frac{V}{(V_A^2 + 2gH_c)^{\frac{1}{2}}}.$$

Its value is about from .95 to .995.

The coefficient of velocity may be obtained by measurements with a Pitot tube, or by measuring the coördinates of the path of the jet, which is the curve of a trajectory (see figure 43), and thence deducing the velocity; and dividing by $(2gh_c)^{\frac{1}{2}}$, as follows:

$$x = Vt; \quad y = \tfrac{1}{2}gt^2; \quad \text{whence } V = \left(\frac{gx^2}{2y}\right)^{\frac{1}{2}}; \quad \text{therefore}$$

$$C_v = \frac{V}{(2gh_c)^{\frac{1}{2}}} = \left(\frac{x^2}{4yh_c}\right)^{\frac{1}{2}}.$$

168. The coefficient of discharge is the ratio between the actual discharge (Q) and the theoretic discharge (Q_t); that is,

$$C = \frac{Q}{a(V^2 + 2gH_e)^{\frac{1}{2}}}.$$

The coefficient of discharge may be computed from volumetric or weight determinations of discharge with a high degree of precision.

169. Coefficient determinations. The methods of determining directly C_v and C_c are both instructive and interesting; but the results obtained are not wholly satisfactory or as precise as an experimental determination of the coefficient of discharge. Even with jets from a circular orifice a close measurement of the contracted section is difficult, owing to the vibrations of the jet. The jets from a rectangular orifice can rarely be measured with any reasonable degree of accuracy.

The actual velocity of a jet has been determined by a Pitot tube with a high degree of precision.

170. An orifice in a thin wall. If an orifice cut in a true plane presents as nearly as may be only a line of contact to the emerging jet, the orifice is designated an *orifice in a thin wall*, *square-edged* or *sharp-edged*. To secure this condition, it is essential only that the thickness of the wall be such that the jet will not, after leaving the inner face, again touch on the side of the opening.

Standard orifice. If an orifice in a thin wall is set far enough from the side of the vessel or channel to secure **full contraction of the jet**, is **round or square**, and has **no dimensions greater than one foot** (for which shapes and dimensions reliable coefficients are available), it is called a standard orifice.

Complete contraction. If an increase in head or any other change in the setting of an orifice will not increase the contraction, it is said to be complete.

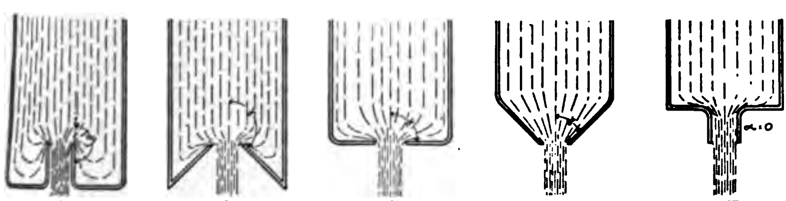
To insure complete contraction of a jet from an orifice in a thin wall, the margin of the orifice should be flat in every direction for at least three times the smallest dimension of the opening; the opening should be placed symmetrically with respect to the sides of the approach channel; and the head ought to be three times the least dimension of the orifice. It should be noted that

complete or nearly complete contraction may be secured in short tubes, and reëntrant tubes (see Chapter X).

171. Orifices in thick walls. If the opening be in a plate of such thickness, even though square-edged, that the stream again touches the sides of the opening after leaving the inner face (see figure 45, *B*); or if the opening be of gradually diminishing area such that the jet clings to the sides (see figure 45, *C* and *D*) the contraction will be partly suppressed, and for a given head the discharge will be more than for the orifice of the same area in a thin plate. The limit at which an orifice becomes a tube, nozzle, or mouthpiece is not well defined. Orifices in a thick wall (excepting nozzles) are not as accurate for measuring water as orifices in a thin wall; because, not only are few reliable coefficients available, but also they are more difficult to secure, owing to great varieties of shape and dimensions possible, and the difficulties of duplicating any shape to conform to those from which the few experimental coefficients available were obtained.

172. Comparison of orifices as to contraction and discharge. Weisbach made a great number of experiments to determine the law according to which the contraction changes with the angle α. Figure 47 *A* to *E* shows the range from the greatest contraction to practically none, the discharge increasing as the contraction decreases. By comparing the following coefficients of discharge with the figures, the difference between various kinds of orifices is obvious. Diameter = .033 foot. Heads 1 to 10.

COMPARISON OF ORIFICES



A *B* *C* *D* *E*

FIG. 47.

	A		*B*		*C*	*D*					*E*
α	180°	157½°	135°	112½°	90°	67½°	45°	22½°	11¼°	5¾°	0°
c	.541	.546	.577	.606	.632	.684	.753	.882	.924	.949	.966

173. For measuring water with a high degree of precision by orifices, only orifices in a thin wall should be employed, using an orifice for which the greatest number of reliable coefficients are available.

For discharging a maximum volume through an opening of a given diameter, orifices with wide and well-rounded lips should be used.

174. The procedure in measuring the flow of water with an orifice comprises:

(1) Testing and setting gauges for measuring head or pressure;

(2) Calibrating the opening and the channel of approach by direct measurements and computing therefrom the areas a and A;

(3) Determining the difference in elevation between the water surface and the orifice; and the pressures p_a and p_c, if other than atmospheric pressure.

For the coefficients given in the tables the head has been measured from the water surface to the center of the plane of the opening.

To be mathematically accurate the head should be measured from the center of the vena contracta, especially in horizontal orifices; because the full velocity does not develop before reaching this point.

The head may be measured by means of a hook gauge, as in figure 44, a piezometer tube, as in figure 43, a mercury column, or a calibrated steam guage. The head should be measured sufficiently distant from the plane of the orifice to avoid inaccuracies due to dropping of the water surface toward the opening.

(4) Selecting the coefficient of discharge suited to the shape and dimensions of the opening and the head; and

(5) Computing the discharge by a suitable formula.

175. More precise formula for vertical circular orifices. Referring to figure 43, BC is an elementary strip (so narrow that its width is negligible) in the opening whose area is $2(r^2-y^2)^{\frac{1}{2}}\,\delta y$; and the head on BC is $(h_c - y)$.

Neglecting the velocity of approach in the channel, and assum-

ing equal intensity of pressure on the water surface and on the jet, the theoretic discharge in cubic feet per second through the strip *BC* is

$$\delta Q_t = 2(2g)^{\frac{1}{2}}(r^2 - y^2)^{\frac{1}{2}}(h_c - y)^{\frac{1}{2}}\,\delta y. \tag{11}$$

Expand equation (11) in series, then integrate between the limits, $y = +r$, and $y = -r$, prefix the coefficient of discharge (C'), and the result is a formula for computing the discharge of circular orifices in cubic feet per second, viz.:

$$Q = C'\pi r^2(2gh_c)^{\frac{1}{2}}\left(1 - \frac{r^2}{32\,h_c^2} - \frac{5\,r^4}{1024\,h_c^4} - \cdots \text{etc.}\right). \tag{12}$$

The following table gives factors equivalent to the value of the series in equation (12) for given ratios of h_c and d, from which it is evident that when h_c equals 3 d a neglect of the series would cause an error of but .1 per cent.

TABLE XI

$\frac{h_c}{d}$	Factor	Log	$\frac{h_c}{d}$	Factor	Log	$\frac{h_c}{d}$	Factor	Log
0.5	0.960	$\bar{1}.9825$	0.9	0.990	$\bar{1}.9955$	1.6	0.997	$\bar{1}.9987$
0.6	0.975	$\bar{1}.9891$	1.0	0.992	$\bar{1}.9964$	2.0	0.998	$\bar{1}.9991$
0.7	0.982	$\bar{1}.9922$	1.2	0.994	$\bar{1}.9976$	3.0	0.999	$\bar{1}.9996$
0.8	0.987	$\bar{1}.9942$	1.4	0.996	$\bar{1}.9983$	10.	1.	0.

This more precise formula need be used only if the head is relatively low (less than 2 d).

In general it is convenient to change C' to C by means of the factors in Table XI, and write equation (12) as follows:

$$Q = C\pi r^2(2gh_c)^{\frac{1}{2}}. \tag{13}$$

Coefficients for vertical circular orifices applicable with formula (13) are given in Tables XV and XVI. Values of C corrected by Table XI are shown in black face type.

In Tables LXVII and LXIX will be found areas of circles.

In Table LXI are given velocities for various heads, and discharges for stream one square inch in area.

Example. What is the discharge through a vertical circular orifice .5 foot diameter under a head on the center of 16 feet, with full contraction of the jet?

By (13)

$$Q = .596 \times .1964 \times 8.02 \times 4 = 3.76 \text{ cubic feet per second.}$$

176. Precise formula for vertical rectangular orifices. Referring to figure 44, BC is an elementary strip in the opening, the area of which strip is $L\delta y$, and the head on BC is y.

Neglecting the velocity of approach in the channel, and assuming equal intensity of pressure on the water surface and on the jet, the discharge through the strip BC in cubic feet per second is

$$\delta Q_t = L(2gy)^{\frac{1}{2}}\,\delta y. \tag{14}$$

Integrating (14) between the limits $y = h_b$ and $y = h_u$, and prefixing the coefficient of discharge (C'), there results the precise formula for computing the discharge for rectangular orifices in cubic feet per second:

$$Q = \frac{2\,C'}{3}L(2g)^{\frac{1}{2}}(h_b^{\frac{3}{2}} - h_u^{\frac{3}{2}}). \tag{15}$$

For this formula many coefficients (C') were originally determined; but it is not as convenient for computation as the less precise expression

$$Q = CLO(2gh_c)^{\frac{1}{2}} = CL(h_b - h_u)(2g)^{\frac{1}{2}}\left(\frac{h_b + h_u}{2}\right)^{\frac{1}{2}}. \tag{16}$$

If then Q is to be computed by formula (16), the coefficient, if it has been determined for (15), should be multiplied by a factor to get the coefficient C for use in (16).

$$\text{This factor} = \frac{\frac{2}{3}L(2g)^{\frac{1}{2}}(h_b^{\frac{3}{2}} - h_u^{\frac{3}{2}})}{L(h_b - h_u)(2g)^{\frac{1}{2}}\left(\frac{h_b + h_u}{2}\right)^{\frac{1}{2}}} = \frac{2(2)^{\frac{1}{2}}(h_b^{\frac{3}{2}} - h_u^{\frac{3}{2}})}{3(h_b - h_u)(h_b + h_u)^{\frac{1}{2}}}$$

This correction is necessary only for very low heads.

The following table gives values of this factor in terms of h_c and O; from which it is evident that when $\frac{h_c}{O}$ equals 3, the use of this factor would affect the result but .1 per cent.

Coefficients for vertical rectangular orifices applicable with formula (16) are given in Tables XVII to XX. Values of C corrected by the factors in Table XII are in black face type.

Table LXX gives squares of numbers.

TABLE XII

$\frac{h_c}{O}$	Factor	Log	$\frac{h_c}{O}$	Factor	Log	$\frac{h_c}{O}$	Factor	Log
.5	0.943	$\bar{1}.9744$	.80	0.982	$\bar{1}.9922$	1.6	0.996	$\bar{1}.9982$
.54	0.955	$\bar{1}.9800$	.90	0.986	$\bar{1}.9940$	2.0	0.997	$\bar{1}.9989$
.58	0.963	$\bar{1}.9835$	1.00	0.989	$\bar{1}.9952$	2.5	0.998	$\bar{1}.9993$
.60	0.966	$\bar{1}.9848$	1.2	0.993	$\bar{1}.9967$	3.0	0.999	$\bar{1}.9995$
.70	0.976	$\bar{1}.9895$	1.4	0.995	$\bar{1}.9977$	10.0	1.000	0.

Example. What is the discharge through a vertical orifice 1.0 foot square under a head on the center of 4.0 feet; with full contraction of the jet?

$$Q = .602 \times 1^2 \times 8.02 \times 2 = 9.66 \text{ cubic feet per second.}$$

177. **Horizontal orifices.** The formulas for vertical orifices apply theoretically to horizontal orifices. Few coefficients have, however, been directly determined for them; coefficients intended for vertical orifices must be therefore ordinarily used for horizontal orifices, but with less assurance of accuracy. It is fair to assume that the head if measured to the orifices should be corrected to the center of the vena contracta. See figure 48.

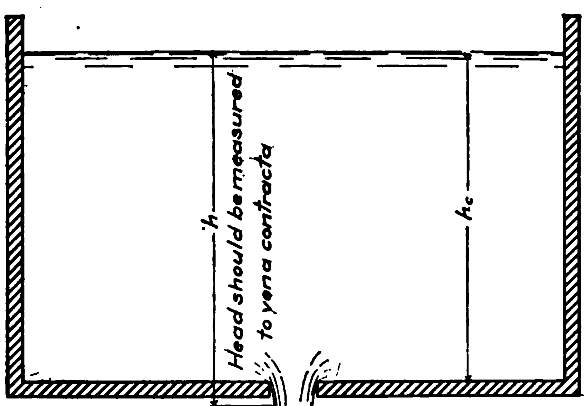


Fig. 48. — Horizontal Orifice.

Table XXI contains a few experimental coefficients for horizontal orifices.

178. **Concerning coefficients of discharge.** The number of reliable directly determined coefficients for orifices, while relatively

large, does not completely cover a very extensive range of heads; and some of those given in the tables were determined under conditions that did not allow complete contraction.

Smith's coefficients for round and square orifices here given (Tables XV and XVII) were based chiefly upon his own, Lesbros's, and Poncelet and Lesbros's experiments, but few of the experiments included heads exceeding 6 feet. In Mair's experiments (see Table XVI) the heads were not more than 2 feet. Bovey's experiments on very small orifices (Tables XVIII and XIX) were for heads up to 20 feet. Ellis's experiments included heads up to 20 feet. In brief, coefficients in use for heads exceeding 6 feet in most cases, and 10 to 20 feet in rare cases, are based on mathematical extensions of curves drawn for low heads.

Nevertheless, a comparative plotting of existing data seems to indicate, after making allowance for considerable difference in the precision attained by the different experimenters, that every orifice may have its critical head, above which the coefficient is nearly constant. Hence, Smith's tables if used for high heads will probably not be greatly in error. Recent experiments by Judd and King (Table XVI) tend to confirm this assumption. In the experiments of Lesbros, Poncelet and Lesbros, Smith, Mair, Bovey, Drummond, and King the discharge was measured by volume. Ellis used a weir, but it was not well set, and his apparatus for measuring the head was defective.

Coefficients given in Tables XV to XXI are given only for heads practically within the range of the original experiments, excepting only Smith's coefficients, in Tables XV and XVII.

Several tables are given bodily. The other experiments have been regrouped according to the outlines of the orifices; and in most cases have been replotted in order to determine values of C for heads varying by increments of one foot. The values of C thus taken from the plotted curves have also been checked by interpolation with the original experiments. It is believed that no violence has been done to the original values of the coefficients; and the result offers an easy means of comparison.

For orifices in a thin wall with full contraction of the jet, and under similar conditions, it is probably true that:

(*a*) With similar shapes, the head being constant, the coefficient decreases as the area increases;

(*b*) For any given orifice, if the contraction is complete, the coefficient decreases as the head increases up to some constant value; for heads too low for complete contraction, the coefficient decreases as the head decreases;

(*c*) For equal areas, and with the same head, coefficients for circular orifices are less than for square orifices, and for square orifices less than for rectangular orifices in which the length and depth are unequal;

(*d*) For rectangular orifices of equal area and under the same head, the coefficient increases as the ratio between the length and breadth, or the ratio between the breadth and length, increases; there being practically no difference whether the longer or shorter side is vertical, provided there is full contraction.

179. **The effect of suppressing contraction.** The suppression of contraction may be made partial or complete by widening or rounding the lips of the orifice, by reducing the distance between the edges of the opening and the sides or bottom of the channel, by reducing the head below the height required for complete contraction, or by placing two or more orifices side by side in the same wall.

For **complete suppression**, the coefficient of discharge is practically unity. In an orifice in a thin wall suppression can not be complete.

For **partial suppression**, other conditions being the same, the coefficient of discharge is less than unity, more than the coefficient for the same orifice with full contraction; and slight changes in form may produce wide variations in the coefficient. The published experiments are insufficient to warrant the framing of a satisfactory law for computing the results of suppressing contraction; but they indicate that the discharge is not increased in direct proportion to the percentage of perimeter on which the contraction is suppressed.

The following experiments are useful in estimating the results of suppression. The data available are very meager and of limited application. The effect of suppression in various forms of orifices will be considered again in Chapter X.

Experiments on suppression of contraction. Darcy and Bazin *

* Darcy and Bazin, *Recherches Hydrauliques*, vol. I, pp. 52 to 63.

experimented on 12 orifices each 0.656 foot square, and set as shown in figure 49. The openings were cut in flat copper plates

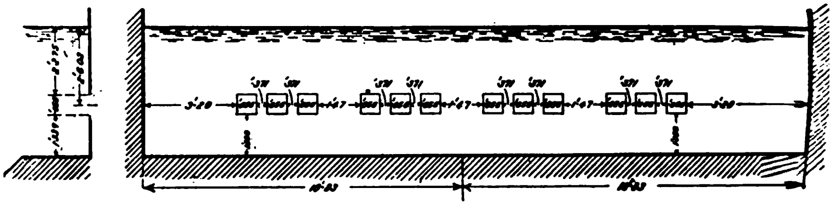


FIG. 49. — Orifices used by Darcy and Bazin.

each orifice was *provided with a gate which when opened fully cleared the opening.*

The results of these experiments were as follows:

Number of orifices open	1	2	3	4	5 to 12
Coefficient C* for such orifices as were open	.633	.642	.646	.649	.650

With one gate, the fifth from the right, raised .328 feet, the others mentioned fully opened

Number of gates fully opened	1	2	3	4	5 to 12
Coefficient C† for the partly open gate . .	.650	.657	.660	.662	.663

From Lesbros's experiment, H. Smith, Jr., ‡ has derived the following two tables, XIII and XIV, to show the percentage increase in the value of C for different amounts of contraction.

TABLE XIII

SQUARE ORIFICE .656 FEET × .656 FEET

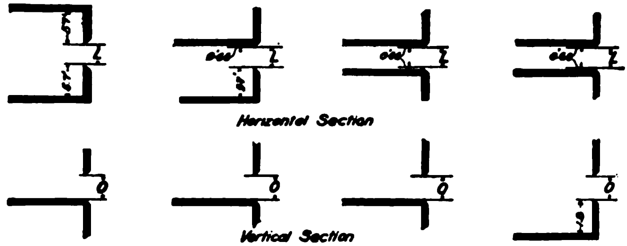
AMOUNT OF CONTRACTION	PERCENTAGE INCREASE IN C FOR HEADS			
	1	2	3	5
Contraction nearly suppressed on one side	1.2	1.0	1.2	1.3
Contraction entirely suppressed on one side . . .	3.8	3.3	3.1	3.5
Contraction nearly suppressed on two sides. . . .	5.7	4.5	4.0	4.0
Contraction entirely suppressed on one side; Contraction nearly suppressed on another side . .	5.8	5.3	5.3	5.6
Contraction entirely suppressed on two opposite sides	7.3	6.0	5.6	5.6
Contraction entirely suppressed on one side; Contraction nearly suppressed on two other sides . .	13.3	10.9	9.9	9.8
Contraction entirely suppressed on three sides . .	15.6	13.4	11.9	11.6

* Experimental value ; discharge determined volumetrically.

† Calculated by Darcy and Bazin from experiments.

‡ *Hydraulics*, p. 65.

TABLE XIV



Dimension of Orifice		Heads, Ft.			Heads, Ft.			Heads, Ft.			Heads, Ft.		
L. Ft.	O. Ft.	1	3	5	1	3	5	1	3	5	1	3	5
.656	.656	3.8	3.1	8.5	5.7	4.0	4.0	5.8	5.3	5.6	13.3	9.9	9.8
.656	.328	5.4	5.2	5.4	3.2	2.4	3.1	7.0	6.7	7.0	10.7	9.8	10.0
.656	.164	6.3	6.5	7.4	1.6	1.4	2.4	7.3	7.5	8.4	8.9	8.6	9.5
.656	.098	7.8	7.8	8.5	3.4	2.1	2.4	8.1	8.8	9.2	9.6	9.2	9.5
.656	0.33	8.9	11.1	12.4	4.5	5.1	5.6	8.5	11.1	12.2	8.5	11.1	12.7

From Bidoni's * experiments, the following rule has been derived for small orifices, with heads not exceeding 3 feet.

If S = the proportion of the perimeter of an orifice on which suppression occurs.

The percentage increase in C for rectangular orifices = 15.2 S.

The percentage increase in C for circular orifices = 12.8 S.

This formula is not precise; and S for rectangular orifices must be less than .75, and for circular orifices less than .85.

180. Submerged orifices. When the opening is entirely covered by water on the downstream face, the orifice is said to be submerged; and the head (h_c) becomes the difference in the elevation between the water surfaces above and below the orifices. See figure 50.

h_c' being the difference in elevation from the center of the orifice to the upstream water surface, and h_c'' to the downstream surface, Formula (9), neglecting velocity of approach, becomes

$$Q = Ca(2g)^{\frac{1}{2}}\left[h_c' - h_c'' + \frac{p_u - p_o}{\gamma}\right]^{\frac{1}{2}}. \qquad (17)$$

* D'Aubuisson, *Traité d'Hydraulique.*

If $p_u = p_o$, $\quad Q = Ca(2g)^{\frac{1}{2}}(h_c' - h_c'')^{\frac{1}{2}}$. (18)

The discharge for submerged orifices is slightly less than for orifices of similar size and form with free discharge under the same head. The difference according to Francis is probably as

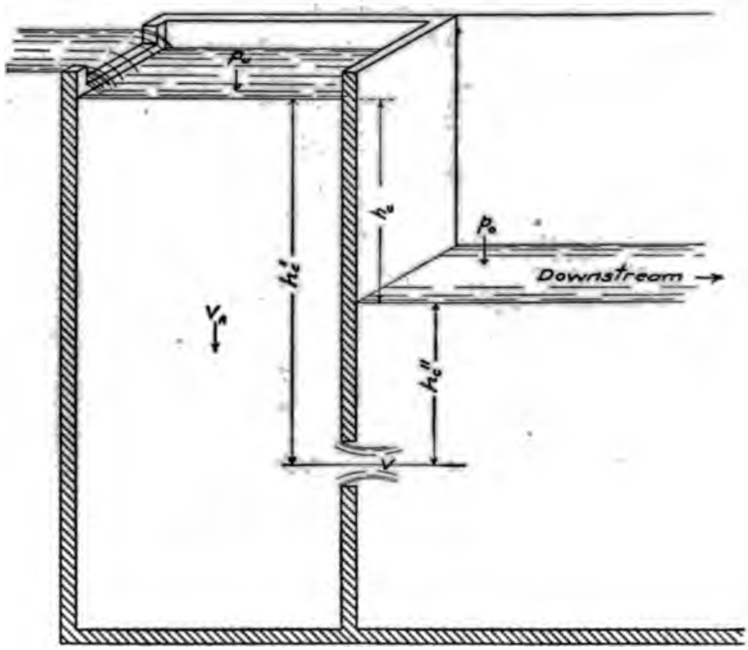


Fig. 50. — Submerged Orifice.

much as 3 per cent for very low heads, but for a head of 10 feet and over the difference is probably negligible, decreasing also with increase in the size of the opening. For orifices more than one square foot in area, and heads greater than three times the least dimensions, the available coefficients indicate that no correction is necessary.

A few coefficients for submerged orifices are given in Table XXI.

Example. A sharp-edged orifice 1.0 foot square is set with its center 12 feet lower than the upstream water surface, and 3 feet lower than the downstream water surface. Compute the discharge.

$Q = .604 \times 1^2 \times 8.02(12 - 3)^{\frac{1}{2}} = 14.53$ cubic feet per second.

81. Partially submerged orifices. For such openings the discharge can not be closely estimated except by special calibration for each case.

182. Discharge under a dropping or rising head. The time required either to empty or to fill a vessel through an orifice

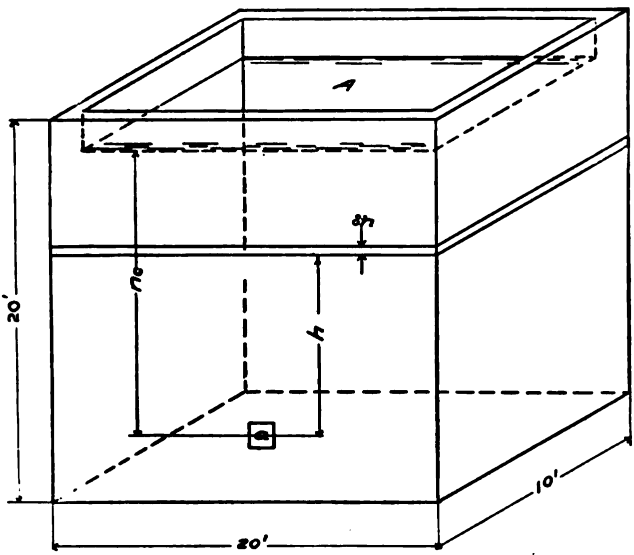


FIG. 51.

under a continuously varying head may be computed as follows. See figure 51.

Let $A=$ the cross-sectional area in square feet of a prismatic vessel.

The total volume above the center of the orifice $= Ah_c$.

Let $h=$ head at any instant. If, in any time δt, the head is lowered δh feet, then the volume discharged equals $A\delta h$, and the rate of discharge by formula (10) $= Ca(2gh)^{\frac{1}{2}}$.

Therefore, $$\delta t = \frac{A\delta h}{Ca(2gh)^{\frac{1}{2}}} = \frac{A}{Ca(2g)^{\frac{1}{2}}} \times \frac{\delta h}{(h)^{\frac{1}{2}}}. \tag{19}$$

Integrating (19) within the limits of h, corresponding to time $=0$ and time $=T$, viz., h_c and h_1,

$$\int_0^T \delta t = \frac{A}{Ca(2g)^{\frac{1}{2}}} \times \int_{h_1}^{h_c} \frac{\delta h}{(h)^{\frac{1}{2}}}. \quad \text{Therefore } T = \frac{2A}{Ca(2g)^{\frac{1}{2}}}(h_c^{\frac{1}{2}} - h_1^{\frac{1}{2}}). \tag{20}$$

For emptying the vessel down to the center of the orifice, th[at] is when $h_1 = 0$; $T = \frac{2\,A}{Ca(2\,g)^{\frac{1}{2}}}(h_e)^{\frac{1}{2}}$. (

Obviously this is just twice the time required to discharge [the] same volume of water under a constant head h_e.

Selection of coefficients for discharge under a dropping hea[d]. Usually to take the mean of the coefficients for the two limitin[g] heads is sufficient. If great accuracy is required, the interva[ls] of time required to lower the head by short drops in successio[n] may be separately computed; and their sum will be the tota[l] time of emptying.

Example. Given a rectangular prismatic vessel 10 feet × 20 feet × 20 feet (high), with an orifice in a thin wall .5 foot square, set with its lower edge 4 feet above the bottom. If flow begins when the water surface is 9 feet above the center of the orifice, what time will be required to lower the water surface 9 feet?

$$T = \frac{2 \times 200}{.598 \times .5 \times .5 \times 8.02}(9)^{\frac{1}{2}} = 1000 \text{ seconds.}$$

The time required to fill or empty a canal lock, if the orifices are submerged, may also be computed by formula (21), the head used being the difference between the head bay level and the tail bay level, which is usually called the lift.

Example. Given a lock chamber 80 feet × 30 feet; with one orifice 2 feet high by 4 feet long for filling, and one of the same size for emptying, the total lift being 9 feet. How many seconds will be required to fill the lock from tail bay level to head bay level? $C = .6$. See figure 52.

By Formula (21), $T = \frac{2 \times 30 \times 80 \times 3}{.6 \times 2 \times 4 \times 8.02} = 374$ seconds.

To find the time required to raise the water surface level from tail bay level to 5 feet above it.

By Formula (20),

$$T = \frac{2 \times 30 \times 80}{.6 \times 2 \times 4 \times 8.02}[(9)^{\frac{1}{2}} - (4)^{\frac{1}{2}}] = 124.7 \text{ seconds.}$$

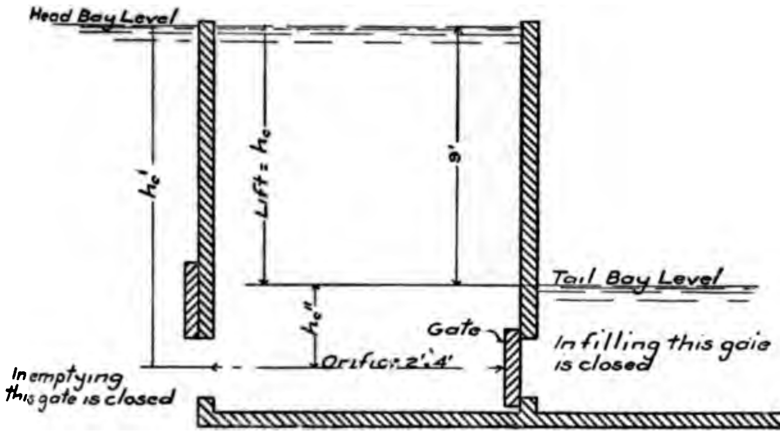


FIG. 52. — Submerged Discharge in a Lock.

183. **The miner's inch.** The miner's inch is the volume of flow of water that will be discharged through an opening one inch square under a head, not necessarily the same in every locality, but usually about .55 feet. The term, which was originally used somewhat loosely to measure water for mining purposes in the western part of the United States, has been in late years defined by law in equivalent cubic feet per second, ranging in value in different states from about .02 to nearly .03 cubic feet per second. It is an awkward term, but its use is often necessary on account of custom and law. See Table XX for coefficients for rectangular orifices under low heads.

184. **The loss of head** in a jet from an orifice may be determined theoretically as follows:

The actual velocity $V = C_v(2\,gH_c)^{\frac{1}{2}}$ (H_c being the total hydrostatic head).

Hence $$\frac{V^2}{2\,g} = C_v^2 H_c\text{; and } H_c = \frac{V^2}{2\,g C_v^2}.$$

Since if there were no loss $\dfrac{V^2}{2\,g}$ would equal H_c,

the lost head, $h_\lambda = H_c - C_v^2 H_c = (1 - C_v^2)H_c$; or in terms of velocity head

$$h_\lambda = \frac{V^2}{2\,gC_v^2} - \frac{V^2}{2\,g} = \frac{V^2}{2\,g}\left(\frac{1}{C_v^2} - 1\right) = \xi_o \frac{V^2}{2\,g}. \tag{22}$$

This expression is applicable to all forms of orifice.

Example. If $C_v = .975 \; h_\lambda = \frac{V^2}{2g}\left(\frac{1}{.975^2} - 1\right) = .052\frac{V^2}{2g}$,

$C_v =$	.995	.99	.985	.98	.975	.97	.95	.90	.82	.80	.75	.72	.70
$\xi_0 =$	.01	.02	.031	.041	.052	.063	.108	.235	.487	.563	.778	.929	1.04

It is fairly evident from a careful study of the discharge from orifices in a thin wall, especially in some recent experiments, that C_v is much nearer .99 than .975; and that the usual estimated allowance for loss in such orifices is too great.

TABLE XV

Smith's Coefficients of Discharge for Vertical Circular Orifices

Applicable strictly to vertical circular orifices in a flat, rigid, thin wall, with full contraction of the jet; and for the dimensions and heads given. Direct interpolation should be made for intermediate values. "For heads of over 100 feet use $C = .592$."

Head from Center of Orifice	Diameter of the Orifice in Feet												
	.02	.03	.04	.05	.07	.10	.12	.15	.20	.40	.60	.80	1.0
Feet													
(?) .3				.637	.628	.621	**.612**	**.607**					
.4			.637	.631	.624	.618	.612	**.605**					
.5		.643	.633	.627	.621	.615	.610	.605	**.599**	**.593**	**.585**		
.6	.655	.640	.630	.624	.618	.613	.609	.605	**.600**	**.594**	**.588**	**.581**	
.7	.651	.637	.628	.622	.616	.611	.607	.604	.601	**.595**	**.590**	**.585**	.580
.8	.648	.634	.626	.620	.615	.610	.606	.603	.601	**.596**	**.591**	**.587**	.583
.9	.646	.632	.624	.618	.613	.609	.605	.603	.601	**.597**	**.592**	**.589**	.585
1.0	.644	.631	.623	.617	.612	.608	.605	.603	.600	**.597**	**.593**	**.590**	.586
1.2	.641	.628	.620	.615	.610	.606	.604	.602	.600	**.598**	**.594**	**.592**	.589
1.4	.638	.625	.618	.613	.609	.605	.603	.601	.600	.599	**.595**	**.593**	.591
1.6	.636	.624	.617	.612	.608	.605	.602	.601	.600	.599	**.596**	**.594**	.592
1.8	.634	.622	.615	.611	.607	.604	.602	.601	.599	.599	.597	**.594**	.594
2.0	.632	.621	.614	.610	.607	.604	.601	.600	.599	.599	.597	**.595**	.594
2.5	.629	.619	.612	.608	.605	.603	.601	.600	.599	.599	.598	**.596**	.595
3.0	.627	.617	.611	.606	.604	.603	.601	.600	.599	.599	.598	.597	.596
3.5	.625	.616	.610	.606	.604	.602	.601	.600	.599	.599	.598	.597	.596
4.	.623	.614	.609	.605	.603	.602	.600	.599	.599	.598	.597	.597	.596
5.	.621	.613	.608	.605	.603	.601	.599	.599	.598	.598	.597	.596	.596
6.	.618	.611	.607	.604	.602	.600	.599	.599	.598	.598	.597	.596	.596
7.	.616	.609	.606	.603	.601	.600	.599	.599	.598	.598	.597	.596	.596
8.	.614	.608	.605	.603	.601	.600	.599	.598	.598	.597	.596	.596	.596
9.	.613	.607	.604	.602	.600	.599	.599	.598	.597	.597	.596	.596	.595
10.	.611	.606	.603	.601	.599	.598	.598	.597	.597	.597	.596	.596	.595
20.	.601	.600	.599	.598	.597	.596	.596	.596	.596	.596	.596	.595	.594
50. (?)	.596	.596	.595	.595	.594	.594	.594	.594	.594	.594	.594	.593	.593
100. (?)	.593	.593	.592	.592	.592	.592	.592	.592	.592	.592	.592	.592	.592

TABLE XVI

MAIR'S COEFFICIENTS OF DISCHARGE FOR VERTICAL CIRCULAR ORIFICES *

Square-edged clean-cut holes bored in thin gun-metal plates

Head from Centre of Orifice	Approximate Diameter of Orifice in Inches								
	1	1¼	1½	1¾	2	2¼	2½	2¾	3
	Absolute Area in Square Feet								
	.00546	.00852	.012281	.016749	.021806	.027576	.033898	.040933	.049139
Feet									
0.75	.616	.614	.616	.610	.616	.612	.607	.607	.609
1.00	.613	.612	.612	.611	.612	.611	.604	.608	.609
1.25	.613	.614	.610	.608	.612	.608	.605	.605	.606
1.50	.610	.612	.611	.606	.610	.607	.603	.607	.605
1.75	.612	.611	.611	.605	.611	.605	.604	.607	.605
2.00	.609	.613	.609	.606	.609	.606	.604	.604	.605

JUDD AND KING'S COEFFICIENTS OF DISCHARGE FOR VERTICAL CIRCULAR ORIFICES IN A THIN WALL †

Heads from Center of Orifice	Diameter of Orifice in Inches					
	¾	1	1½	2	2½	
Feet 4 to 94	.611	.610	.609	.608	.596	Average .607

WEISBACH'S COEFFICIENTS OF DISCHARGES FOR A VERTICAL CIRCULAR ORIFICE IN A THIN WALL. DIAMETER = .033 FEET (.1 *m*) ‡

Head, .066 ft.	C, .711	Head, 340 ft.	C, .600

SMITH'S COEFFICIENTS OF DISCHARGE FOR THE FLOW OF MERCURY AND OIL THROUGH A CIRCULAR ORIFICE IN A THIN WALL. DIAMETER = .02 FOOT §

Head in feet. . . .	.5	1.0	1.5	2.0	2.5	3.0	3.5
C for Mercury . . .	.618	.607	.601	.598	.597	.595	.594
C for Lubricating Oil	.750	.735	.728	.724	.721	.720	.718

* J. G. Mair, *Proceedings Institute of Civil Engineers*, vol. 84, p. 427.
† *Eng'g News*, 1906. ‡ *Weisbach's Mechanics*. § H. Smith, Jr., *Hydraulics*.

TABLE XVII

Smith's Coefficients of Discharge for Vertical Square Orifice

Applicable strictly only to vertical square orifices in a flat, rigid, thin wall, with f contraction of the jet and for dimensions and heads given. Direct interpolati should be made for intermediate values.

Head from Center of Orifice	Side of Square in Feet												
	.02	.03	.04	.05	.07	.10	.12	.15	.20	.40	.60	.80	1
Feet													
(?) .3				.642	.632	**.623**	**.616**	**.610**					
.4			.643	.637	.628	.621	**.615**	**.609**					
.5		.648	.639	.633	.625	.619	.614	**.609**	**.603**	**.597**	**.588**		
.6	.660	.645	.636	.630	.623	.617	.613	.610	**.604**	**.598**	**.592**	**.580**	
.7	.656	.642	.633	.628	.621	.616	.612	.609	.605	**.600**	**.594**	**.589**	.5
.8	.652	.639	.631	.625	.620	.615	.611	.608	.605	**.600**	**.596**	**.591**	.5
.9	.650	.637	.629	.623	.619	.614	.610	.608	.605	**.601**	**.598**	**.593**	.5
1.0	.648	.636	.628	.622	.618	.613	.610	.608	.605	**.602**	**.599**	**.595**	.5
1.2	.644	.633	.625	.620	.616	.611	.609	.607	.605	**.603**	**.600**	**.598**	.5
1.4	.642	.630	.623	.618	.614	.610	.608	.606	.605	.604	**.601**	**.599**	.5
1.6	.640	.628	.621	.617	.613	.609	.607	.606	.605	.605	**.601**	**.600**	.5
1.8	.638	.627	.620	.616	.612	.609	.607	.606	.605	.605	.603	**.601**	.6
2.0	.637	.626	.619	.615	.612	.608	.606	.606	.605	.605	.604	**.601**	.6
2.5	.634	.624	.617	.613	.610	.607	.606	.606	.605	.605	.604	**.602**	.6
3.0	.632	.622	.616	.612	.609	.607	.606	.606	.605	.605	.604	.603	.6
3.5	.630	.621	.615	.611	.609	.607	.606	.605	.605	.605	.604	.603	.6
4.0	.628	.619	.614	.610	.608	.606	.606	.605	.605	.605	.603	.603	.6
5.	.626	.617	.613	.610	.607	.606	.605	.605	.604	.604	.603	.602	.6
6.	.623	.616	.612	.609	.607	.605	.605	.605	.604	.604	.603	.602	.6
7.	.621	.615	.611	.608	.607	.605	.605	.604	.604	.604	.603	.602	.6
8.	.619	.613	.610	.608	.606	.605	.604	.604	.604	.603	.603	.602	.6
9.	.618	.612	.609	.607	.606	.604	.604	.604	.603	.603	.602	.602	.6
10.	.616	.611	.608	.606	.605	.604	.604	.603	.603	.603	.602	.602	.6
20.	.606	.605	.604	.603	.602	.602	.602	.602	.602	.601	.601	.601	.6
50. (?)	.602	.601	.601	.601	.601	.600	.600	.600	.600	.600	.599	.599	.5
100. (?)	.599	.598	.598	.598	.598	.598	.598	.598	.598	.598	.598	.598	[illegible]

"For heads over 100 feet use $C = .598$."
From Hamilton Smith, Jr., *Hydraulics*, p. 58.

TABLE XVIII

COEFFICIENTS OF DISCHARGE FOR VERTICAL RECTANGULAR ORIFICES IN A THIN WALL WITH FULL OR NEARLY FULL CONTRACTION. LONGER DIMENSION HORIZONTAL

Length in feet L	2	2	0.656	2.0	0.656	0.234	0.3	0.656	0.656	0.468	0.656	1.9685	4.17
Height in feet O	2	1	0.328	0.5	0.164	0.0585	0.05	0.0984	0.0656	0.0292	0.0328	0.0656	0.167
Ratio $\frac{L}{O}$	1 to 1	2 to 1	2 to 1	4 to 1	4 to 1	4 to 1	6 to 1	20 to 3	10 to 1	16 to 1	20 to 1	30 to 1	29 to 1
Authority	Ellis	Ellis	Poncelet and Lesbros	Ellis	Poncelet and Lesbros	Bovey	Smith	Poncelet and Lesbros	Poncelet and Lesbros	Bovey	Poncelet and Lesbros	Lesbros	Smith
Head in feet 0.5			.613		.635		.635	.635	.652		(.1) .690 (.5) .662	.634	
1.		.597	.616	.611	.631	.613	.627	.632	.645	.664	.652	.633	$h_c = .583$
2.	.611	.598	.617	.611	.629	.636	.620	.629	.638	.651	.641	.630	$C = .616$
3.	.597	.599	.615	.611	.627	.632	.618	.628	.634	.646	.634	.628	
4.	.606	.599	.613	.610	.624	.629	.617	.626	.629	.642	.627	.625	
5.		.597	.611	.609	.620	.628	.617	.623	.624	.639	.620	.623	
6.		.598	.610	.608	.617	.627				.637	.614	.620	
7.		.598		.608		.626				.636			
8.		.599		.607		.625				.635			
9.		.600		.607		.624				.634			
10.		.601		.606		.624				.633			
20.				.600		.621				.629			

TABLE XIX

COEFFICIENTS OF DISCHARGE FOR VERTICAL RECTANGULAR ORIFICES IN A THIN WALL, WITH FULL OR NEARLY FULL CONTRACTION
LONGER DIMENSION VERTICAL

Length (ft.) L	.0329	.0585	.0656	.0656	.0165	.0292	[illegible]56
Height (ft.) O	.132	.234	.164	.656	.2635	.468	1.[illegible]68
Ratio $\frac{L}{O}$	1 to 4	1 to 4	2 to 5	1 to 10	1 to 16	1 to 16	1 to 30
Authority	Bovey	Bovey	Lesbros	Lesbros	Bovey	Bovey	Les[illegible]ros
Head in feet							
1	[illegible]	[illegible]	645	.644	.671	.663	.6[illegible]0
2	[illegible]	[illegible]	.63[illegible]	.639	.657	.650	.6[illegible]5
3	[illegible]	[illegible]	.6[illegible]	.633	.649	.645	.6[illegible]9
4	[illegible]	[illegible]	[illegible]	.627	.643	.641	.6[illegible]8
5	[illegible]	[illegible]	[illegible]	.622	.639	.639	.6[illegible]6
6	[illegible]	[illegible]	[illegible]	.616	.636	.637	
7	[illegible]	[illegible]			.633	.636	
8	[illegible]	[illegible]			.632	.634	
9	[illegible]	[illegible]			.631	.633	
10	[illegible]	[illegible]			.629	.632	
20	[illegible]	.620			.622	.629	

TABLE XX

DRUMMOND'S COEFFICIENTS OF DISCHARGE FOR VERTICAL RECTANGULAR ORIFICES CUT WITH SQUARE EDGES IN THIN BRASS PLATES

Length in feet L	2") .167	2") .167	½") .042	2") .167	1") .083	4") .333	½") .042	1") .08[illegible]
Height in feet O	2") .167	½") .042	2") .167	1") .083	2") .167	½") .042	4") .333	1") .08[illegible]
Ratio $\frac{L}{O}$	1 to 1	4 to 1	1 to 4	2 to 1	1 to 2	8 to 1	1 to 8	1 to [illegible]
Authority	Thomas Drummond *							
Head								
6″ = 0.500′	.614	.634	.634	.621	.619	.637	.638	.621
7″ = 0.583′	.613	.634	.633	.619	.618	.637	.636	.619
8″ = 0.667′	.612	.632	.632	.618	.619	.634	.635	.618
10″ = 0.833′	.611	.628	.628	.616	.618	.634	.635	.617
12″ = 1.000′	.610	.627	.626	.615	.617	.631	.633	.615

* *Trans. Canadian Soc. C. E.*, vol. 14, 1900, p. 131.

TABLE XXI

Coefficients of Discharge for Submerged Orifices in a Thin Wall | **Partly Submerged Orifice in a Thin Wall**

Shape	Circle	Circle	Square	Square	Rectangle	Circle	Square	Square	Circle	Depth of Submergence, Feet
Dimensions in feet	$d = .05$	$d = .1$	.05 × .05	.1 × .1	$L = 3.0$ $O = .05$	$d = 1.0$	1. × 1.	4. × 4.	$d = 2.$	
Vertical (V) or Horizontal (H)	V	V	V	V	V	H	H	V	H	
Authority	H. Smith, Jr.					Ellis		Mead	Ellis	
Heads, feet										
.05								.631		
.1								.611		
.2				.607				.609		
.3		.600		.607				.614		
.4		.600	.609	.606						
.5	.599	.600	.609	.605	.621					
1.	.597	.600	.607	.604						
2.	.595	.599	.605	.603		.608	.598		.590	0
3.	.595	.598	.604	.604		.607	.600		.595	0
4.	.595	.598	.604	.604	.620	.607	.602		.600	.02
5.					.620	.606	.602		.606	.12
6.					.620	.605	.603		.610	.20
8.					.619	.602	.604		.612	.30
10.					.618	.600	.605		.615	.40
20.						.600	.605			

Problems

1. With a head of 300 feet in the supply pipe, what time would be require to put 2000 gallons of water into a steam boiler through a 3-inch diameter orifice 2 feet below the surface, when the gauge reads 60 pounds per square inch?

2. What head is required to discharge 360 cubic feet per minute through an orifice 4 inches high and 6 inches wide?

3. The injection orifice of a marine engine's condenser is 10 feet below sea level; the vacuum gauge on the condenser reads 24 inches. With what velocit will the water enter the opening? Specific gravity of sea water = 1.025.

4. The injection orifices of the jet condenser of a marine engine are 6 fee below the surface of sea; the vacuum gauge on the condenser reads 20 inches Compute the velocity of flow and area of a circular orifice needed to delive 200,000 gallons per hour.

5. A vertical tank has an orifice 3 inches in diameter, set 4.02 feet abov bottom. Head on orifice equals 49 feet. Find distance from bottom of tank to place where stream strikes the ground. $C_v = .975$.

6. Compute the discharge from a circular orifice 3 inches in diameter unde a head of 16 feet.

7. Compute the discharge from a rectangular orifice 2 feet wide by 1 fo high; head is 2 feet on top edge of orifice.

8. Compute the discharge through a circular orifice 9 inches in diamet under a head of 67 feet.

9. Compute the discharge through a rectangular orifice 1 foot wide by 1 feet high under a head of 16 feet.

10. Compute the discharge from a vertical orifice 6 inches high by 8 inch wide, when the head upon the upper edge is 31 feet.

11. Find the probable time necessary to empty a cylindrical tank, 12 fe in diameter and 20 feet high, through a circular orifice 8 inches in diamet placed in the side with its upper edge 1 foot above the bottom.

12. A prismatic basin 25 feet by 20 feet contains water 50 feet deep and h an opening 1 foot square in the side with its lower edge at the bottom. H long will be required to lower the water surface 43 feet?

13. Submerged orifice. Heads constant. Depth of water on one side feet; on other side 13 feet. Orifice 1 foot square. Compute the discharge.

14. A canal lock 400 feet long by 70 feet wide has a lift of 10 feet. Th are two sluices each 10 feet wide by 2 feet high. The head on the top edge the sluice is 12 feet. How long will it take to fill the lock?

15. The inner faces of the walls of a lock chamber have a batter of 1 i to 1 foot. Its length is 500 feet and its width at top of water is 80 feet. T two sluices for emptying are 8 feet wide by 2 feet high. The top of th opening is 20 feet below water in lock and 8 feet below water in canal bel How many minutes will emptying require?

CHAPTER X

ORIFICES (Continued)

MOUTHPIECES. SHORT TUBES. CONICAL TUBES. SLUICES

185. An orifice extended either inwardly or outwardly in a direction nearly parallel to the axis of the jet in a manner that will modify its flow is usually called a mouthpiece, a short tube, a conical tube, a sluice, or a nozzle, according to the shape and dimensions of the ajutage. Most of the types here mentioned, though illustrating important principles, have little value as measuring devices on account of the lack of precise coefficients, and because simpler forms of measuring apparatus can be used. Nozzles, however, are so useful as water-measuring devices, as well as for directing jets of water, that they will be separately discussed in Chapter XI.

186. Borda's mouthpiece. A short cylindrical tube projecting inwardly of such dimensions that the jet secures full contraction and does not again touch the tube is usually called "Borda's mouthpiece." The inner edge of the tube must be relatively thin; and its length should be about equal to the radius of the opening. See figure 53.

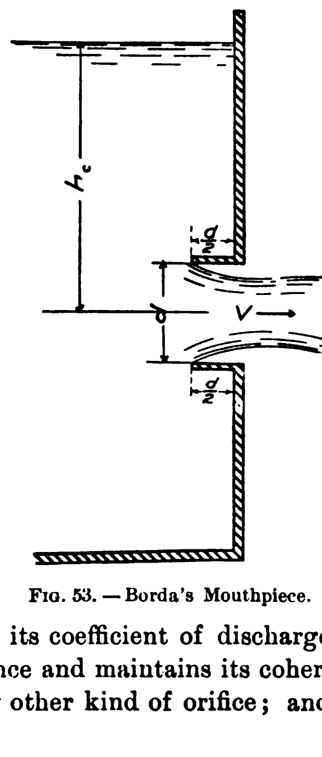


FIG. 53. — Borda's Mouthpiece.

The coefficient of velocity for such a tube is probably nearly .99; but the contraction of the jet is even more pronounced than for an orifice in a thin wall with full contraction; hence its coefficient of discharge is less. The jet is clearer in appearance and maintains its coherence for a longer distance than for any other kind of orifice; and

its coefficient of discharge is supposed to be nearly constant for any head. It is in effect an orifice in a thin wall.

There have been, however, but few experimental determinations of the coefficient of discharge; its mean value from the experiments of Borda, Bidoni, and Weisbach is .53. Hence

$$Q = .53\,\frac{\pi d^2}{4}(2\,gh_e)^{\frac{1}{2}}. \qquad (1)$$

187. The reëntrant short tube. If a cylindrical tube set as in figure 53 has a length of from 2 to 3 diameters instead of $\frac{1}{2}$ a diameter, the jet, after contracting, will again enlarge as in the standard short tube (§ 188) and fill the bore; and the discharge will be greater than for Borda's mouthpiece.

Coefficients of discharge. The mean value of the coefficient of discharge (C) for reëntrant tubes is usually given as about .72.

Recent experiments by H. J. I. Bilton * give the following values of C for heads from .5 foot upwards; higher limits not stated. Length of all tubes $2\frac{1}{2}$ diameters.

Diameter of tube in inches,	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$
Coefficient of discharge,	.91	.87	.85	.83	.81	.79	.77	.76	.75

Therefore $$Q = .72 \text{ to } .91(2\,gh_e)^{\frac{1}{2}}\,\frac{\pi d^2}{4}. \qquad (2)$$

The head lost in a reëntrant tube, taking $C = .72$. As the stream does not contract on leaving the tube, the coefficient of discharge is taken to be identical with the coefficient of velocity; then (see § 184),

$$h_h = \frac{V^2}{2\,g}\left(\frac{1}{.72^2} - 1\right) = .93\,\frac{V^2}{2\,g}.$$

188. Standard short tubes. A cylindrical tube with the entrance end set flush in a flat wall, having a length of from 2 to 3 diameters, is commonly called a standard short tube. Figure 54 shows such a tube.

Let V = velocity at EF;

V_c = velocity at the contracted section (CD);

p_0 = intensity of pressure in the tube at CD;

p_a = atmospheric pressure on water surface, and on EF.

* *Eng'g. News*, July 9, 1908.

D the stream is con- to about the same dimensions as if it discharging freely r from an orifice in wall; but if the of sufficient length, about 2 to 3 diam- he jet will, after con- ;, expand and again tube at the dis- end *EF*, thus sealing he atmosphere the round *CD*. The air ned in this space is re rarefied by the water, which acts ir pump. Thus the ;y of pressure (p_0) point being less than heric pressure, the V_c will be in excess velocity due to the

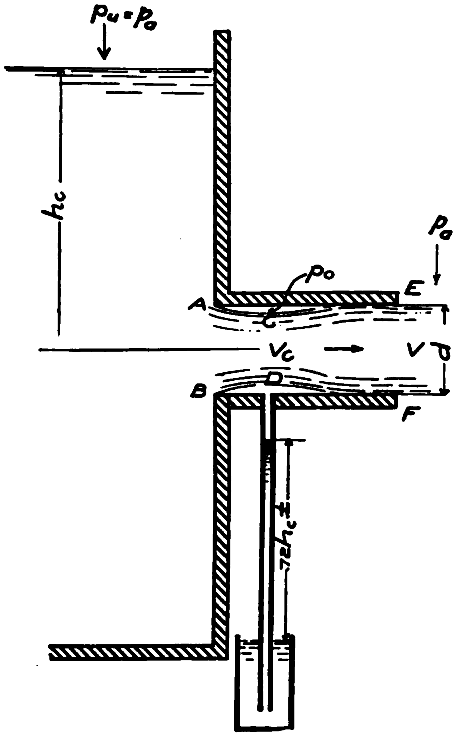


Fig. 54. — Standard Short Tube.

ed head h_c. V_c may be computed as follows:
Bernoulli's theorem

$$h_c + \frac{p_a}{\gamma} = \frac{V_c^2}{2g} + \frac{p_0}{\gamma} + \frac{V_c^2}{2g}\left(\frac{1}{C_v^2} - 1\right) = \frac{p_0}{\gamma} + \frac{V_c^2}{2gC_v^2}. \qquad \text{(A)}$$

ce

$$V_c = C_v\left[2g\left(h_c + \frac{p_a}{\gamma} - \frac{p_0}{\gamma}\right)\right]^{\frac{1}{2}}, \qquad \text{(B)}$$

$$Q = C_c a C_v\left[2g\left(h_c + \frac{p_a}{\gamma} - \frac{p_0}{\gamma}\right)\right]^{\frac{1}{2}} = C_c a V_c. \qquad \text{(C)}$$

ormula (C) does not include losses due to sudden enlarge- etween *CD* and *EF*, and as C_c and p_0 are difficult to deter- he simple orifice formula is commonly used; viz.:

$$Q = CaV = Ca(2gh_c)^{\frac{1}{2}}. \qquad \text{(D)}$$

coefficient of discharge for the tube as a whole.

Coefficients of discharge for short tubes have not been determined. Weisbach * gave values of C for tubes in length, under heads .75 to 1.97 feet as follows:

Diameters, feet,	.033	.066	.098
C,	.84	.83	.82

For a mean value of C, Weisbach gave .815. The v ally used is .82.

Hence $$Q = .82\frac{\pi d^2}{4}(2\,gh_c)^{\frac{1}{2}}.$$

The head lost in a short tube computing as for an ori

Head lost $h_\lambda = \frac{V^2}{2\,g}\left(\frac{1}{.82^2} - 1\right) = .487\,\frac{V^2}{2\,g}$; nearly enou

Since the stream on leaving the tube at EF has no c the coefficient of discharge is taken to be identical coefficient of velocity at EF, and so used in computin (§ 184).

This apparatus does not increase the actual energy; makes available a part of the energy which is unused the contraction of the jet.

The intensity of pressure p_0 can never be such as wil discharge through the contracted section to exceed $\frac{\pi c}{4}$ actually, it is always less, on account of losses due to f sudden change of section; nor can p_0 be less than 1 per square inch below atmospheric pressure. There is theoretic relation between h_c and p_0 which may be follows:

By Bernoulli's theorem

$$h_c + \frac{p_a}{\gamma} = \frac{V^2}{2\,g} + \frac{p_a}{\gamma} + \frac{V^2}{2\,g}\left(\frac{1}{C^2} - 1\right) = \frac{V^2}{2\,gC^2} + \frac{p_a}{\gamma}.$$

Hence $$\frac{V^2}{2\,gC^2} = h_c.$$

From (C) and (D), $$Q = C_c a V_c = a V = Ca(2\,gh_c$$

Then $$V_c = \frac{C}{C_c}(2\,gh_c)^{\frac{1}{2}};\ \text{and}\ \frac{V_c^2}{2\,g} = \left(\frac{C}{C_c}\right)^2 h_c.$$

* *Mechanics*, Coxe's Translation, p. 854.

From (A) and (E), $$\frac{p_0}{\gamma}+\frac{V_c^2}{2gC_v^2}=\frac{V^2}{2gC^2}+\frac{p_a}{\gamma}. \tag{H}$$

Substituting from (F) and (G), $$\frac{p_0}{\gamma}=\frac{p_a}{\gamma}+h_c-\left(\frac{C}{C_cC_v}\right)^2 h_c. \tag{5}$$

If $C=.82$, $C_c=.63$, and $C_v=.995$ (C_c and C_v apply to the area at CD), equation (5) becomes $\frac{p_0}{\gamma}=33.9-.72\,h_c$.

If a water barometer were inserted at CD, the water in its tube should rise to $.72\,h_c$. If the tube were shorter than $.72\,h_c$, the water would be sucked up through the tube and discharged at the outlet.

In experiments by Venturi and Weisbach, the water rose from .7 to .8 h_c.

A short tube has little practical value as a water-measuring device.

189. Long tubes, or short pipes. A similar contraction takes place at the entrance of tubes of greater length than 3 diameters; and the amount of the contraction will depend upon the form of the entrance. In general, for want of very precise information, the assumption must be made that the jet inside any pipe will form as if issuing into the air from an orifice shaped like the pipe entrance; from what is known of jets from orifices we may assume a suitable coefficient and compute the diameter of the contracted section, and therefrom deduce the lost head in entrance and enlargement. If the pipe is longer than 3 diameters, the friction head lost in the pipe must also be added to the head lost in the tube.

Then $$V=\left[h_c-\frac{V^2}{2g}\left(\frac{1}{C^2}-1\right)-f\frac{L}{D}\frac{V^2}{2g}\right]^{\frac{1}{2}}. \tag{6}$$

See Chapter XV for flow in pipes.

190. Bell-mouthed orifices. If the entrance to an orifice is gradually rounded, the contraction of the jet will be almost entirely suppressed; that is, C_c will be nearly unity; in any case the discharge will be materially increased.

The area of a bell-mouthed orifice is measured at its smaller or outlet end. For the orifice shown in figure 55, which was made to

conform as nearly as possible to the shape of a jet from ar in a thin wall,

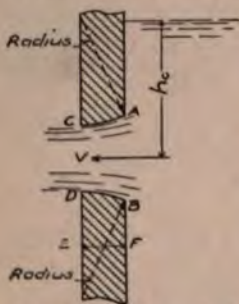


Fig. 55. — Bell-mouthed Orifice.

$AB = .042$ foot, $CD = .033$

$EF = .0165$ foot, radius $= .054$

Weisbach * found the following coe of discharge:

h_c in feet,	.066	1.64	11.48	55.77
C,	.959	.967	.975	.994

Compare these coefficients with th Weisbach's orifice of same diameter ir wall. (See Table XVI.)

The orifice shown in figure 56 is Ellis's † horizontal squ merged orifice with a wooden mouthpiece attached. The of the opening was a quarter ellipse of which the semidi were .5 foot (vertical) and .33 foot (horizontal).

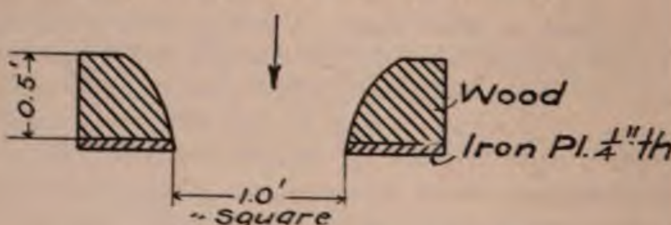


Fig. 56. — Horizontal Bell-mouthed Orifice.

The coefficients of discharge ‡ for this orifice were as foll

h_c in feet,	2	4	6	8	16	18
C,	.950	.948	.945	.943	.943	.944

Compare these with the coefficients for the same orifice without the mouthpiece. (See Table XXI.)

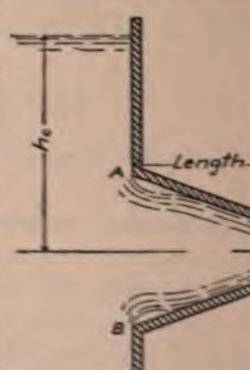


Fig. 57. — Conical Convergi

191. Conical converging tubes. See figure 57. Castel's experiments give

* Weisbach's *Mechanics*, Coxe's Translation, p. 821.

† *Trans. Am. Soc. C. E.*, 1876, Vol. 5, p. 61.

‡ Actual experimental values plotted; and a mean curve drawn from which these coefficients were taken.

the most complete set of coefficients for conical converging tubes. The quantity discharged in determining these coefficients was measured volumetrically. The area a was at CD, the smaller end. The following Table XXII contains part of Castel's coefficients for use in the formula:

$$Q = C\frac{\pi d^2}{4}(2gh_c)^{\frac{1}{2}}$$

TABLE XXII

CASTEL'S * COEFFICIENTS FOR CONVERGING TUBES

LENGTH = .131 FOOT		d = .05 FOOT	
a	C	a	C
0	.829	16° 36′	.938
1° 36′	.866	19° 28′	.924
3° 10′	.895	23° 00′	.914
4° 10′	.912	30° 00′	.895
5° 26′	.924	40° 20′	.870
7° 52′	.930	48° 50′	.847
13° 24′	.946		

LENGTH = .164 FOOT		d = .066 FOOT	
a	C	a	C
2° 50′	.914	13° 40′	.956
5° 26′	.930	15° 2′	.949
6° 54′	.938	18° 10′	.939
10° 30′	.945	23° 4′	.930
12° 10′	.950	35° 52′	.920

LENGTH = .328 FOOT		d = .066 FOOT	
a	C	a	C
11° 52′	.965	16° 35′	.951

Freeman's experiments. J. R. Freeman † made experiments on a nozzle attached as shown in figure 72, Chapter XI, except that the tin cone was removed. The discharge was measured volumetrically.

The coefficients of discharge determined were as follows:

Observed head h_c,	120.1	119.7	46.7
C,	.987	.985	.990

For further discussion of conical converging tubes, see Nozzles, Chapter XI.

192. **Experiments by Brownlee** ‡ on the tube shown in figure 58 gave the following results:

h_c,	1	2	4	6	8	10	12.5
C,	.941	.946	.959	.961	.963	.965	.966

* Taken from Weisbach's *Mechanics*.

† *Trans. Am. Soc. C. E.*, vol. 24, 1891, p. 499.

‡ *Inst. Engrs. and Shipbldrs. in Scotland*, vol. 19, p. 100.

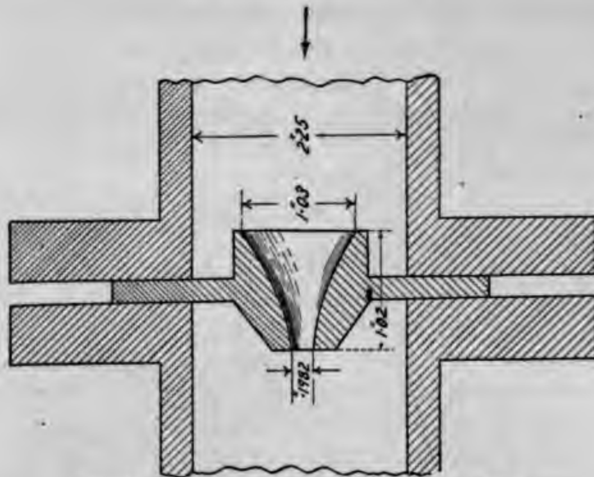
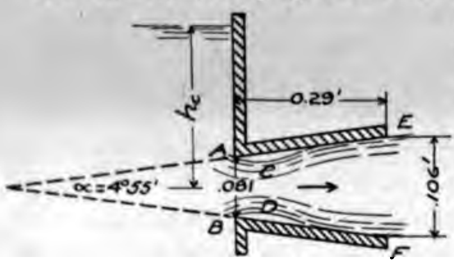


Fig. 58.—Conical Converging Tube set in a Pipe.

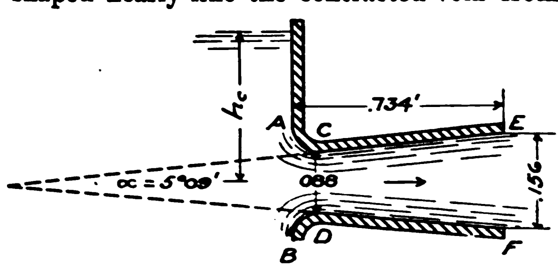
193. Conical diverging tubes. If a conical diverging tube has a sharp edge at entrance, the jet will contract in the tube just beyond the entrance; again expand; and if the angle of divergence is not too great, not more than about 8°, will again fill the tube at the outlet. Figure 59 shows a tube of this character, used by Weisbach.*



For $h_c = 0.82$ to 10.82 feet, taking for a the area at AB; $C = .946$.

Fig. 59.—Conical Diverging Tube.

If the entrance is shaped nearly like the contracted vein from an orifice in a thin wall, the stream will follow very closely the walls of the tube; and the discharge will be greater than when the inlet edges are square.



Taking as a the area at CD; $C = 1.55$.

Fig. 60.—Conical Diverging Tube.

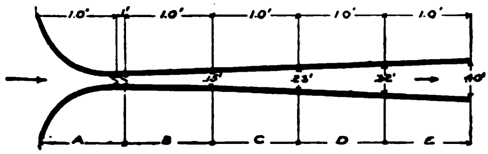
Figure 60 shows a tube used by Eytelwein (taken from Weisbach).†

* Weisbach's *Mechanics*, Coxe's Translation, p. 861. † *Ibid.*, p. 862.

In either of the above two cases the intensity of pressure at the contracted section is less than atmospheric; and the discharge is usually greater than would be computed for a given head on the center of the contracted section if the area of that section is used in computing the discharge, and less than if the area at the large end (*EF*) is called the area.

194. Compound tubes. The following figure and table of coefficients illustrate the effect of a partial vacuum at the contracted section upon the discharge from a tube; the area of the tube was measured at its smallest section.

The tube shown in figure 61 is from experiments of J. B. Francis.* The smallest diameter was .1 foot.



Parts	Head, Ft.	C
A	.088 to 1.51	0.81 to 0.94
A & *B*	.020 " 1.47	1.15 " 1.59
AB & *C*	.014 " 1.81	1.42 " 2.16
ABC & *D*	.014 " 1.86	1.88 " 2.48
Whole tube	.014 " 1.41	1.44 " 2.42

All for diameters = 0.1 foot.

Fig. 61. — Francis's Compound Tube.

"Boyden's diffuser" for outward flow turbines is a practical application of the phenomenon observed in these tubes, and serves the same purpose for an outward flow turbine that a draft tube does for an inward or mixed flow; that is, it acts as an air pump at the discharge orifices of the runner. See figure 167.

195. Sluices in open channels. Sluice gates in open channels in which the area of the opening is nearly as large as the cross-sectional area of the channel have been proved to be unsatisfactory devices for measuring flow, because of the relatively great effect of velocity of approach, and of partial and unknown suppression of contraction. Few coefficients are available, and these are usually not applicable to other than the opening for which they were obtained.

For any actual sluice satisfactory coefficients may be obtained for different positions of the gate, and for variations of upstream and downstream surface water levels, by current meter or rod float or other measurements.

The following examples will illustrate the wide divergence in coefficients for sluices.

* *Lowell Hydraulic Experiments,* 4th ed., pp. 212 *et seq.*

Bellasis * gives the following coefficients for sluice gates:

Kind of Opening	Description	Width of Opening	Height of Opening	Head	(
Sluice See figure 62	As shown with boards *CF* or *DE* added	2.0 ft. Same	1.31 to 0.10 ft. Same	.33 to 9.8 ft. measured from upper edge	.61 t (aver .64 t (aver Max. : 1 ft.
Sluice	In wood 1.77 ft. thick at bottom, 0.87 ft. elsewhere	4.265 4.265	1.7 ft. 0.39 ft.	6 to 14 ft. measured from center	.6: .8(
Iron Gates	Working in masonry heads No contraction at bottom and sides	4 to 10 ft.	3 to 2 ft.	.25 to 4.8	.72 t (aver

Weisbach gives the sluices shown in figure 63.

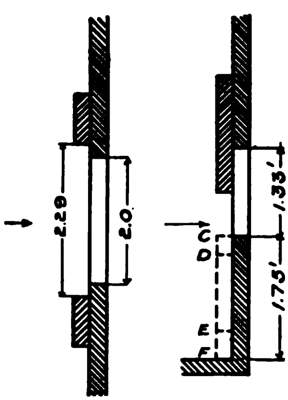


Fig. 62.

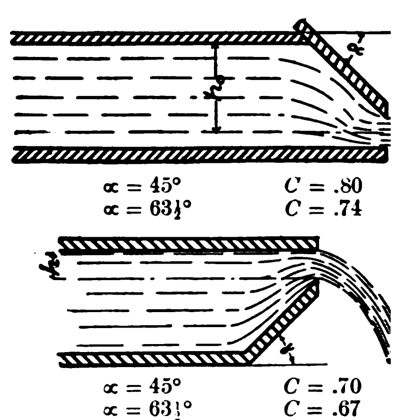


Fig. 63.

* *Hydraulics*, p. 68.

TABLE XXIII

merged Sluice. VALUE OF COEFFICIENTS* OF DISCHARGE FOR FLOW THROUGH HORIZONTAL SUBMERGED TUBE, 4 FEET SQUARE, FOR VARIOUS LENGTHS, LOSSES OF HEAD, AND FORMS OF ENTRANCE AND OUTLET

ES OF HEAD IN FEET	FORMS OF ENTRANCE AND OUTLET	LENGTH OF TUBE, IN FEET						
		0.31	0.62	1.25	2.50	5.00	10.0	14.0
.05	*A*	.631	.650	.672	.769	.807	.824	.838
	a	.672			.742	.810		.848
	b	.740			.769	.832		.862
	c	.834			.769	.875		.890
	c′							.875
	d	.948			.943	.940	.927	.931
.10	*A*	.611	.631	.647	.718	.763	.780	.795
	a	.636			.698	.771		.801
	b	.685			.718	.791		.813
	c	.772			.718	.828		.841
	c′							.830
	d	.932			.911	.809	.892	.893
.15	*A*	.609	.628	.644	.708	.758	.779	.794
	a	.630			.689	.767		.803
	b	.677			.708	.787		.814
	c	.765			.708	.828		.839
	c′							.829
	d	.936			.910	.899	.893	.894
.20	*A*	.609	.630	.647	.711	.768	.794	.809
	a	.632			.694	.777		.819
	b	.678			.711	.796		.833
	c	.771			.711	.838		.856
	c′							.846
	d	.948			.923	.911	.906	.905
.25	*A*	.610	.634	.652	.720	.782	.812	.828
	a	.634			.705	.790		
	b	.683			.720	.809		
	c	.779			.720	.854		
	c′							
	d	.965			.938	.928		
.30	*A*	.614	.639	.660	.731	.796	.832	.850
	a	.639						
	b	.689						
	c	.788						
	c′							
	d	.980						

Results of experiments made at the hydraulic laboratory of the University of Wisconsin by C. B. Stewart, on the discharge through orifices and tubes four feet square and of various thicknesses or lengths and with various conditions of contraction. In computing *C*, velocity of approach was neglected.

* Mead's *Water Power Engineering*, p. 45.

M

3. Compute the discharge through a short tube 2 inches in diameter under a head of 2 feet. Determine the loss of head. Compute the intensity of pressure inside the tube around the contracted area.

4. If the coefficient of contraction (C_c) of the contracted section of the vein discharging through a short tube is .68, and C_v is .98, find negative pressure head in terms of h_c.

5. Compute the discharge through the bell-mouthed horizontal orifice 1 foot square under a head of 10 feet; also compute for an orifice in a thin wall of the same dimensions, and under the same head.

6. Compute the discharge through a bell-mouthed orifice (figure 55) .033 foot in diameter, under a head of 100 feet, and compare this with the discharge through an orifice in a thin wall of like dimensions, and under the same head.

7. Compute the discharge under a head of 4 feet through a conical converging tube ½ inch diameter having the convergence that will give the maximum discharge.

8. A tube shaped somewhat like Francis's compound tube (figure 61) has the following dimensions: diameter of inlet, 1 foot; diameter of throat, .2 foot; diameter at outlet, .8 foot. The length from inlet to throat is 3 feet, and from throat to outlet 9 feet. Determine the mean velocity in the throat, the intensity of pressure in the throat, and the probable discharge through the tube, if the head is 16 feet.

9. A vertical bell-mouthed pipe, such as shown in figure 64, has a head of 1¾ inches. Compute the discharge.

CHAPTER XI

NOZZLES; FLOW OF WATER THROUGH FIRE HOSE

197. A nozzle is a conical converging tube attached to the outlet end of a pipe having the same internal diameter as the base of the nozzle.

Forms of nozzles. Nozzles are commonly classified as smooth and ring nozzles.

Smooth nozzles are gradually tapering bores having no interior constrictions to cause contraction of the jet. Figure 65 shows a smooth nozzle of the best type; its coefficient C is about .977.

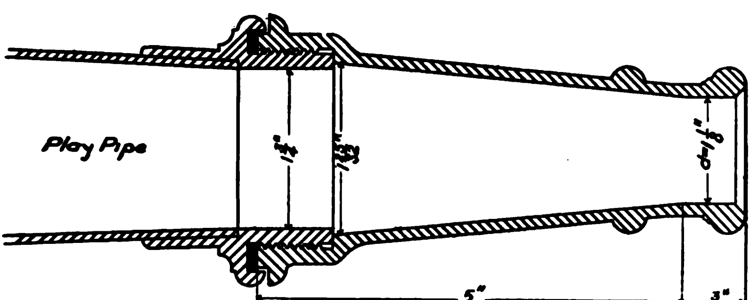


FIG. 65. — Smooth Nozzle.

Ring nozzles are gradually tapering bores into which are fitted orifices or short tubes which are not only of smaller diameter than the smaller end of the bores, but which, on account of their form, will cause marked contraction of the jets as they leave the nozzles. The coefficients of discharge depend upon the contraction of the jet, which depends upon the form of the edge of the orifice, and the ratio of its area to that of the nozzle just behind the ring. As the relative area of the hole is decreased, the coefficients decrease (see Table XXVI), until, if the diameter of hole and channel of approach have a ratio of about 1 to 7, the nozzle virtually becomes an orifice in a thin wall.

Figure 66 illustrates a square ring nozzle, for which $C = .7$–

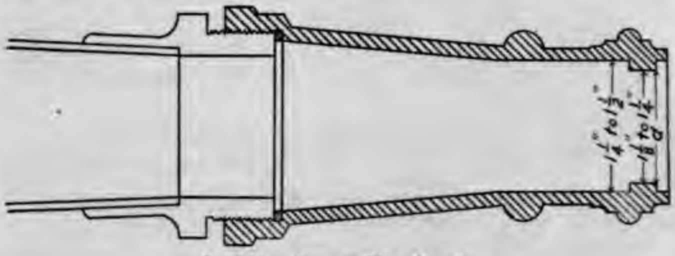


FIG. 66. — Square Ring Nozzle.

Figure 67 illustrates an ordinary knife-edge or undercut ri nozzle, for which $C = .713$.

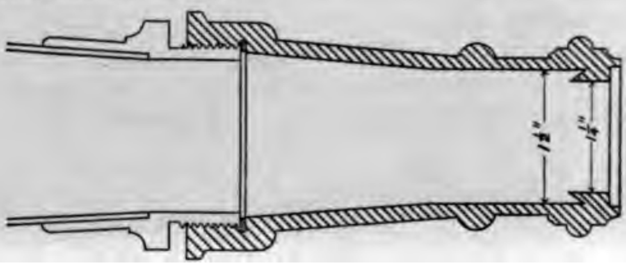


FIG. 67.— Undercut Ring Nozzle.

Figure 68 illustrates an extreme form of knife-edge or undercut ring nozzle; practically a Borda's mouthpiece fitted into : ring nozzle. The coefficient of this pai ticular nozzle is .582.

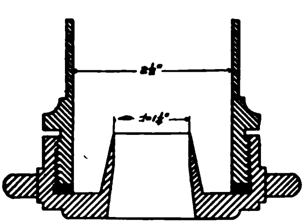


FIG. 68. — Extreme Form of Undercut Ring Nozzle.

Nozzles are merely special forms c orifices in which the contraction of th jet is wholly or partially suppressed Orifice formulas apply, therefore, t nozzles; but must include the effect c the head due to velocity of approacl which is usually a considerable part c the total head.

198. Formula for discharge in terms of the pressure head an the diameters at the base and orifice of the nozzle may be derive from Bernoulli's theorem as follows.

Figure 69 shows a nozzle arranged for measuring discharge.

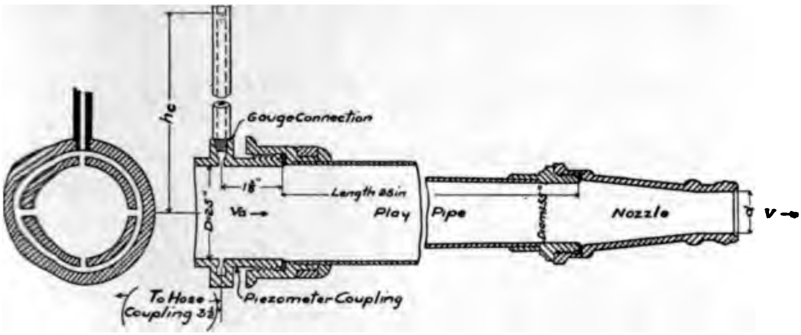


FIG. 69.—Nozzle with Play Pipe and Piezometer Arranged for Measuring Discharge.

Let D = the diameter in feet of the pipe at the section where the pressure head is measured; viz. just upstream from the base (entrance) of the nozzle or of the play pipe if one is used.

A = the area in square feet corresponding to diameter D.

d = the diameter in feet of the nozzle orifice.

a = the area in square feet corresponding to diameter d.

V = the actual velocity in feet per second at the smallest section of the jet.

V_t = the theoretic velocity in feet per second corresponding to the total head.

V_A = the actual velocity in feet per second in the section where the pressure head is measured; *velocity of approach*. On figure 69 written V_a.

C = coefficient of discharge = $C_v C_c$.

C_v = coefficient of velocity.

C_c = coefficient of contraction.

h_c = piezometer reading, in feet of water, at the base of the nozzle, reduced to the elevation of the center of the nozzle orifice.

p_c = piezometer reading in pounds per square inch corresponding to h_c.

H_c = piezometer reading (h_c) + velocity head $\left(\frac{V_A^2}{2g}\right)$; the total hydrostatic head (in feet).

$p = \left(h_c + \frac{V_A^2}{2g}\right).433$ = total head in pounds per square inch.

Q = the actual discharge in cubic feet per second.

By Bernoulli's theorem, first neglecting irrecoverable loss energy,

$$\frac{p_a}{.433} + h_c + \frac{V_A^2}{2g} = \frac{p_a}{.433} + \frac{V_t^2}{2g} = \frac{p_a}{.433} + H_c.$$

But if the discharge is steady

$$Q = AV_A = C_c a C_v V_t = CaV_t.$$

Therefore $$V_A = \frac{CaV_t}{A} = \frac{Cd^2V_t}{D^2}.$$

Substitute in (1) the value of V_A from (3),

Then $$h_c + \frac{C^2d^4V_t^2}{D^4\,2g} = \frac{V_t^2}{2g}.$$

Transposing,

$$\frac{V_t^2}{2g} = \frac{h_c}{1 - C^2\frac{d^4}{D^4}} = H_c; \text{ and } V_t = \left[\frac{2gh_c}{1 - C^2\frac{d^4}{D^4}}\right]^{\frac{1}{2}} = (2gH_c)^{\frac{1}{2}}.$$

Therefore, the actual velocity

$$V = C_v\left[\frac{2gh_c}{1 - C^2\frac{d^4}{D^4}}\right]^{\frac{1}{2}} = C_v(2gH_c)^{\frac{1}{2}}.$$

And the actual discharge

$$Q = C\,8.02\,\frac{\pi d^2}{4}\left[\frac{h_c}{1 - C^2\frac{d^4}{D^4}}\right]^{\frac{1}{2}} = C\,6.3\,d^2\left[\frac{h_c}{1 - C^2\frac{d^4}{D^4}}\right]^{\frac{1}{2}}$$
$$= C\,6.3\,d^2(H_c)^{\frac{1}{2}}.$$

Note. In § 211 are given certain formulas in terms of diameters in i and other units common to fire stream practice.

Experimental coefficients of discharge have been determine Hamilton Smith, Jr., Ellis, Weston, Freeman, and others. experiments previous to Freeman's were limited to a few of many types of nozzles in use.

199. Freeman's Experiments.* In 1888 John R. Freeman ducted at Lawrence, Mass., an extensive set of experimer upon the discharge of water through fire hose, and through no

* *Trans. Am. Soc. C. E.*, Vol. 21, pp. 303–482.

† For the Associated Factory Mutual Insurance Companies.

7/8 to 1½ inches in diameter, of many types. His observations were made with a high degree of precision, and settled many important questions concerning fire stream discharge. The chief subjects investigated were:

Comparative efficiency of different kinds of nozzles.

Coefficients of discharge of nozzles.

Loss of pressure due to friction in various kinds of fire hose.

Loss of pressure due to curves in fire hose.

Loss of pressure due to reduction of waterway at couplings.

The effects of area and form of nozzle and of variations in pressure, upon the height and distance reached by jets.

Distribution of velocities in jets.

In 1890* Freeman made a second set of experiments upon smooth cone nozzles 1¾, 2, and 2½ inches in diameter to determine the coefficients of discharge, and demonstrate the value of nozzles as water meters.

200. **The arrangement of the apparatus** used in both series by Freeman was essentially as shown in figure 70. This was modified somewhat in different experiments.

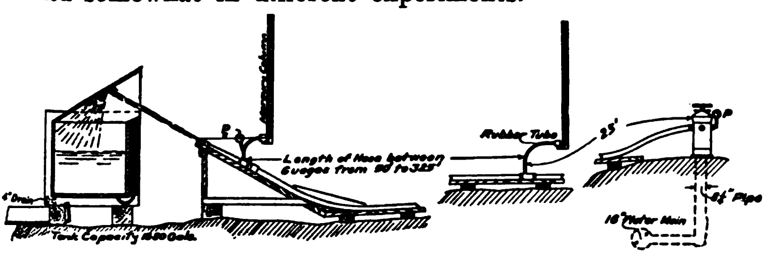


FIG. 70. — Layout of Apparatus in Freeman's Experiments.

The arrangements of nozzles for the 1890 experiments are shown in figures 71 and 72.

At B in figure 71 in some observations a screen of wire netting was inserted; in others it was absent. Also three longitudinal diaphragms or rifle-blades about 16 inches long were inserted in some experiments, in others not. Again the pipe BC was for some observations removed and a piezometer coupling only about three inches long with six equally spaced holes was attached directly to the siamese coupling. None of these changes

* *Trans. Am. Soc. C. E.*, Vol. 24, pp. 492–527.

affected the discharge coefficients more than about 1 per cent and usually much less.

In other experiments the nozzles were detached from the pipe

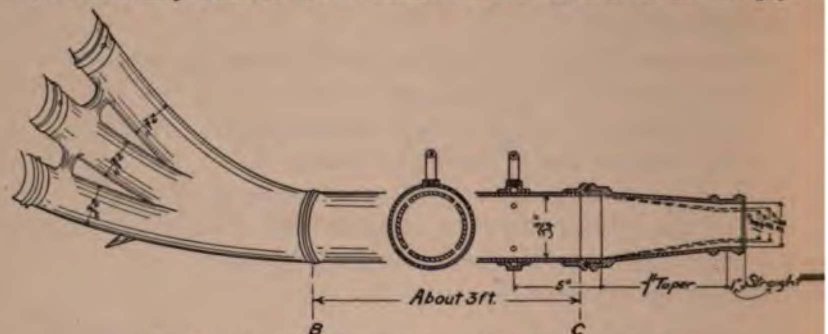


FIG. 71. — Nozzle and Piezometer on a Siamese Coupling.

and siamese, and attached to a cast-iron cylinder, as shown in figure 72, with a view to eliminating possible error due to the high velocity past the piezometer.

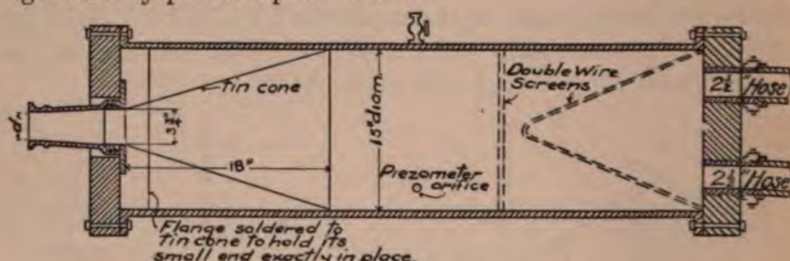


FIG. 72. — Nozzle Set on an Iron Cylinder.

201. The quantities discharged were measured by volume in a rigid wooden tank lined with sheet zinc. The error in measurements of quantity probably did not exceed $\frac{1}{3}$ of 1 per cent.

The time of each experiment was determined by a stop watch in the first series, and an electric chronograph in the second set. The duration of each experiment was controlled by deflecting the nozzle by hand. The error in time of filling the tank probably did not exceed $\frac{1}{5}$ of 1 per cent.

The discharges in gallons per minute in the first set of experiments were probably measured with an error not exceeding $\frac{1}{2}$ of 1 per cent; and the average error was less. In the second set the limit of error was about $\frac{1}{10}$ of 1 per cent.

202. Measurement of pressure or head. The gauges used for measuring intensities of pressure were open mercury columns in glass tubes 20 feet in height and $\frac{3}{16}$-inch internal diameter. Measurements were checked by Bourdon gauges; and for some rougher determinations they were used alone. Figure 73 shows one of the mercury gauges.

"These mercury columns were each 20 feet in height, reading to about 117 pounds per square inch pressure. Their scales were graduated to read directly in pounds per square inch, and to include the correction for lowering of surface of mercury in cistern.

"The mercury was procured from Mr. Huddleston, the well-known barometer maker, who stated it to be pure. The value 13.563 for specific gravity of mercury at 60 degrees Fahrenheit was adopted. (Authority, Regnault, P. A., 62–50. See Clark's *Constants of Nature.*) The weight of a cubic inch of water at 0 degrees C. was taken as 0.03610 pound, on authority of J. B. Francis, *Lowell Hydraulic Experiments*, page 29; and, therefore, by comparison we derive weight of 1 cubic inch of mercury at 60 degrees Fahrenheit = 0.4896 pound. Height of mercury column corresponding to 1 pound per square inch = 2.0425 inches. The diameter of glass tube was about $\frac{3}{16}$ inch or about the same as for an ordinary barometer and diameter of cistern 3 inches.

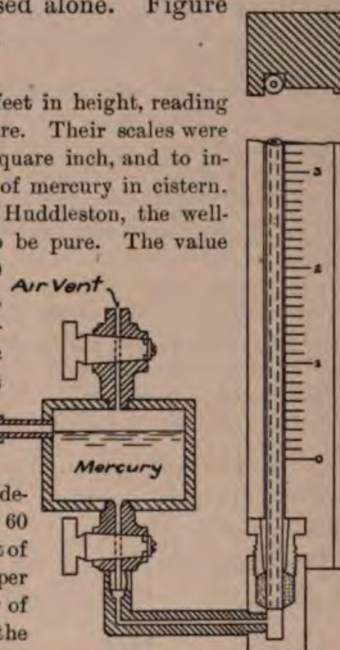


Fig. 73. — Mercury Gauge.

"Therefore, to compensate for lowering of mercury surface in cistern, scale of this instrument was graduated with 1 pound per square inch = 2.0343 inches.

"Error due to capillarity in the $\frac{3}{16}$-inch tube was avoided by setting scale to read zero when pressure was zero, amount of mercury in cistern having been previously adjusted so that its surface was just at level of entrance into cistern of tube transmitting the pressure. Since there was occasionally a slight loss of mercury while forcing water through tubes and cistern, for making sure of absence of air bubbles in tubes, note was made before and after each set of experiments of reading of mercury column with pressure removed, and mercury was added to bring column up to the zero level if necessary, or a slight correction made to the observed reading.

"For the benefit of future experimenters in hydraulics, I would say that these semiportable mercury columns proved very satisfactory in their operation. Very little of the difficulty of leaky joints noted by M. Darcy was met with, mainly because we profited by his experience, used a less cumbrous form

of apparatus, and avoided joints wherever possible, making glass tube in onl two pieces, and joining these by heavy rubber tube containing cloth insertion tightly wired on. Joints in iron fittings need to be very much closer to hol mercury than to hold water, and as the ordinary taper-screw pipe joint will b found to give trouble, nothing but an accurate 'shoulder fit' should be use Cisterns were made of wrought iron with welded joints, and interior of cister and iron connections was japanned after putting together, to increase tightne and prevent rust.

"As stated above, the graduation assumed the diameter of the glass tubes t be $\frac{3}{16}$ inch. These tubes were afterwards accurately calibrated by notin weight of mercury required to fill them to certain heights, and correspondin corrections deduced. For gauge A maximum correction at 100 pounds = − .(pound, and for gauge B + .01 pound. This result was exactly confirmed b placing the two gauges side by side, and subjecting both to identical pressur through nearly their whole range.

"In a gauge of this construction the lowering of the surface of mercury i cistern lowers the datum of the gauge; but the variation due to this cause f 50 pounds pressure, for instance, would be only .02 pound per square inc and thus almost inappreciable. Variation of 20 degrees Fahrenheit in temper ture of mercury, from 60 to 80 degrees, would affect indication of gauge onl about $\frac{1}{1000}$ part, while it would affect a water column $\frac{3}{1000}$."

All things considered, it would seem reasonable to expect thes mercury gauges, including corrections applied, to measure a pre sure correctly within $\frac{1}{10}$ of 1 per cent.

Piezometer couplings were used whenever the internal pressur in a pipe was to be measured. Such a coupling, frequently calle a piezometer ring should be used i precise determina tions of pressur are desired. It ca be connected to an form of gauge Figure 74 show the coupling use in the first set of experiments, and figure 71 shows those used i the second set. Still another was used similar to that in 71, bu shorter and having six holes instead of four.

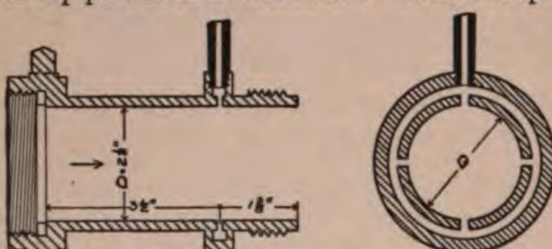


FIG. 74. — A Piezometer Coupling.

Measurement of loss of pressure. The fire hose was laid on straight smooth plank walk in lengths of 50 to 325 feet, usuall 150 to 300. One piezometer coupling with gauge was locate about 25 feet from the hydrant, another at the base of the pla

pipe. (See figure 70.) The difference in pressure between the two gauges was observed for various quantities of water flowing through the hose as measured in the tank. This difference in pressure, for any given flow, when corrected for differences in elevation between gauges and hose, is the head or pressure lost; here expressed in pounds per square inch. The total difference divided by the length of hose between piezometers is the head or pressure lost per linear foot of hose, which included the loss in couplings.

203. Comparative efficiency of various nozzles. The smooth cone nozzle with a simple play pipe, as shown in figure 69, was, for throwing streams and measuring flow, found superior to any other type.

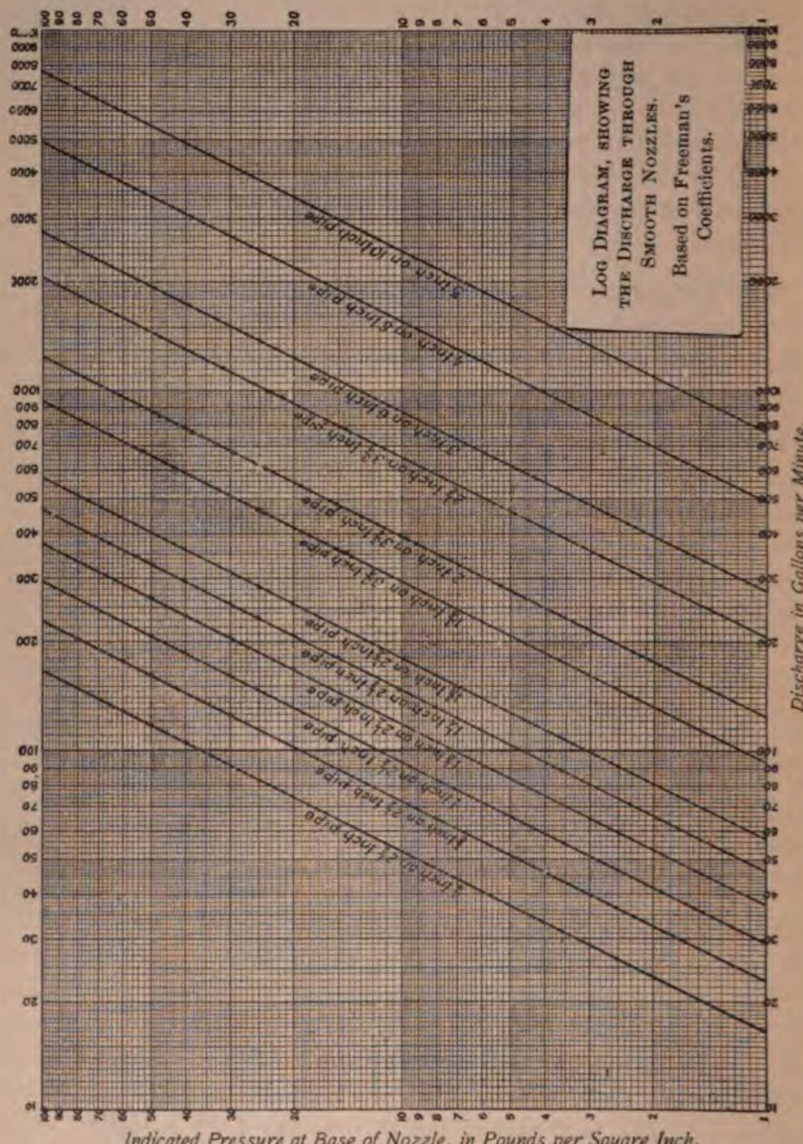
204. Determination of coefficients of discharge. The coefficients of discharge were found by dividing the actual discharge in gallons per minute by 29.83 $d^2(p^{\frac{1}{2}})$, d is in inches (§ 211).

Coefficients of discharge for smooth nozzles. Even though the nozzles were of different interior form and length, the coefficients are nearly uniform. The nozzles used in the experiments of 1888 were selected at random from stock. Those used in 1890 were especially made, but were of about the same quality.

Figure 75 is a logarithmic diagram showing the relation between indicated pressure and discharge for various cone nozzles.

For smooth cone nozzles of various sizes, the coefficients given in Table XXV were determined. The coefficients are applicable with or without a play pipe if the pipe and the nozzle base are of the same diameter and free from restrictions or enlargements.

205. For ring nozzles the contraction varied; therefore the coefficient changed with every variation in the ratio of nozzle orifice area to the area of the nozzle just behind the ring. The coefficients derived from Freeman's experiments on ring nozzles were graphically shown by a curve from which the values in Table XXVI were read, which, until more complete experiments are presented, may be found of use in indicating the probable value of coefficients for similar cases. Freeman's experiments included many other forms of nozzles, but the results are too voluminous to be given.



Log Diagram, showing the Discharge through Smooth Nozzles.
Based on Freeman's Coefficients.
Discharge in Gallons per Minute.
Indicated Pressure at Base of Nozzle, in Pounds per Square Inch.

FIG. 75.

TABLE XXV

Coefficients of Discharge for Smooth Nozzles

Nominal Diameter, Inches	Actual Diameter, Inches	Number of Observations	Observed Pressures, Pounds Sq. In. at Base of Play Pipe	Discharge Measured in Tank, Gals. per Minute	Coefficients of Discharge (Mean Value)	Remarks on Arrangement of Nozzles and Connections
			Experiments of 1888			
0.75	0.751	4	25 to 60	83.4 to 129.3	.983	As shown in figures 69 and 70
0.875	0.880	7	30 to 80	125.0 to 205.2	.982	As shown in figures 69 and 70
1.000	1.001	3	30 to 50	161.0 to 208.5	.972	As shown in figures 69 and 70
1.125	1.126	15	20 to 50	169.0 to 266.5	.976	As shown in figures 69 and 70
1.25	1.250	18	15 to 51	180.4 to 344.2	.971	As shown in figures 69 and 70
1.375	1.375	12	20 to 30	252.0 to 310.6	.959* (.977)	As shown in figures 69 and 70 (If larger play pipe were used)
	1.55	3	12 to 20	261.0 to 338.5	.976	Play pipe without nozzle of size shown in figure 69
			Experiments of 1890			
1.75	1.748	13	14.16 to 56.42	350.4 to 699.6	.999	Connected to pipe or siamese by piezometer coupling (figure 71) in the various ways described in § 200
2.00	1.998	17	14.88 to 48.62	474.2 to 864.8	.996	
2.50	2.4995	13	13.80 to 31.25	763.6 to 1156.0	.997	
1.75	1.748	5	20.14 to 57.48	406.1 to 689.3	.995	Connected directly to cast-iron cylinder shown in figure 72
2.00	1.998	9	5.03 to 51.45	265.5 to 850.	.995	
2.50	2.4995	5	19.91 to 37.86	820.7 to 1132.0	.987	
For large sizes up to 6-inch; heads more than 10 feet; and angle of convergence 10° to 15°					.995	Suggested by Freeman

* Throat of play pipe was too small for nozzle of this size; hence low coefficient.

Coefficients for square ring nozzles are given in Table XXVI. Values of C printed in italics are below the limits of Freeman's experiments.

TABLE XXVI

$$\text{Ratio} = \frac{\text{Diameter of outlet of nozzle}}{\text{Diameter of nozzle area just behind the ring}}.$$

Ratio	.1	.2	.3	.4	.5	.6	.7	.8	.85	.9	.95	1.0
C	*.60*	*.61*	*.61*	*.62*	.63	.65	.68	.71	.73	.77	.87	.975

206. Loss of pressure due to friction in hose. The character of the interior surface of the hose as to smoothness is of the utmost importance in determining friction losses; in unlined linen hose the loss per foot was found to be about 2.5 times that for smooth rubber hose of the same diameter in which the friction loss was least.

The precise diameter of the hose must be used in calculating velocities in hose (or pipes), as Freeman found considerable difference between the nominal and actual diameters of hose.

The friction loss varied nearly, but not precisely, as the square of the velocity of flow.

The following Table XXVII summarizes the head lost in friction with different quantities of discharge for various kinds of hose; also gives coefficients C_z for use in the Chezy formula for flow of water in pipes. C_z is used here only to distinguish it from the nozzle coefficient.

The letters in Table XXVII refer to the following kinds of hose: *

A, solid rubber, very smooth. C, woven cotton, rubber-lined, a heavy regular fire hose, interior surface almost free from ridges. D, nearly the same as C, but of lighter weight. E, knit cotton, rubber-lined, medium smooth. I, like E, but rougher. M, same as D, but of smaller diameter. K, woven cotton, rubber-lined, cheap, with medium thin lining, when under pressure interior is full of small ridges. L, unlined linen hose, good quality.

207. Loss of pressure due to curves in fire hose. For fire hose laid in its ordinary smooth curves, but not cramped or kinked, the friction loss will be about 6 per cent greater than in perfectly straight hose.

* See photographs of casts of interior surface and a more complete description of each kind, on pages 347 to 353 in Freeman's original article.

TABLE XXVII — FRICTION HEAD IN FIRE HOSE

Kind of Hose	Nominal Diameter, Inches	Actual Average Diameter, Inches Δ	Diameter of Couplings, Inches	Mean Velocity in Hose, Ft. per Sec. V	Discharge, Gallons per Minute G	Lost Head per 100 Ft. Length Corrected for Expansion. Lbs. per Sq. In.	S Lost Head per Foot of Length in Feet	C_s in Chezy Formula $V=C_s\sqrt{RS}$	Mean Value f = Lost Head per 100 Ft. $G=240$ $\Delta=2\frac{1}{2}$ Lbs. per Sq. In.
A	2.5	2.65	2.52	12.50	214.9	8.06	.1863	123.3	13.4
				13.96	240.0	10.00	.2311	123.6	
				17.00	292.2	14.73	.3404	124.0	
				20.00	343.8	20.40	.4714	124.0	
C	2.5	2.47	2.53	13.40	200.1	10.66	.2464	119.1	14.1
				16.07	240.0	15.00	.3467	120.4	
				20.00	298.7	22.80	.5269	121.5	
D	2.5	2.49	2.47	13.20	200.3	10.50	.2427	117.7	14.2
				15.81	240.0	14.50	.3351	120.0	
				19.00	288.4	20.48	.4733	121.3	
				21.00	318.7	24.70	.5708	122.1	
E	2.5	2.68	2.50	7.50	131.9	3.50	.0809	111.6	16.0
				10.00	175.8	6.23	.1440	111.6	
				13.65	240.0	11.30	.2611	112.3	
				17.00	298.9	17.01	.3931	114.8	
I	2.5	2.69	2.51	11.50	203.7	10.20	.2357	100.1	19.4
				13.55	240.0	13.50	.3120	101.7	
				16.00	283.4	18.96	.4382	102.2	
				18.00	318.9	22.35	.5165	105.8	
M	2.0	2.12	2.07	14.00	154.0	14.92	.3448	113.4	14.6
				17.00	187.0	21.19	.4897	114.8	
				21.81	240.0	33.20	.7673	118.4	
K	2.5	2.53	2.48	3.50	54.8	1.13	.0261	94.3	25.5
				5.00	78.4	2.52	.0582	90.3	
				7.50	117.5	5.99	.1384	87.8	
				10.00	156.7	10.60	.2450	88.0	
				12.00	188.0	14.82	.3425	89.3	
				15.31	240.0	24.10	.5570	89.3	
				17.00	266.4	29.20	.6748	90.1	
				19.00	297.7	35.76	.8264	91.0	
L	2.5	2.60	2.50	3.50	57.9	1.79	.0414	73.9	33.2
				5.00	82.7	3.63	.0839	74.1	
				7.50	124.1	7.99	.1846	74.9	
				10.00	165.5	13.50	.3120	76.9	
				14.50	240.0	27.20	.6286	78.5	$f=\dfrac{204600}{C_s^2}$
				17.00	281.3	37.03	.8558	78.9	
				19.00	331.0	50.30	1.1624	79.6	

The following table summarizes the effect in increasing hea
when hose was laid in curves of 2, 3, or 4 feet radii.

TABLE XXVIII—LOSS OF HEAD DUE TO CURVES

LENGTH OF HOSE, 57.2 FEET

Kind of Hose	Actual Diameter, Inches	Mean Flow, G, reduced to	Loss in Head for Flow in Col. 3, Pounds per Sq. In.	Increased Loss in Head due to Curvatures	Increased Loss in Head divided by Velocity Head	Degrees of Curvatures	Alignment except as note Straight
Sample D	2.49	258	10.04	0.47	0.24	180°	4 curves, 2 feet radius, eac
Sample D	2.49	293	12.66	0.44	0.17	180°	45°, each 2 feet straigh
Sample D	2.49	323	15.18	0.61	0.20	180°	between each curve
Sample D	2.49	258	10.13	0.56	0.29	360°	1 circle, 2 feet radius t
Sample D	2.49	293	12.90	0.68	0.27	360°	center of hose
Sample D	2.49	323	15.40	0.83	0.27	360°	
Sample D	2.49	258	10.23	0.66	0.34	360°	4 curves, 2 feet radius, 90°
Sample D	2.49	293	12.96	0.74	0.29	360°	each, 2 feet straight be-
Sample D	2.49	323	15.33	0.76	0.25	360°	tween each curve (X)
Sample D	2.49	258	10.33	0.76	0.39	360°	4 curves, 3 feet radius, 90°
Sample D	2.49	293	12.95	0.73	0.29	360°	each, 3 feet straight be-
Sample D	2.49	323	15.62	1.05	0.24	360°	tween each curve (Y)
Sample D	2.49	258	10.43	0.86	0.44	360°	4 curves, 4 feet radius, 90°
Sample D	2.49	293	13.19	0.97	0.39	360°	each, 4 feet straight be
Sample D	2.49	323	15.73	1.15	0.38	360°	teen each curve (Z)
Sample A	2.64	304	9.04	0.49	0.23	360°	Like X
Sample A	2.64	304	9.28	0.73	0.35	360°	Like Y
Sample A	2.64	304	9.32	0.77	0.37	360°	Like Z

208. The loss of head due to reduction of the area of the strea
at the couplings was found to be about equal to the sum of th
loss in sudden contraction on entering the bushing, and the lo
in sudden enlargement in leaving the bushing (see Chapter XV

For example: for a $2\frac{1}{4}$-inch hose coupling on a $2\frac{1}{2}$-inch hose f
a discharge of 240 gallons per minute the loss would be about .
pound per bushing, or since hose is usually in 50-foot lengths, .
pound per 100 feet.

For washers smaller than the hose the loss is about 1.25 tin

the loss of head due to sudden contraction of stream, assuming the coefficients of contraction as for ring nozzles (see Chapter XV).

For example: a $2\frac{1}{4}$-inch washer in a $2\frac{1}{2}$-inch hose discharging 200 gallons per minute would cause a loss of 1.04 pounds per 100 feet of hose; and when discharging 300 gallons per minute a loss of 2.34 pounds per 100 feet. With a 2-inch washer the loss would be 4.16 pounds in the first case, and 9.34 pounds in the second case.

209. **Vertical and horizontal distance attained by jets from nozzles.** Theoretically, the height attained should be equal to the pressure head plus the velocity head; actually it is less. For nozzles up to $1\frac{1}{2}$ inches with the same head the larger nozzles will give a higher jet. For horizontal distance, see Table XXIX.

The height J of the extreme drops in still air from nozzles $\frac{3}{4}$ to $1\frac{3}{8}$ inches diameter may be computed by the following formula:

$$J = H_c - .00135 \frac{H_c^2}{C_c^{\frac{1}{2}} d}. \quad (d \text{ is in inches.})$$

The height of a thoroughly first-class fire stream will be:

When J in feet equals	50	75	100	125	150
Height in per cent of J	82 %	79 %	73 %	67 %	63 %

210. **Distribution of velocity in jets from a nozzle.** By experiments with a Pitot tube the distribution of velocity was found to be uniform throughout the jet from the best nozzles except near the periphery of the jet. The actual velocity on the central portion of the jet was equal to the full theoretic velocity due to the head H_c.

211. **Freeman's formulas.** In the following formulas derived by Mr. Freeman for use in computing fire stream discharge, the symbols used were as follows; all diameters of pipes and orifices are given in inches. Other symbols are identical with those given in § 198.

P = intensity of pressure at the top of the hydrant in pounds per square inch.

If the nozzle is higher than the top of the hydrant, subtract from the hydrant pressure the difference in elevation in feet multiplied by .433; if the nozzle is lower, add the difference.

C = coefficient of discharge of hydrant nipple: for shar[p] square corner, .82; for well-rounded entrance, 1.0; fo[r] ordinary hydrant the value determined was .71.

L = length of the hose in feet.

d = diameter nozzle orifice in inches.

D = diameter of channel where pressure is measured in inche[s].

D_h = diameter of hydrant nipple in inches.

F = the loss of head per 100 feet of hose with 240 gallo[ns] per minute flowing for any given kind of hose; depen[ds] upon diameter, roughness, obstructions, and curv[es] pounds per square inch.

For unlined linen hose $2\frac{1}{2}$ inches diameter $F = 30$.
Inferior rubber-lined hose $2\frac{1}{2}$ inches diameter $F = 26$.
Smoothest rubber-lined hose $2\frac{1}{2}$ inches diameter $F = 13$

$$F = \frac{97.66}{\Delta^5} f + 1 + .5, \text{ for 100 feet of hose, when } G = 240.$$

f = friction loss in pounds per square inch, for 100 feet, $G = 240$, and $\Delta = 2\frac{1}{2}$; values are given in Table XXVII.

The pressure in pounds per square inch lost per 100 feet i[n] sinuosities is about 1 pound, when $G = 240$.

The pressure in pounds per square inch lost per 100 feet fo[r] ordinary obstructions is about .5 pound, when $G = 240$.

Δ = actual diameter of hose in inches.

G = discharge in gallons of 231 cubic inches per minute.

Given d, D, and h_e. Required Q.

$$Q = .04374\ Cd^2\left[\frac{h_e}{1 - C^2\left(\frac{d}{D}\right)^4}\right]^{\frac{1}{2}} \tag{8}$$

Example. Given, a $1\frac{1}{4}$-inch smooth nozzle attached to $2\frac{1}{2}$ pipe and the observed head in a piezometer at the base of the nozzle 100 feet. Compute the discharge in cubic feet per second.

$$Q = .04374 \times .971 \times 1.25^2\left[\frac{100}{1 - .971^2\left(\frac{1.25}{2.50}\right)^4}\right]^{\frac{1}{2}} = .684.$$

Given d, D, and H_e. Required Q.

$$Q = .04374\ Cd^2(H_e)^{\frac{1}{2}}; \text{ where } H_e = \frac{h_e}{1 - C^2\left(\frac{d}{D}\right)^4}. \tag{9}$$

Example. The total head at the base of a 1¼-inch smooth nozzle attached to 2½-inch pipe is 106.26 feet. Compute the discharge in cubic feet per second.

$$Q = .04374 \times .971 \times 1.25^2\,(106.26)^{\frac{1}{2}} = .684.$$

Given d, D, and p_c. Required Q.

$$Q = .06645\ Cd^2\left[\frac{p_c}{1 - C^2\left(\frac{d}{D}\right)^4}\right]^{\frac{1}{2}}. \tag{10}$$

Example. Given a 1¼-inch smooth nozzle attached to a 2½-inch pipe, the pressure on a Bourdon gauge at the base of the nozzle being 43.3 pounds. Compute the discharge in cubic feet per second.

$$Q = .06645 \times .971 \times 1.25^2\left[\frac{43.3}{1 - .971^2\left(\frac{1.25}{2.50}\right)^4}\right]^{\frac{1}{2}} = .684.$$

Given d, D, and p_c. Required G.

$$G = 29.83\ Cd^2\left[\frac{p_c}{1 - C^2\left(\frac{d}{D}\right)^4}\right]^{\frac{1}{2}}. \tag{11}$$

Example. Given same conditions, as in the last preceding example, compute the discharge in gallons per minute.

$$G = 29.83 \times .971 \times 1.25^2\left[\frac{43.3}{1 - .971^2\left(\frac{1.25}{2.50}\right)^4}\right]^{\frac{1}{2}} = 307.$$

Given d, D, and p. Required G.

$$G = 29.83\ Cd^2(p)^{\frac{1}{2}}; \text{ where } p = \frac{p_c}{1 - C^2\left(\frac{d}{D}\right)^4}. \tag{12}$$

Given the hydrant pressure (P). Required the nozzle pressure (p).

$$p = \frac{P}{C^2 d^4\left[\frac{1}{D_h^4}\left(\frac{1}{C^2} - 1\right) + \frac{LF}{6472}\right] + 1}. \tag{13}$$

From which also P may be found if p is given.

For lengths greater than 200 feet, and nozzles not larger than 1⅛ inches, the following less precise formulas may be used; p being thus found, the discharge may be computed.

For smooth nozzles,

$$(C = .974);\ p = \frac{P}{\frac{LFd^4}{6822} + 1}. \qquad (1$$

For square nozzles,

$$(C = .74);\ p = \frac{P}{\frac{LFd^4}{11819} + 1}. \qquad (1$$

Example. The pressure at a fire hydrant is 100 pounds p square inch. To the hydrant is attached 300 feet of best quali rubber-lined $2\frac{1}{2}$-inch hose. Compute the pressure at the base the nozzle (*a*) with a $1\frac{1}{4}$-inch smooth nozzle.

$$p = \frac{100}{\frac{300 \times 13 \times 1.25^4}{6822} + 1} = 42.$$

(*b*) With a $1\frac{1}{4}$-inch square ring nozzle.

$$p = \frac{100}{\frac{300 \times 13 \times 1.25^4}{11819} + 1} = 55.4.$$

Discharge of hose with open butts, that is, hose without nozzle o play-pipe.

$$P = G^2\left(\frac{.001123}{D_h^4 O^2} + .0000001736\,LF\right). \qquad (16$$

$$G = \left[\frac{P}{\frac{.001123}{D_h^4 O^2} + .0000001736\,LF}\right]^{\frac{1}{2}}. \qquad (17$$

Example. Compute the discharge through 400 feet of unline $2\frac{1}{2}$-inch linen hose with open butt, which is attached to a fi hydrant in which the pressure is 100 pounds per square incl Let $D_h = 2\frac{1}{2}$ and $O = .82$.

$$G = \left[\frac{100}{\frac{.001123}{2.5^4 \times .82^2} + .0000001736 \times 400 \times 30}\right]^{\frac{1}{2}} = 217.$$

212. Application. Nozzles may be used satisfactorily to mea ure the discharge from pipes under pressure wherever a pipe c

be installed of sufficient length to establish uniform flow before the water reaches the nozzle. Nozzles are especially useful in measuring discharge through pipes, fire hose, or in testing the performance of pumps or impulse water wheels. Such a test involves in many cases the loss of all the water measured; and in all cases, except in the case of impulse wheels in service, a waste of the energy of the jet. With nozzles to which Freeman's coefficients are applicable, discharge measurements can be made with an error not exceeding 2 per cent. The apparatus necessary to make measurements of this kind is easily transported, and the expense of making them is slight.

Fire stream table. Table XXIX, which is for the most part taken from a similar table by Freeman, will give results sufficiently accurate for fire stream computations. The values of F used are given on p. 180; for intermediate values interpolate directly. Values of G vary as the square root of p_e or p; values in other columns vary directly as p_e or p.

Examples. (*a*) Given $p_e = 80$, for a $1\frac{1}{8}$-inch smooth nozzle attached to 400 feet of best quality rubber-lined hose. What pressure P is required at the hydrant, assuming the nozzle orifice and hydrant top to be at the same elevation?

From table, the lost head for first 100 feet $= 112$; for each additional 100 feet, 25.5. Therefore $P = 112 + 3 \times 25.5 = 188$ pounds per square inch.

(*b*) Given a 1-inch smooth nozzle with 300 feet of the best quality rubber-lined hose, $p_e = 50$, what P will be required at the hydrant?

By direct interpolation between 40 and 60 pounds the loss for the first 100 feet as shown by the table is 62.5; for each additional 100 feet the loss is 9.8, hence:

$$P = 62.5 + 2 \times 9.8 = 82.1 \text{ pounds per square inch.}$$

(*c*) What is the discharge through a $1\frac{3}{8}$-inch ring nozzle under 70 pounds indicated pressure? The discharge is approximately proportional to the square root of p_e or p. From table, for 100 pounds $G = 429$.

Then for 70 pounds, $G = (\frac{70}{100})^{\frac{1}{2}} \times 429 = .837 \times 429 = 359$.

TABLE XXIX

Fire Stream Table

This table was computed by Freeman's formulas. Nozzles are assumed to be attac to 2½-inch play pipe, and hose 2½ inches diameter.

p_e	p	Vertical Jet		Hor. Dist. Jet Elevated 30° to 45°		G	Pounds Pressure Required at Hydrant to Maintain Pressure (p) at Nozzle					
							For First 100 Ft.			Each Additional 100		
Indicated (Gauge) Pressure at Base Play Pipe	Effective (Static) Pressure at Base Play Pipe	Av. Highest Drops, Still Air	Height Good Fire Stream Modified by Wind	Av. Extreme Drops Level of Nozzle Still Air	Limit of Good Effective Fire Stream, Still Air	Gallons per Minute Discharged	Unlined Linen Hose	Inferior Rubber-lined Cotton Hose Rough Inside	Best Quality Rubber-lined Cotton Hose Smooth Inside	Unlined Linen Hose	Inferior Rubber-lined Cotton Hose Rough Inside	Best Quality Rubber-lined
$C = .974$ ¾-inch Smooth Nozzle or ⅞-inch Ring Nozzle												
Lbs. per Sq. In.		Ft.	Ft.	Ft.	Ft.		Lbs. per Square Inch					
20	20.2	40	33	72	29	73	23	23	22	2.8	2.4	
40	40.3	78	60	112	40	104	46	46	43	5.6	4.9	
60	60.5	104	72	136	54	127	69	68	65	8.4	7.3	
80	80.7	123	79	153	62	147	93	91	86	11.2	9.7	
100	100.8	134	83	167	68	164	116	114	108	14	12.1	
$C = .974$ ⅞-inch Smooth Nozzle or 1-inch Ring Nozzle												
20	20.3	41	34	74	33	100	26	25	23	5.2	4.5	
40	40.6	81	62	124	49	142	52	50	46	10.6	9.2	
60	60.9	112	77	153	61	174	78	75	69	15.9	13.7	
80	81.2	132	85	172	70	201	103	101	91	21.2	18.3	
100	101.5	144	90	186	76	224	129	126	114	26.3	22.7	
$C = .974$ 1-inch Smooth Nozzle												
20	20.5	43	35	77	37	132	30	29	25	9.1	7.9	
40	41.0	83	64	133	55	186	60	58	50	18.0	15.7	
60	61.5	117	79	167	67	228	90	87	75	27.0	23.4	
80	82.1	140	89	189	76	263	120	115	100	36.0	31.3	
100	102.6	152	96	205	83	295	150	144	125	45.3	39.2	
$C = .974$ 1⅛-inch Smooth Nozzle												
20	20.8	43	36	80	38	168	36	34	28	14.7	12.8	
40	41.7	84	65	142	59	238	73	69	56	29.5	25.6	
60	62.5	122	83	178	72	291	109	103	84	44.1	38.3	
80	83.3	146	92	203	81	336	145	138	112	59.0	51.1	
100	104.1	158	99	224	89	376	182	172	140	73.6	64.0	

Fire Stream Table — (*Continued*)

$C = .974$ $1\frac{1}{4}$-inch Smooth Nozzle

Lbs. per Sq. In.		Ft.	Ft.	Ft.	Ft.		Lbs. per Square Inch					
20	21.2	44	37	83	40	209	45	42	32	22.7	19.8	9.9
40	42.4	86	67	148	63	296	91	84	65	45.6	39.6	19.8
60	63.7	126	85	186	76	363	136	127	97	68.6	59.6	29.8
80	84.9	150	95	213	85	419	181	169	129	91.5	79.5	39.7
100	106.1	161	101	236	93	468	226	211	162	114	98.7	49.5

$C = .974$ $1\frac{3}{8}$-inch Smooth Nozzle

20	21.8	45	38	85	42	257	58	53	39	34.5	29.9	14.9
40	43.7	89	69	152	66	363	116	107	77	68.6	59.7	29.8
60	65.5	131	87	192	79	445	174	160	116	104.0	89.7	44.8
80	87.3	154	97	220	88	514	232	214	154	137.9	119.7	59.8
100	109.1	165	103	243	96	574	285	267	193	161.6	149.0	74.5

$C = .74$ $1\frac{1}{8}$-inch Ring Nozzle

20	20.5	42	34	76	36	126	29	28	25	8.3	7.2	3.6
40	40.9	82	62	131	53	179	59	56	49	16.7	14.5	7.2
60	61.4	115	76	164	65	219	88	84	74	25.0	21.7	10.8
80	81.8	138	86	186	73	253	117	113	98	33.4	29.0	14.5
100	102.3	150	93	202	80	283	146	141	123	41.7	36.3	18.1

$C = .74$ $1\frac{1}{4}$-inch Ring Nozzle

20	20.7	43	35	78	37	157	34	33	27	12.8	11.2	5.6
40	41.4	83	63	138	55	222	69	65	54	25.5	22.3	11.1
60	62.1	119	79	172	68	272	103	98	81	38.3	33.5	16.7
80	82.8	143	88	196	77	314	137	130	108	51.4	44.7	22.3
100	103.5	154	95	215	84	351	171	163	135	64.1	55.8	27.0

$C = .74$ $1\frac{3}{8}$-inch Ring Nozzle

20	21.1	43	36	79	38	192	41	39	30	19.2	16.7	8.3
40	42.1	85	64	144	59	271	82	77	61	38.3	33.3	16.6
60	63.2	123	81	180	72	332	124	116	91	57.3	50.0	25.0
80	84.2	147	90	206	81	383	165	155	122	76.5	66.6	33.3
100	105.3	158	97	227	88	429	206	193	152	96.0	83.4	41.7

Problems

1. Develop the formula for discharge from a nozzle and find velocity of and coefficient of velocity C_v, for a $1\frac{3}{8}$-inch nozzle attached to a 2.5-inch p pipe and discharging 310.6 gallons per minute under an indicated pressure of pounds per square inch.

2. Compute the discharge in gallons per minute through a smooth nozzle inches in diameter under a pressure at the base of the nozzle of 60 pounds p square inch. Diameter of play pipe $2\frac{1}{2}$ inches.

3. A $1\frac{1}{4}$-inch smooth nozzle is attached to a $2\frac{1}{2}$-inch play pipe. Indicat pressure at base of play pipe is 80 pounds per square inch. Find discharge nozzle in gallons per minute.

4. Find discharge in gallons per minute through $1\frac{1}{4}$-inch smooth nozz attached to 4-inch pipe. Pressure head at base of nozzle is 70 pounds p square inch.

5. If the hydrant pressure is 100 pounds per square inch, find the pressu at the base of a $1\frac{1}{8}$-inch smooth nozzle attached to a $2\frac{1}{2}$-inch rubber-lined hos of best quality, 400 feet long. Also find discharge in gallons per minute.

6. What will be the discharge in gallons per minute from a smooth nozz $1\frac{1}{4}$ inches in diameter attached to a $2\frac{1}{2}$-inch hose, the indicated pressure bein 80 pounds per square inch at the base of the nozzle? To what vertical heigl would the stream rise?

7. What is the head at a fire hydrant when the discharge from a $1\frac{3}{8}$-in ring nozzle at a distance of 400 feet is 257 gallons per minute through a $2\frac{1}{2}$-in hose?

8. What head will be required in a fire hydrant to give a discharge of 2 gallons per minute at a distance from the hydrant of 450 feet, through $2\frac{1}{2}$-in rough rubber-lined pipe and $1\frac{1}{4}$-inch ring nozzle?

9. Given a hydrant pressure of 80 pounds per square inch. What will the discharge in gallons per minute through 700 feet of best quality rubb lined fire hose, $2\frac{1}{2}$ inches in diameter, and a $1\frac{1}{8}$-inch smooth nozzle, supposi that the nozzle is held 10 feet above the elevation of the hydrant?

10. The head on a fire hydrant is 234 feet. To it is connected 500 feet smooth rubber-lined hose, $2\frac{1}{2}$ inches in diameter, with a $1\frac{1}{4}$-inch smooth nozz attached. Find the discharge in gallons per minute.

11. A fire stream nozzle is held 15 feet higher than the elevation of t hydrant to which the hose is attached. Nozzle is $1\frac{1}{4}$-inch smooth, hose is $2\frac{1}{2}$-in inferior rubber-lined and 300 feet long. Discharge is 250 gallons per minut (*a*) Find necessary pressure at hydrant. (*b*) Also determine the dischar with the nozzle removed from the hose.

CHAPTER XII

WEIRS

213. A weir is an overfall opening or notch used in determining the volume of flow from measurements of the depths (*heads*) of water running over its crest or sill, of which the length and shape are known. Usually a weir is a dam, or a notch in the top of a dam, or in a bulkhead built across a stream. Weirs are usually classified according to their outlines and the profiles of their crests.

Rectangular weirs are the most common in outline; but trapezoidal, triangular, and even very irregular forms are often used. Rectangular weirs are further distinguished as contracted or suppressed.

A contracted weir, or one with end contractions, has a length of crest less than the width of channel in which it is set. (See figure 76.)

A suppressed weir is one in which the end contractions are suppressed; that is, it has a crest equal in length to the width of the channel in which it is set. (See figure 77.)

A sharp-crested or thin-edged weir is one which has a crest on which the overfalling sheet of water, called the *nappe* (overhanging sheet), touches only a sharp, regular edge on its upstream face and springs entirely clear of the downstream parts of the weir. The actual thickness of the crest for high heads may be considerable, and yet allow the nappe to spring clear; but for very low heads the crest should be of metal and very thin. The upstream face should be vertical; and the downstream face so nearly vertical that the nappe cannot touch it.

Weirs of irregular section are those in which the profiles of the weirs and crests (measured in a vertical plane parallel to the direction of flow) are such as will cause the nappe, after passing the upstream edge, to adhere to or again touch the crest, or the weir face; or will produce a form of nappe in any way different

from that of a vertical, sharp-crested weir of equal he
under equal heads.

Free overfall implies that the nappe is formed with atı
pressure on all its surfaces, and is subjected to no modif
ditions after passing the crest.

Submergence implies that the nappe falls into water
either above the crest level, or so near the crest level
form of the nappe is thereby modified.

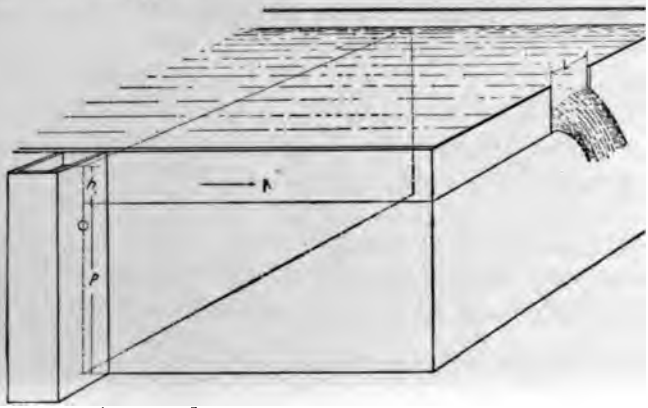


Fig. 76. — Rectangular Weir with Two End Contractions.

214. Nomenclature. The symbols here given and e
with the aid of figures 76 and 77, will be used througho
cussing weir

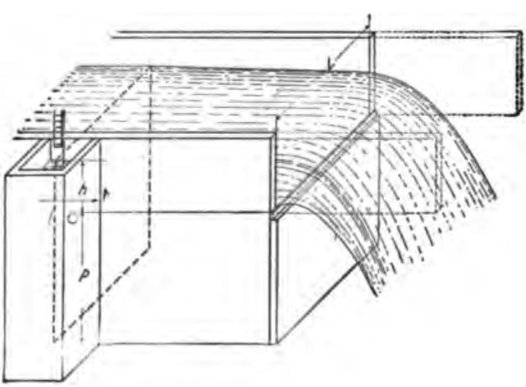


Fig. 77. — Suppressed Weir.

Figure 7
sents a rec
vertical we
level crest
end contract
having free
Figure 7
sents a su
weir with le
prolonging
downstream
the crest
shown in light broken lines, very slightly decreases the d
but is common practice.

h = the observed head on the crest, being the difference in elevation in feet between the top of the crest and the surface of the water in the channel, at a point upstream, which should, if possible, be taken just beyond the beginning of the surface curve.

H = the observed head corrected to include the effect of the velocity of approach.

L = the length of the crest in feet.

p = the height of the crest, being the difference in elevation in feet between the top of the crest and bottom of the channel; strictly used only in rectangular channels, and measured in the same stream cross section as the head.

A = the cross-sectional area of the channel where h is measured, which is called the channel of approach.

V_A = the mean velocity of approach in feet per second.

h_v = the head due to the mean velocity of approach $= \frac{V_A^2}{2g}$.

N = the number of end contractions.

Q = the actual discharge in cubic feet per second.

G = the area of the channel of approach divided by the width of the nappe: for suppressed weirs, $G = \frac{A}{L} = (p + h)$; and for contracted weirs, $G = \left[\frac{A}{L - \frac{NH}{10}}\right]$. § 220.

Rectangular Weirs

215. Theory of weir measurements. A rectangular weir is in theory a rectangular vertical orifice lacking an upper edge. The early hydraulicians called it an "uncovered orifice"; and upon this conception present weir formulas are based. Using formulas for rectangular vertical orifices, (14) and (15) in § 176 and making $h_b = h$ and $h_u = 0$, the formula for computing weir discharge in cubic feet per second, neglecting velocity of approach, becomes

$$Q = \tfrac{2}{3}\, cL(2g)^{\frac{1}{2}}h^{\frac{3}{2}} = \tfrac{2}{3}\, cLh(2gh)^{\frac{1}{2}}. \qquad (1)$$

c = coefficient of discharge, determined by experiment.

To determine the volume of flow it is therefore necessary t measure L and h, choose a suitable coefficient, and solve equatic (1) or an equivalent formula. The length of the crest and tl head may be measured with any desired degree of precision.

When a not very high degree of precision is required, standard conditions exist, a suitable coefficient may be readi selected; and the computation of discharge is simple. Whe however, a high degree of precision is required, or the standa conditions do not exist, a reasonable choice of a coefficient fro among the large number available, some apparently contradictor may present many difficulties.

The flow of water over weirs is a combination of the flow in open channel and the flow through a form of orifice. Many el ments tend to modify this flow, some of which in certain cas can be eliminated by duplicating experimental conditions; b in other cases must be considered more or less carefully as th desired degree of precision may demand. The important modif ing elements are:

(1) The velocity of approach;

(2) The contractions of the nappe;

(3) The conditions surrounding the nappe below the cres whether it has free overfall, is depressed, or is "wetted under neath."

216. The velocity of approach. The channel by which the wate reaches the weir is called the *channel of approach.* Its cross-se tional area (A) should be measured in the same vertical plane the head, and at right angles to the direction of flow. The mea velocity of flow in the channel of approach is called *the velocity o approach* (V_A). Hence

$$V_A = \frac{\text{discharge in cubic feet per second}}{\text{area of channel of approach}} = \frac{Q}{A} \text{ feet per second.}$$

The effect of velocity of approach is to increase the dischar above that due to the observed head (h), an amount that can l determined only approximately for several reasons.

The head due to the mean velocity of flow (h_v), which equa $\frac{V_A^2}{2g}$, is not a precise measure of the kinetic energy of the movi water in the channel. For in few, if in any, streams is the veloci

uniform throughout any cross section; and, if the actual velocities represented by V_1, V_2, V_3, V_n were determined in equal subdivisions of the cross sections by, say, a current meter, the summation of their velocity heads $\left(\frac{V_1^2}{2g} + \frac{V_2^2}{2g} + \cdots \frac{V_n^2}{2g}\right)$ would not equal but usually exceed $\frac{V_A^2}{2g}$. (See figure 78.) In every case this correction would depend very much upon the distribution of the velocities throughout the channel of approach.

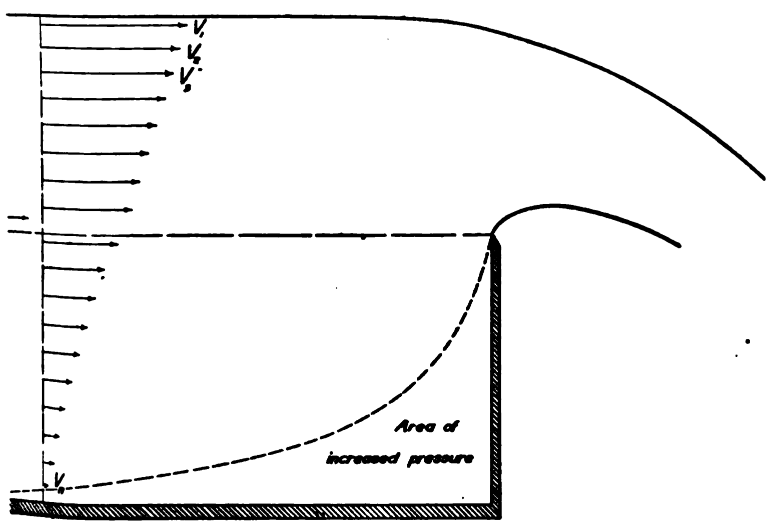


FIG. 78.

Whatever value may be assigned to the velocity of approach, its full value is probably not available at the crest of the weir, some energy being lost in change of velocity and direction, in eddy currents, and in unknown ways.

If the velocity of approach is great and the correction becomes an important part of the total head (H), its effect cannot easily be distinguished from the effect of other factors, notably the effect of modified crest contraction.

The correction for the effect of velocity of approach is applied by experimenters in their formulas either by increasing the observed head by some function of the velocity head, assigning a constant value to the coefficient for a given weir as did Francis and Fteley and Stearns; or by including this correction in a

weir coefficient of varying value as Bazin did. In usin
weir formula or coefficients of any experimenter, his own m
of correcting for velocity of approach, upon which the
of his coefficients depends, should be faithfully duplicated.

217. The contractions of the nappe. The form of the na
determined by three factors, viz. the upper surface curv
crest contraction establishing the shape of the lower s
curve, and the lateral or end contractions, if any.

The area of the section where the crest contractions ar
end contractions (if any) reach a maximum in weirs is proba
important in establishing the volume of flow through a w
the area of the vena contracta is in an orifice, being in
cases the effective area of the stream.

The surface curve. The free surface of the water in the
nel as it approaches the crest curves downward at an incr
rate of curvature over and beyond the weir, thus formin
upper boundary surface of the nappe.

The crest contraction. The lower surface of the nappe de
itself from the crest at some angle varying with the cond
and form of weir, rises to a summit, then descends. The di
tion in sectional area of the nappe due to the influence o
lower surface curve is termed *the crest contraction.*

Complete crest contraction is said to exist when a further in
of the ratio of the height of weir to the observed head
will not increase the contraction; a ratio of $\frac{3}{1}$ is usually suffi

218. The effects of the upper and lower surface curv
modifying the volume of flow are important; but excep
Bazin's experiments* on suppressed weirs, little accurate k
edge concerning this subject is available.† Bazin showed
series of measurements of the profiles of nappes under diff
heads on vertical and inclined weirs, and by simultaneous
urements of the volume of flow on his standard weir, tha
crest contraction furnishes fundamental data for comp

* *Annales des Ponts et Chaussées*, Mém. et Doc., 1890, 1st semestre, pp.

† See also Poncelet and Lesbros, *Expériences Hydrauliques*, plate 7; also
and Stearns, *Trans. Am. Soc. C. E.*, Vol. 12, table XIII.

the discharge, that $\frac{\epsilon}{h}$ (see figure 79) roughly represents a sort of coefficient of contraction, and that the section of the nappe just above the summit of the lower curve is the effective area of discharge.

He showed further that Boussinesq's* formula,

$$Q = .52(2g)^{\frac{1}{2}}(h-\epsilon)^{\frac{3}{2}},$$

was approximately correct, but that the determination of ϵ was too delicate an operation for practical use.

Bazin's experiments, while too voluminous for reproduction here, are most pertinent to an understanding of weir discharge, and some of his conclusions will be briefly stated.

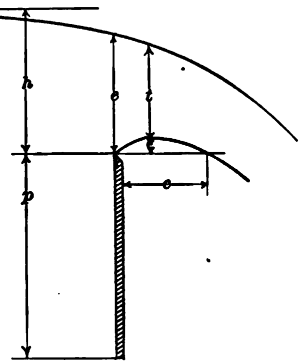


FIG. 79.

219. Bazin's conclusions concerning contractions. If by the use of coördinates the profiles of the nappes are plotted for a given weir of constant height (p), the forms of each curve will be almost identical for all heads, although the absolute dimensions change with the head. Replacing the abscissas x by $\frac{x}{h}$, and the ordinates y by $\frac{y}{h}$, for any value of $\frac{x}{h}$ the corresponding value of $\frac{y}{h}$ will be practically constant.

For similar forms of weir an increase in the ratio $\frac{h}{p}$ will increase the amplitude of the upper surface curve and decrease the crest contraction.

Inclining the weir face upstream (up to 1 on 1) has but little effect on the upper surface curve, but increases the crest contraction.

Inclining the weir face downstream decreases the height of the upper curve, elongating it both upstream and downstream. At slopes of 1 on 4 the crest contraction will be practically eliminated; and the opening ceases to be a weir.

* *Comptes Rendus de l'Académie des Sciences*, 4, juillet, 1887.

The thickness relative to the head $\frac{e}{h}$ of the stream at the weir crest which is nearly constant for vertical weirs and for weirs inclined (up to 1 on 1) upstream, diminishes very rapidly as the weir is inclined downstream.

$\frac{e}{h}$ for the same inclination increases with the ratio $\frac{h}{p}$.

The relative crest contraction $\frac{\epsilon}{h}$ decreases from about .16 for weirs inclined 1 on 1 upstream to .003 for weirs inclined 1 on 4 downstream.

Maximum crest contraction occurs at a distance of .25 h downstream for weirs with vertical faces or with upstream inclination; at .17 h for downstream inclination of 1 on 1, and at .05 h for 1 on 4.

The relative chord length $\frac{c}{h}$ decreases with $\frac{\epsilon}{h}$.

The relative thickness of the nappe $\frac{t}{h}$ above the summit of the lower curve decreases as the weir face is inclined upstream up to 1 on 1, increases as the weir face is inclined downstream up to 2 on 1, and decreases with further downstream inclination.

Increase or decrease in the rate of discharge follows closely upon increase or decrease of $\frac{t}{h}$.

The following ratios given by Bazin show how the changes in the form of surface curves caused by inclination of the face modify the flow as compared with a vertical weir.

	Inclination of Weir	
Toward Upstream	1 horizontal 1 vertical	.93
	2 horizontal 3 vertical	.94
	1 horizontal 3 vertical	.96
	Vertical	1.00
Toward Downstream	1 horizontal 3 vertical	1.04
	2 horizontal 3 vertical	1.07
	1 horizontal 1 vertical	1.10
	2 horizontal 1 vertical	1.12
	4 horizontal 1 vertical	1.09

The foregoing ratios are used only as illustrating the principles of the effect of the upper surface curve and the crest contraction;

but in developing existing weir formulas, no successful attempt has been made to separate these effects from that of velocity of approach, and they are ordinarily combined.

220. End contractions. If the length of the weir crest is less than the width of the channel of approach, the nappe is contracted laterally after passing the crest and its width is reduced, and with it the discharge is diminished. Whether this contraction occurs in the same section as the summit of the lower surface curve so far as is known, has never been determined.

Complete end contraction. If a weir has sharp-edged ends, an increase in the distance between an end and the adjacent side of the channel, up to a certain limit, will increase the degree of contraction. When this limit has been reached, the contraction is said to be complete. Francis said that the distance should be at least equal to the head; in the light of later experiments it appears that the distance should be at least 2 to 3 times the head.

Corrections for end contraction. For incomplete end contractions there is no satisfactory method of corrections; hence in precise measurements the contraction should be completed or eliminated.

For complete end contraction there is only one satisfactory method available, which was proposed by J. B. Francis,* as a result of his observations. The correction is based upon the assumption that the only effect of end contractions is to reduce the width of the nappe; and it consists in substituting for the measured length of crest L, in the formula for computing the discharge, the value $\left(L - \frac{NH}{10}\right)$. Later Fteley and Stearns † by a series of experiments corroborated this conclusion and recommended the continuation of the use of this factor.

221. Conditions surrounding the nappe below the crest. Prolonging the sides of the channel downstream above the level of the crest, as shown by dotted lines in figure 77, was found by Francis to diminish the flow about $\frac{1}{240}$. He recommended not prolonging the sides of the channel beyond the crest. Both Fteley and Stearns, and Bazin, on the other hand, used such prolongation in all their experi-

* *Lowell Hydraulic Experiments*, p. 74.
† *Trans Am. Soc. C. E.*, Vol. 12, pp. 108–114.

ments on suppressed weirs; and this is now the more comm practice, excepting where there is difficulty in getting perfe aëration under the nappe.

Special provision for free access of air underneath the sheet mu be made, if the sides of the channel are prolonged downstrea below the crest level. If the space underneath the nappe is cut from atmospheric pressure, the imprisoned air becomes more rarefi as the flow continues, the water rises behind the nappe, and wh the rarefaction is sufficient the nappe will be "depressed" towa the weir. If the head becomes high enough, all the air will be pelled and the space under the nappe is occupied by a mass turbulent water. This condition is called by Bazin "wetted und neath." The effect of depressing and wetting is, in general, expand the cross-sectional area of the nappe and increase the charge. The results of Bazin's researches * on this subject are voluminous to be reproduced and can not be satisfactorily c densed.

At very low heads, perhaps half an inch or less, the na usually adheres to the crest and the lower face of the weir, m ing the measurement of discharge uncertain.

If free access of air at the ends is provided, the downstre water level may rise very nearly to or perhaps slightly above crest without affecting the discharge.

Figure 80, taken from Bazin's experiments, shows certain c ditions that may occur.

222. The procedure to be followed in weir measurem comprises:

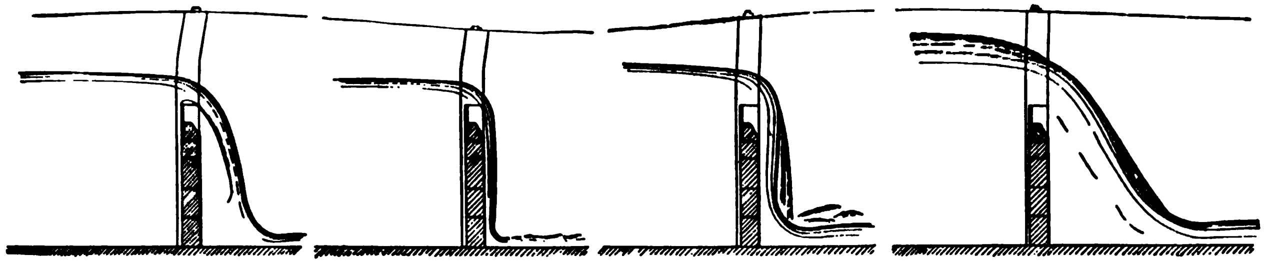
(1) Constructing and setting up the weir and the gauge measuring the head; reproducing, if possible, the experimen conditions of the formula to be used.

(2) Measuring the length of the crest and determining irregularities if any.

(3) Taking a profile of the crest if not sharp-edged.

(4) Determining by actual measurements the cross-sectio area of the channel of approach.

* *Annales des Ponts et Chaussées*, Mém. et Doc., 1891, 2d semestre, pp. 445– same, 1894, 1st semestre, pp. 249–357, latter includes profiles of nappes.

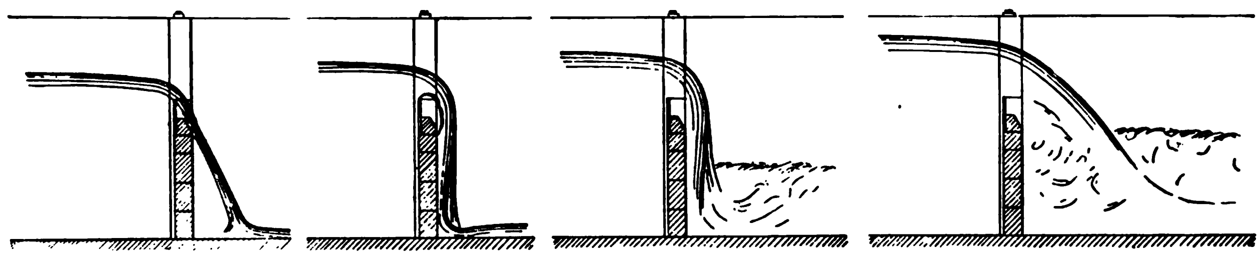


A — Nappe depressed.

B — Adhering nappe as formed under a low head.

C — Adhering nappe as formed under a greater head

D — Nappe of form known as wetted underneath.



E — Nappe adhering at crest and detached below.

F — Adhering nappe detached at crest of weir.

G — Adhering nappe with backing up of water in channel below weir.

H — Wetted underneath form with backing up of water in channel below weir.

FIG. 80. — Forms of Nappe occurring if Air is cut off from beneath the Sheet. As compared with Free Overfall, the Discharge may be increased with *A* as much as 6 %; with *D*, 15 %; with *G*, 25 %.

(5) ...blishing by leveling the relative elevations of t... crest of th... weir, and the zero of the gauge.

(6) When the desired regulation of flow is established, determining the head by hook gauge or other observations at interval as frequent as the conditions require.

(7) If possible, measure the actual velocity in the channel o... approach by a current meter or some other direct method; and

(8) Compute the discharge by the formula selected.

Three of these operations require especial consideration, viz. *construction and setting*, *the measurement of the head*, and the *selection of the formula*.

223. Construction and setting of weirs. In order to eliminat... as far as possible factors for which precise allowance can not b... made, the construction and setting should meet the followin... conditions:

(1) A sharp-crested weir with complete crest contraction shoul... be used.

(2) The crest should be level, and its ends vertical.

(3) The end contractions should be complete, or, if suppresse... entirely suppressed.

(4) The upstream face should be vertical; the downstream s... designed that the nappe has free overfall.

(5) Free access for air under the nappe should be made certai...

(6) The weir should be set at right angles to the direction ... flow.

(7) The channel of approach should be straight for at lea... 25 feet above the weir, of practically uniform cross section, a... of slight slope (preferably none).

(8) Screens of coarse wire or baffles of wood should be set ... the channel, if necessary, to equalize the velocities in differe... parts of the channel, but not nearer the crest than 25 feet.

(9) The channel of approach should have a large cross-sectio... area in order to keep the velocity of approach low.

224. Measurement of the head. Precise determinations of t... head on the crest (h) necessitate accurate observations of a suit... ble gauge, preferably a hook gauge, so set as to give the tr... head.

The hook gauge * (see figure 81) is a graduated sliding scale, with a sharp hook on the lower end, fitted with a vernier, and attached vertically to a frame. In order to make an observation the point of the hook is slowly raised by a screw from below the water surface until the point slightly elevates the surface of the water, but does not pierce it; then, if with a slight lowering of the point, this distortion of the surface just disappears, the point is assumed to be at the water level. The relation of the zero of the scale to the elevation of the crest must be found by leveling or by a still water comparison. If possible, the zero of the scale should coincide with the average elevation of the crest. The head is usually taken to be the arithmetical mean of all observations for any one condition of steady flow.

FIG. 81. — Hook Gauge.

Other forms of gauges, such as graduated tubes and stationary graduated wooden scales, are frequently used, but can not be read so closely; dial floats, although extremely useful for continuous observations or registration, are usually subject to errors caused by inertia and by lost motion in the mechanism.

Errors of observation. Ordinarily, by means of the vernier, differences of .001 of a foot may be read and smaller subdivisions estimated. The hook gauge observation is liable to slight errors of setting and reading, which are ordinarily not important, but may be in the cases of low heads. For example, an error of .001 of a foot with a head of .1 foot will cause an error in discharge computations of 1.5 per cent; the same error in observation with a 1.0-foot head will cause an error of only .15 per cent.

Errors in head observations caused by improper gauge location. The obvious position for a gauge is directly in the channel of approach on one or both sides, and just beyond the beginning of the surface curve; and unless a high degree of precision is required, it may be so placed. Even with a steady flow, however, the water surface may oscillate more rapidly than it can be fol-

* Called Boyden's Hook Gauge, after its inventor. See *Lowell Hydraulic Experiments*, p. 18.

lowed, even with a hook gauge. Therefore, to remedy this difficulty, as far as possible, the gauge should be set in a recess or still box outside of, but in communication with, the channel; and by throttling the communication between the channel and the still box, the oscillations can be further reduced to facilitate observations. The still box is obviously a piezometer.

The location of a still box should meet the following essential conditions:

(1) The cross-sectional area of the communicating opening or pipe must be sufficient to allow free communication with the channel even when throttled.

(2) The channel end of this opening must be set into and exactly flush with the flat walls of the channel, or into a flat surface laid parallel to the direction of flow; and the pipe itself must be normal to the direction of flow. (See discussion of piezometers, Chapter VII.)

(3) The channel end of this opening must be located far enough upstream to avoid the slope of the surface curve, and not far enough to increase the observed head by the natural slope of the stream.

The area of increased pressure (see figure 78), which forms above the bottom, beginning at the upstream face of the weir and extending upstream, perhaps about to the beginning of the surface curve,* once thought to be a location at which the observed head would include the velocity head, has been proved to be a poor location for the opening.

Avoid perforated pipes, no matter where the holes are bored, laid transversely or longitudinally in the stream at different depths; avoid so-called piezometers of any form which project in any direction into the stream. After the Lowell hydraulic experiments were made, Francis sometimes used pipes with holes bored in a vertical plane in order to secure an average pressure across the stream, in recognition of the fact that the surface is not transversely level. Since his time, this has been shown to be a vicious practice, which may introduce more errors than it was designed to obviate.

The essential conditions of location of a still box will in general

* See Fteley and Stearns, *Trans. Am. Soc. C. E.*, Vol. 12, p. 42, Plate IV.

be met if its opening is set well upstream from the beginning of the surface curve, and at or a few inches below the crest level.

If Francis's, Fteley and Stearns's, Bazin's, or any particular experimenter's formula is to be used, his location should be duplicated.

225. Weir formulas. The practice of weir measurements in this country depends upon the original experiments of James B. Francis; of Fteley and Stearns; of H. Bazin (in France); of George W. Rafter, Gardiner S. Williams, R. E. Horton, and others at the Cornell Hydraulic Laboratory; and the investigations of H. Smith, Jr. (about the year 1880) of all the available data then existing; and the production therefrom of valuable and widely used coefficients.*

In order to furnish the student with a basis for a choice of formulas, the authors have endeavored, in so far as the nature of this book permits, to restate from the original records the essential conditions of the experiments and the methods by which the more important formulas were derived.

Sharp-crested rectangular weirs with free overfall and the derivation of their formulas will be first considered; and then the values of the coefficients by which the same formulas and occasional new ones have been made to suit weirs of irregular section and outline with submerged or other modified conditions.

Weir Formulas

226. The general weir formula may be expressed by the equation $Q = CLH^{\frac{3}{2}}$. To this form all the equations in use may be reduced; but it is better practice, in view of the several methods of correcting for the velocity of approach followed by the various experimenters, to use their form of equation.

The Francis formula.

$$Q = 3.33\,(L - .1\,NH)H^{\frac{3}{2}}. \qquad (2)$$

$$H^{\frac{3}{2}} = [(h + h_v)^{\frac{3}{2}} - h_v^{\frac{3}{2}}].$$

See § 233.

* For the most complete compilation and examination of existing weir data, as well as for hitherto unpublished results, the invaluable W. S. & I. Paper No. 200, by R. E. Horton, *Weir Experiments, Coefficients*, etc., should be consulted.

The Fteley and Stearns formula.

$Q = 3.31\, LH^{\frac{3}{2}} + .007\, L.$

$H = (h + 1.50\, h_v)$ for suppressed weirs.

$H = (h + 2.05\, h_v)$ for contracted weirs.

For contracted weir make $L = (L - .1\, NH)$.

See §§ 237–242.

The H. Smith, Jr., formula.

$Q = (c_s \text{ or } c_c)\, \frac{2}{3}\, L(2\, g)^{\frac{1}{2}} H^{\frac{3}{2}}.$

$H = (h + 1\frac{1}{3}\, h_v)$ for suppressed weirs.

$H = (h + 1.4\, h_v)$ for contracted weirs.

See § 243.

The Bazin formula.

$Q = mLh(2\, gh)^{\frac{1}{2}}$ (for suppressed weirs only).

m = coefficient including effects of crest contraction an
velocity of approach.

See §§ 252–255.

Francis's Experiments on Sharp-crested Vertical W with Free Overfall

227. James B. Francis made at Lowell, Massachusetts, du the period of 1848 to 1852, elaborate investigations in the of water over weirs which, although limited in range, have ra if ever, been excelled in precision. They were published an his other notable hydraulic researches in the *Lowell Hydr Experiments.*

228. Development of formula.* In order to establish a for for computing the discharge over sharp-crested weirs with contractions, Francis caused a constant but unknown volu of flow to pass over various weirs of different lengths from 3 17 feet, and having from 2 to 8 lateral (end) contractions. I simultaneous measurements of head and observations of the ef of lateral contractions, he adopted the following formula:

$$Q = C(L - 0.1\, NH)\, H^{\frac{3}{2}}. \ddagger$$

* *Lowell Hydraulic Experiments*, pp. 71–102.

† Maintained constant by a constant head on set of wheel orifices.

‡ Same, p. 118. He stated that the 1.47th power, while inconvenient t was more exact than the 1.5 power used.

Determination of the coefficient *C*. To establish a value for the coefficient *C*, Francis made a series of 88 experiments measuring the volume of flow in a large basin.*

229. Weir settings.† The same weir was used in all experiments, and was set at the end of a lock chamber about 22.5 feet long hav-

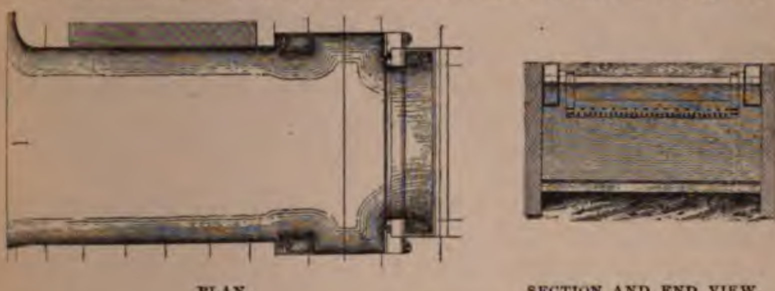


FIG. 82.—Francis's Contracted Weir.

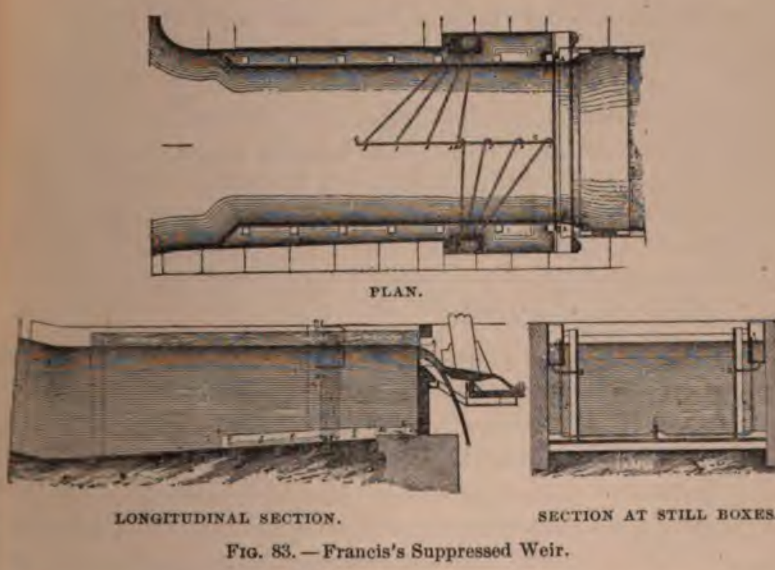


FIG. 83.—Francis's Suppressed Weir.

* These experiments were made at Lowell, Massachusetts, at the "lower locks," so-called, of the water power corporation known as the Proprietors of the Locks and Canals on the Merrimack River, which company bore the expense of the investigation.

† Figures 82 and 83 are reproduced from Plate XIV, *Lowell Hydraulic Experiments.*

ing a plank bottom and masonry side walls. The weir with en contractions is shown in figure 82. The weir without end con tractions (suppressed weir) was obtained by building two paral timber walls inside the original channel, each 20 feet in leng and separated by a space equal to the length of the weir. Figu 83 shows the exact arrangements for the suppressed weir exper ments. For experiments 36–43, 62–66, 79–84, and 85–88 a leve false bottom 20 feet long and 2 feet lower than the crest was added; and for experiments 34–35, and 85–88, a two-foot partition with sharp edges was set in the middle of the crest to cause two additional end corrections.

230. Measurements of head. The head was observed by two hook gauges, one on each side of the channel, set in still boxes which were 18 inches long by 11 inches wide as shown in figures 82 and 83. Communication with the channel was made for the contracted weir measurements by a one-inch diameter hole in the bottom of each box, located 6 feet upstream from the weir and 4 inches lower than the level of the crest. For the suppressed weir, communication was established by pipes *B*, opening into the sides of the channel one foot lower than the level of the crest, or by the single opening for the pipes 4 and 5 which were set in the board *C*. The pipes 1, 2, 3, 6, 7, and 8 shown in figure 83 were not used in this set of experiments. All three openings used were therefore 6 feet upstream from the weir. To prevent rapid oscil lations, the openings were throttled by a perforated plug set on the inside of the still boxes.

231. The volume of flow was measured by the rise of the water surfaces in a rectangular wooden basin built into the lower locks into which, when the desired rate of flow was established by means of head gates, the stream over the weir was deflected by swinging hood. (See figure 83.) In the periods between experi ments the hood caused the water to be wasted through a by-pas channel. The volume of the measuring basin, which was deter mined by careful measurements, was over 12,000 cubic feet, an was nearly filled during each experiment; in every case a cor rection was made for the slight amount of leakage.

The duration of each experiment was measured on a marin chronometer. The beginning and end of each experiment was an

nounced by a bell operated by an electric circuit which was opened and closed when the swinging arm carrying the level reached the position at which half of the stream was flowing into the measuring basin.

The duration of individual experiments was from 182 to 822 seconds. The extreme error in the observations used in the determining the value of the Francis coefficient C probably did not exceed one half of one per cent.

232. The conditions, setting, and the results of the eighty-eight experiments are summarized in Table XXX.

233. Francis's formula. Francis fixed on 3.33 as the mean value of C, making the general formula

$$Q = 3.33(L - 0.1\ NH)H^{\frac{3}{2}}.$$

The corrected H is to be computed by the expression

$$H^{\frac{3}{2}} = (h + h_v)^{\frac{3}{2}} - h_v^{\frac{3}{2}},$$

which, although differently derived,* is equivalent to using the precise formula for rectangular orifices (Chapter IX, formula 14) and integrating it between the limits, $(h + h_v)$ and $(0 + h_v)$.

Francis's Formulas

For contracted weirs, neglecting velocity of approach:

$$Q = 3.33(L - .1\ Nh)h^{\frac{3}{2}}. \qquad (7)$$

For contracted weirs, head corrected for velocity of approach:

$$Q = 3.33(L - .1\ NH)\left[(h + h_v)^{\frac{3}{2}} - h_v^{\frac{3}{2}}\right]. \qquad (8)$$

Note. The use of h instead of H in the factor $(L - .1\ NH)$ used in correcting for end contractions is as precise as ordinary practice warrants.

For suppressed weirs, neglecting velocity of approach:

$$Q = 3.33\ Lh^{\frac{3}{2}}. \qquad (9)$$

For suppressed weirs, head corrected for velocity of approach:

$$Q = 3.33\ L\left[(h + h_v)^{\frac{3}{2}} - h_v^{\frac{3}{2}}\right]. \qquad (10)$$

The correction for the velocity of approach may be made by successive approximations by solving Q by formula (7) or (9), then dividing Q thus found by the area of the channel of approach,

* *Lowell Hydraulic Experiments*, pp. 116–118.

TABLE XXX

Dimensions in feet. Velocities in feet per second.

Experiment Nos.	Number of End Contractions	Length of Weir in Feet	Width of Channel at Hook Gauge	Depth from Level of Crest to Bottom of Channel: At Hook Gauge	Depth from Level of Crest to Bottom of Channel: At Crest	Location of Piezometer for Meas. Hd.: Up-stream	Location of Piezometer for Meas. Hd.: Below Crest Level	Range of Observed Head: From	Range of Observed Head: To	Range of Velocity of Approach: From	Range of Velocity of Approach: To	Mean Value of C
1–4	2	9.997	13.96	5.048	4.60	6	.33	1.524	1.569	.768	.789	3.3181
5–10	2	9.997	13.96	5.048	4.60	6	.33	1.237	1.255	.590	.600	3.3338
11–33	2	9.997	13.96	5.048	4.60	6	.33	.916	1.069	.395	.486	3.3223
56–61	2	9.997	13.96	5.048	4.60	6	.33	.777	.819	.317	.541	3.3246
72–78	2	9.997	13.96	5.048	4.60	6	.33	.592	.655	.218	.251	3.3275
36–43	2	9.997	13.96	2.014	2.014†	6	.33	1.028	1.079	.950	1.005	3.3527
62–66†	2	9.997	13.96	2.014	2.014†	6	.33	.771	.889	.669	.796	3.3403
79–84†	2	9.997	13.96	2.014	2.014†	6	.33	.631	.660	.519	.550	3.3262
34–35	4	7.997*	13.96	5.048	4.60	6	.33	1.010	1.026	.353	.360	3.3601
85–88	4	7.997*	13.96	2.014	2.014†	6	.33	.669	.688	.438	.453	3.3368
44–50	None	9.995–7	9.992‡	5.048	4.60	6		.975	.987	.538	.546	3.3409
51–55‡	None§	9.995–7	9.992‡	5.048	4.60	6	1 to 4.8	.992	1.006	.548	.559	3.3270
67–71‡	None	9.995–7	9.992‡	5.048	4.60	6		.736	.815	.366	.421	3.3393

Arithmetical mean of values of $C = 3.3318$.

* Virtually 2 weirs of equal length with a partition 2 feet wide between them.

† Bottom level for 23 feet upstream from weir crest.

‡ Sides parallel for 20 feet upstream from weir crest.

§ Sheet prevented from expanding downstream by prolonging the sides downstream above crest level.

which gives a trial value of V_A from which $h_v\left(=\frac{V_A^2}{2g}\right)$ is determined. A second determination of Q by formula (8) or (10) will usually be as precise as is necessary.

This computation will be much simplified by the table of weir discharge per foot of crest (Table LXIV) or discharge diagram (figure 85), the tables of velocity heads (Table LXII), and the table of values of $h_v^{\frac{3}{2}}$ (Table LXXIV).

Messrs. Hunking and Hart * suggested a simpler equivalent for H, viz. making $H^{\frac{3}{2}}$ equal to $Kh^{\frac{3}{2}} = (h + h_v)^{\frac{3}{2}} - h_v^{\frac{3}{2}}$. (11)

The value of K will very nearly be, $1 + .2489\left(\frac{h}{G}\right)^2$, or nearly nough for most cases, $1 + \left(\frac{h}{2\,G}\right)^2$. For G see § 214.

The Francis formulas are strictly applicable only to vertical harp-crested rectangular weirs with complete contractions and vith free overfall and

When the head (H) is not greater than one third the length (L);

When the head is not less than .5 foot nor more than 2 feet;

When the velocity of approach is 1 foot per second or less;

When the height of the weir is at least 3 times the head.

In all probability the formulas are usable with higher heads than 2 feet but not much lower than .5 foot as shown by Fteley and Stearns's experiments.

Example. Given a weir with two contractions, length = 10.24 feet; height of crest = 3.8 feet; width of channel of approach = 15 feet; observed head = 1.2 feet. Compute the discharge in cubic feet per second, making correction for velocity of approach. From Table LXIV or diagram (figure 85), $Q = 4.38$ (approx.) cubic feet per second per foot of crest.

Hence $$Q = 4.38(10.24 - .2 \times 1.2) = 43.8 \text{ (approx.)}.$$

Whence $$V_A = \frac{43.8}{(3.8 + 1.2)\,15} = .59 \text{ feet per second,}$$

and $$h_v = \frac{.59^2}{2g} = .0054 \text{ (from Table LXII).}$$

* *Journal Franklin Inst.*, August, 1884, pp. 121–126.

Then $(h + h_v)^{\frac{3}{2}} = (1.2054)^{\frac{3}{2}} = +\,1.3234$ (from Table LXXIV

and $\qquad -h_v^{\frac{3}{2}} = -\ \ .0005$

$$1.3229$$

Hence $Q = 3.33 \times 10 \times 1.3229 = 44.053$ cubic feet per secon

In this example the correction due to velocity of approach but .25 cubic feet or about .6 of one per cent.

By Hunking and Hart's method of correcting for velocity approach,

$$G = \frac{15 \times (3.8 + 1.2)}{(10.24 - .2 \times 1.2)} = 7.5\,; \text{ And } K = 1 + \left(\frac{1.2}{2 \times 7.5}\right)^2 = 1.0064$$

Then $Q = 3.33(10.24 - .24) \times 1.2^{\frac{3}{2}} \times 1.0064 = 44.054.$

cubic feet per secon

Fteley and Stearns's Experiments on Sharp-crested Weirs with Free Overfall

234. In 1877–79 Alphonse Fteley and Frederic P. Stearns made investigations in the flow of water over weirs which n only verified Francis's results, but also contributed new and acc rate data concerning velocity of approach and the flow of wat over weirs, especially with low heads.

The conditions of these experiments may, in so far as they app to sharp-crested vertical weirs, be summarized as shown in Tal XXXI.

235. Measurement of head. In all the five sets of experime the head was measured by hook gauges set in still boxes wh were connected with the channel by pipes. Although the act form of piezometer openings varied, the essential condition t the opening be at or below the crest in and normal to a flat s face parallel to the direction of flow, was in all cases maintain The location of each opening is stated in the table.

236. Measurement of volume of flow. In the experiments (in cated in Table XXXI as A) upon the suppressed weir 5 feet length, the volumes were measured by observing the rise of

* *Trans. Am. Soc. C. E.*, Vol. 12, 1883, pp. 1–114. Experiments were made and at the entrance to the Sudbury Aqueduct of the Boston (now Metropolit Waterworks.

TABLE XXXI

	Year	Number of Experiments	Number of End Contractions (N)	Length of Weir in Feet (L)	Width of Channel of Approach in Feet	Length of Channel of Approach of Rectangular Cross Section & Level Bottom, Feet	Height of Weir Crest above Bottom of Channel, Feet	Range of Observed Heads (h)		Range of Velocity of Approach (V_A)		Piezometer Located (Feet)		Volume of Flow, How Determined
								From	To	From	To	Up-stream	Below Crest Level	
A	1877	31	None	4.994 to 5	5	12	3.17	.074	.820	.023	.639	6	2.4	In basin
B	1878	94	None	5	5	18 to 25	0.5; 1.0; 1.7; 2.6 and 3.56	.19	.94	.077	2.35	6	0.4	Kept constant not measured
C	1878	17	1 1 2	4 3.3 3	5 5 5	18 to 25	.5; 1.0; 1.7; 2.6 and 3.56	.56	.93	.23	1.41	6	0.4	Kept constant not measured
D	1878	54	None 1 2	5 3.3 and 4 2.3 and 3	5 5 5	25	3.56	.15	.95	.054	.556	6	2.4	By suppressed weir
E	1879	10	None	18.966	19	7.5	6.55	.47	1.6	.15	.84	6	0	In basin

On all suppressed weirs, the sides of the channel were prolonged downstream above the level of the crest.

water surfaces in two basins. One had a capacity of about
cubic feet, and the other a capacity of about 6400 cubic fe
These basins were formed by partitioning off convenient portic
of the Sudbury aqueduct.

In the experiments indicated as B and C, a constant but n
determined volume of flow was maintained by establishing
steady head upon certain orifices above the experimental wei

In the experiments indicated as D, the discharge was me
ured on the suppressed weir (5 feet long) previously calibrated

In the experiments (indicated as E) upon the suppressed w
19 feet long, the volume was measured by the rise of water su
faces in another portion of the Sudbury aqueduct, 11,600 feet lo
which had a measuring capacity of 300,000 cubic feet.

The time for all experiments was observed by a stop watch re
ing $\frac{1}{5}$ seconds, checked for long intervals by a chronometer. T
duration of experiments on the 5-foot weir varied from 200
3481 seconds; on the 19-foot weir from 2193 to 14,852 secon
The period of flow was regulated by the use of stop planks a
gates.

The results of Fteley and Stearns's experiments briefly stat
are as follows:

237. Formula for the suppressed weir 5 feet long. From t
31 experiments on the 5-foot weir, the following formula w
derived:

$$Q = 3.33\, LH^{\frac{3}{2}} + .0065\, L. \quad (\text{1}$$

H in this case was computed as equal to $(h + 1.8\, h_v)$.

238. Corrections for velocity of approach. From 94 experime
on suppressed weirs and 17 experiments on contracted weirs,
which a false bottom was used to vary the heights of the cre
the following values of α to be used in the expression $(h + \alpha h$,
by which the corrected head H is to be computed, were dete
mined for suppressed weirs. For weirs with two end contractio
add .55 to the value of α given in Table XXXII.

TABLE XXXII

VALUES OF a FOR SUPPRESSED WEIRS TO BE USED IN THE FORMULA $H = (h + ah_v)$ FOR VARIOUS VALUES OF p

OBSERVED HEAD IN FEET	$p = 0.5$*	$p = 1.0$	$p = 1.7$	$p = 2.6$ AND HIGHER
.2	1.70	1.87	1.66	1.51
.3	1.53	1.83	1.65	1.50
.4	1.53	1.79	1.63	1.49
.6	1.52	1.71	1.60	1.47
.8	1.50	1.65	1.57	1.45
.9	1.49	1.63	1.56	1.44
1.0	1.48	1.61	1.54	1.43
1.2		1.57	1.51	1.41
1.4		1.54	1.48	1.39
1.6		1.51	1.44	1.37
1.8			1.41	1.35
2.0			1.38	1.33

Values below the line are beyond the limits of actual experiment.

The above values are to be used where considerable refinement of calculation is desired.

For average values of a correction for velocity of approach to be used in a general formula, Fteley and Stearns deduced the following expressions:

For suppressed weirs, $H = h + 1.5\ h_v$.

For weirs with two end contractions, $H = h + 2.05\ h_v$.

239. End contractions. Fifty-four experiments were made on the 5-foot weir: first with no end contractions, and afterwards with the same weir shortened to provide one contraction and later two; the discharge was made steady and measured over the 5-foot suppressed weir; then without changing the volume of flow the contractions were set in one at a time and the effect of each change noted by the change in head. From these experiments Fteley and Stearns reach the following conclusions:

The contraction did not increase regularly with the increase in head.

With different forms of weirs the effect of contraction had a variable effect, not sufficiently determined to be defined as a law.

In the Francis factor $\left(L - \frac{NH}{10}\right)$, $\frac{H}{10}$ was found to be n constant value. It varied with perfect contraction from .06 to .124 H.*

As the end contractions can not be properly compensated they should for accurate measurements be avoided; or if unav able, one instead of two should be used.

Being unable to offer a satisfactory substitute, they rec mended Francis's method of making this correction as be reasonably correct.

240. Formula for the suppressed weir 19 feet long. From 10 experiments on the suppressed weir, 18.966 feet in length, following formula was derived:

$$Q = 3.291\, LH^{\frac{3}{2}} + .004\, L. \qquad ($$
$$H = (h + 2.4\, h_v).$$

241. Recomputation of Francis's experimental results. F a recomputation of J. B. Francis's experiments, the follow formula was found to fit his observations:

$$Q = 3.313\, LH^{\frac{3}{2}} + .006\, L. \qquad ($$

242. Fteley and Stearns's general formula. From an exam tion of their own and Francis's results they proposed the gen formula:

$$Q = 3.31\, LH^{\frac{3}{2}} + .007\, L. \qquad ($$
H for suppressed weirs $= (h + 1.5\, h_v)$.
H for contracted weirs (2 ends) $= (h + 2.05\, h_v)$.

For the net length of contracted weirs use $(L - 0.1\, NH)$.

The present practice largely follows the **Fteley and Stea formula for heads from .07 to .5 foot** and **beyond this the Fra formula**; Table LXIV (Appendix) is computed in this man Fteley and Stearns also devoted considerable attention to question of measuring the head somewhere within the area increased pressure and of no velocity (see figure 78) upstre from and just behind the weir, which Francis had mention but they did not recommend measuring the head in this a They suggested that the piezometer orifice may be set at any de

* H being equal to $(h + \alpha h_v)$; α being equal to 1.5 and 2.05 according to t of weir.

if located upstream from the weir at least $2\frac{1}{2}$ times p; but if much nearer than this, the opening should not be much below mid depth.

Figure 85 is a diagram for determining the discharge of weirs. It is based on the Fteley and Stearns formula up to heads of .5

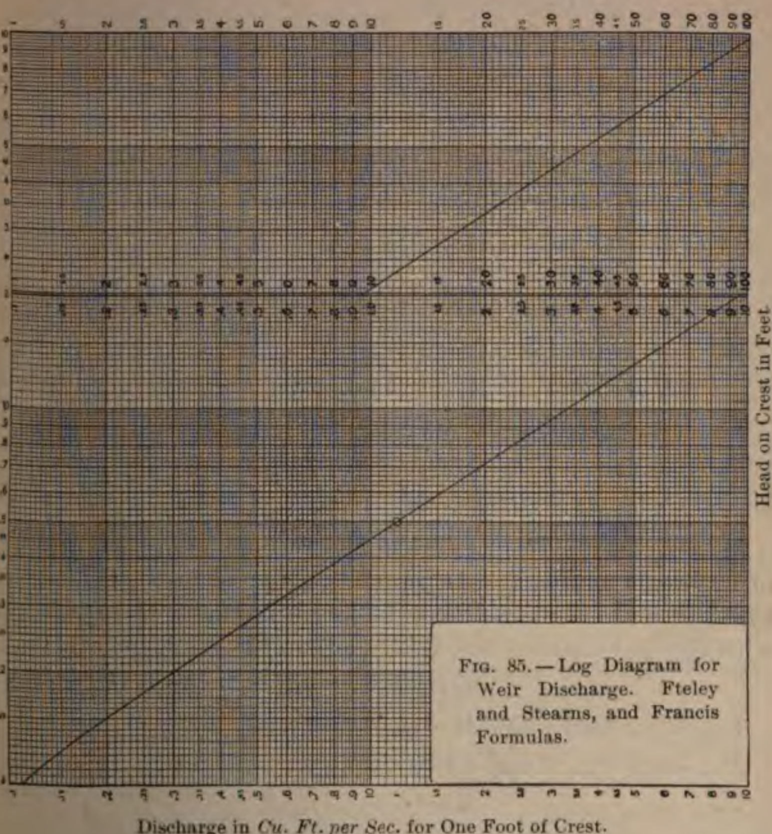


Fig. 85.—Log Diagram for Weir Discharge. Fteley and Stearns, and Francis Formulas.

foot, and the Francis formula for higher heads, for a weir one foot long; for longer weirs the discharge is proportional to the length (corrected for end contractions, if any).

Examples. (*a*) Compute the discharge through a suppressed weir 6 feet long, the crest of which is 1.50 feet above the bottom of the channel, when the observed head is .50 ft.

Neglecting velocity of approach,

$$Q = 3.31 \times 6. \times .5^{\frac{3}{2}} + .007 \times 6. = 7.06.$$

Correcting for velocity of approach,

$$V_A = \frac{7.06}{6 \times (1.50 + .50)} = .59; \; h_v = .0054;$$

$$H = .5000 + 1.5(.0054) = .508;$$

Then $Q = 3.31 \times 6. \times .508^{\frac{3}{2}} + .007 \times 6. = 7.24$ cu. ft. sec.

(*b*) Compute the discharge through a weir with two end co tractions 6 feet long, with its crest 1.50 feet above the bottom the channel; channel 9 feet wide. Observed head .50 foot.

$$Q = 3.31[6 - (.2 \times .5)] \times .5^{\frac{3}{2}} + .007[6 - (.2 \times .5)]$$
$$= 6.94.$$

$$V_A = \frac{6.94}{9 \times (1.5 + .5)} = .39; \; h_v = .0024;$$

$$H = .500 + 2.05 \times .0024 = .5049;$$

$$Q = 3.31 \times 5.9 \times .5049^{\frac{3}{2}} + .007 \times 5.9 = 7.05 \text{ cu. ft. sec.}$$

INVESTIGATIONS OF HAMILTON SMITH, JR.

243. H. Smith, Jr.,* reviewed and recomputed from the resu of Lesbros, Poncelet and Lesbros, Francis, and Fteley and Stear together with 12 experiments of his own on a weir 2.6 feet lo with two end contractions, values of c in the fundamental form

$$Q = c\,\tfrac{2}{3}(2g)^{\frac{1}{2}} L H^{\frac{3}{2}}. \qquad (1$$

For suppressed weirs, $H = (h + 1\tfrac{1}{3}\,h_v)$,

For weirs with 2 end contractions,

$$H = (h + 1.4\,h_v);$$

Whence
$$c = \frac{Q(\text{known by experiments})}{\tfrac{2}{3}(2g)^{\frac{1}{2}} L H^{\frac{3}{2}}}.$$

The values of c thus determined were plotted and throu them curves were drawn for weirs of different form and leng Finally these values of c were tabulated as given in Table LX (Appendix); different coefficients c_s and c_c were established suppressed and contracted weirs.

* *Hydraulics.*

Bazin Experiments on Sharp-crested Weirs with Free Overfall

244. Henri Bazin's experiments, beginning in 1886* and extending over many subsequent years, are the most notable contribution to the science of weir measurements. His early investigations furnished much new data in regard to sharp-crested weirs, especially concerning the effects of variations in height of crest and in velocity of approach; and his subsequent work on weirs of irregular sections and modified nappes opened a field of knowledge almost unexplored.

Bazin's experiments on sharp-crested vertical weirs having no end contractions comprised:

(1) The calibration of a vertical, sharp-crested weir, the nappe falling freely in the air; the volume of flow being measured in a large basin. This became his standard weir.

(2) Calibration of sharp-crested weirs having different heights (p) of crest; the volume of flow being determined by placing them in tandem with the standard weir.

245. **Calibration of the standard weir.** The weir was set in a rectangular channel of smooth-faced concrete 6.56 feet (2 m.) wide,

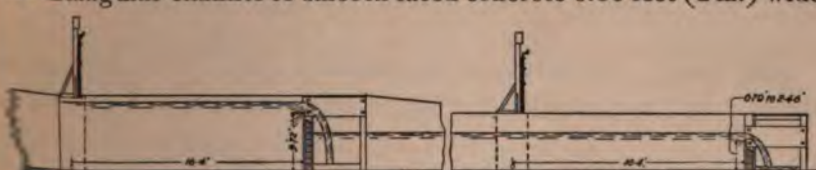


Fig. 86. — Bazin's Weirs set in Tandem.

5.25 feet (1.6 m.) deep, and straight for 49.2 feet (15 m.) upstream from the weir. See figure 86.

At the downstream face of the weir two recesses were built in the sides of the channel to provide free access of air underneath the nappe; but to prevent lateral expansion of the nappe the sides of the channel were prolonged downstream from above the level of the crest.

246. **Measurement of volume.** Below the weir was a channel used as a measuring basin 6.56 feet (2 m.) wide, 3.94 feet (1.2 m.)

* *Annales des Ponts et Chaussées*, Mem. et Doc., 1888, 2d semestre, pp. 393–448.

deep, and 656 feet (200 m.) long, having a bottom slope of ab 1 in 1000, made of concrete with a smooth cement lining. ' volume of the basin at different elevations and the proper all ance for leakage being determined, the rise in the elevation of water surface gave the discharge.

The flow was regulated by means of sluices set upstream. wooden stop-gate was also placed directly on the top of the w The duration of experiments was from 8 to 40 seconds.

247. Measurement of head. 16.4 feet (5 m.) upstream from weir crest there was constructed a lateral chamber 1.64 (.5 m.) square, communicating with the channel of approach b opening .33 foot (.1 m.) in diameter, at the bottom of the cha and normal to and flush with the side wall of the channel.

A hook gauge and a dial float for observing the head were in this chamber. By the dial float the fluctuations of the w surface were observed at very frequent intervals and from t the mean position of the hand of the dial was determined; by the hook gauge the elevation of the water in the chamber responding to that mean position. The hook gauge and crest were referenced to permanent iron bench marks set in walls of the channel.

248. Limits of experiments on standard weir. The rating the standard weir comprised three series of experiments.

Series No.	Number of Experiments	L Length of Weir	p Height of Weir	Range of Observed Head in Feet	Range of Velo of Approach Feet per Sec
1	67	6.56 ft.	3.72 ft.	.194 to 1.012 ft.	.08 to 0.74
2	38	3.28 ft.	3.72 ft.	.188 to 1.339 ft.	.07 to 1.02
3	48	1.64 ft.	3.30 ft.	.191 to 1.780 ft.	.08 to 1.62

249. Formula adopted. Bazin adopted the common form for discharge

$$Q = mLh(2gh)^{\frac{1}{2}}, \qquad ($$

where m is the coefficient of discharge and includes the effec velocity of approach and of all variable factors excepting head.

From the values of Q determined volumetrically, values of m corresponding to the different values of observed heads were computed by the equation

$$m = \frac{Q}{Lh(2gh)^{\frac{1}{2}}}.$$

The mean values of m for series 1 and 2, with values for series 3 (which were less precise) for heads above 1.3 feet, were plotted against the corresponding observed heads. Through the midst of these points a smooth curve was drawn averaging errors of observation; and from this curve values of m were tabulated. See column 2 in Table XXXIII. Bazin found that these values from the curve, on the average, agreed within $\frac{1}{300}$ with the original observations; in a few extreme cases within only $\frac{1}{100}$.

250. Experiments on weirs of different heights (p) of crest. Bazin, having established coefficients for his standard weir, set up, one at a time, four other vertical sharp-crested weirs in tandem with his standard weir. These weirs were erected below the original weir in the channel, which was used in the first experiments as a measuring basin. The head was read on each of these lower weirs, through a hole 16.4 feet upstream in the manner described for the standard weir.* See figure 86. Thus, as the same volume of flow Q passed over both weirs in succession, the value of m (which may be called m') for each of the four lower weirs was computed by comparison as follows:

For the weir p' feet high, $\quad Q = L'm' (2g)^{\frac{1}{2}} h'^{\frac{3}{2}}.$

For the standard weir, $\quad Q = Lm (2g)^{\frac{1}{2}} h^{\frac{3}{2}}.$

Therefore,
$$m' = m\frac{L}{L'}\left(\frac{h}{h'}\right)^{\frac{3}{2}}. \tag{18}$$

The lengths L', L, etc., were constructed as nearly as possible equal to 6.56 feet (2 m.); but as discrepancies occurred, actual lengths had to be used in computations.

Limits of experiments on weirs of various heights. The experiments comprised the following series:

* During these experiments the head was also measured on the standard weir, 16.4 feet upstream, except in series 4, 6, and 8, when it was moved to 32.8 feet upstream.

[illegible] No.	Number of Experiments	p Height of Weir in Feet	Range of Observed Heads (h) in Feet
4 and 5	38	2.46	.26 to 1.44
6 and 7	56	1.64	.26 to 1.41
8 and 9	66	1.15	.29 to 1.38
10	45	.79	.30 to 1.35

251. The values of m', having been computed by comparison with the standard weir, by (18) were plotted to scale against the observed heads; and curves dra[illegible] the standard weir, from which values of the coefficient m' [illegible]lated for each of these four weirs. See columns 3, 4, 5, [illegible] Table XXXIII.

252. **Bazin's general formula.** [illegible] then sought a general formula by which the value of m [illegible] should include a correction for the velocity of approach co[illegible] calculated.

Replacing h by $h + \alpha\dfrac{V_A^2}{2g}$ in form[illegible] (17) and calling μ the value of m when there is no velocit[illegible]f approach, formula (17) becomes:

$$Q = \mu L\left(h + \alpha\frac{V_A^2}{2g}\right)\left[2g\left(h + \alpha\frac{V_A^2}{2g}\right)\right]^{\frac{1}{2}} = \mu L h (2gh)^{\frac{1}{2}}\left(1 + \alpha\frac{V_A^2}{2gh}\right)^{\frac{3}{2}}. \quad \text{(A)}$$

$$\text{Whence, } m = \mu\left(1 + \alpha\frac{V_A^2}{2gh}\right)^{\frac{3}{2}} = \text{(very nearly), } \mu\left(1 + \frac{3}{2}\alpha\frac{V_A^2}{2gh}\right).\text{*} \quad \text{(B)}$$

$$\text{Or since } V_A = \frac{Q}{L(p+h)}; \quad m = \mu\left[1 + K\left(\frac{h}{p+h}\right)^2\right]. \quad \text{(C)}$$

From an elaborate study of his own observations and of Fteley and Stearns's experiments, Bazin established a mean value of α equal to $\frac{5}{3}$; and a mean value of K equal to .55, and adopted as his general formulas:

$$m = \mu\left[1 + .55\left(\frac{h}{p+h}\right)^2\right]. \quad (19)$$

$$Q = \mu\left[1 + .55\left(\frac{h}{p+h}\right)^2\right] L h (2gh)^{\frac{1}{2}}. \quad (20)$$

* For a mathematical deduction of this formula, not given by Bazin, see translation, by Marichal and Trautwine, *Proc. Engr's. Club, Phila.*, pp. 280–281.

TABLE XXXIII

EXPERIMENTAL VALUES OF m* IN BAZIN'S FORMULA

$Q = mLh(2hg)^{\frac{1}{2}}$ for different values of height (p) and head (h). Gotten by a replotting of Bazin's curves. Horizontal lines indicate limits of experiment

OBSERVED HEADS IN FEET	2 $p = 3.72'$	3 $p = 2.46'$	4 $p = 1.64'$	5 $p = 1.15'$	6 $p = .79'$
.164	.4485	.4487	.4490	.4495	.4505
.2	.4428	.4435	.4441	.4451	.4466
.3	.4349	.4359	.4369	.4389	.4442
.4	.4308	.4325	.4349	.4397	.4476
.5	.4283	.4316	.4360	.4428	.4527
.6	.4268	.4321	.4379	.4467	.4584
.7	.4260	.4332	.4403	.4511	.4646
.8	.4258	.4347	.4430	.4555	.4707
.9	.4262	.4361	.4459	.4600	.4769
1.0	.4267	.4377	.4489	.4646	.4832
1.1	.4272	.4393	.4519	.4692	.4896
1.2	.4278	.4410	.4550	.4740	.4961
1.3	.4285	.4428	.4581	.4788	.5026
1.4	.4293	.4444	.4612		
1.5	.4301				
1.6	.4309				
1.7	.4318				
1.8	.4326				
1.9	.4335				
2.0	.4344				
	Series 1–2–3	4 and 5	6 and 7	8 and 9	10

The value of μ. Theoretically when there is no velocity of approach, that is, when the value of p = infinity, m will attain its limiting value, which Bazin designated μ and computed from values of m for the standard weir ($p = 3.72$); very nearly

$$\mu = \frac{m}{1 + .55\left(\frac{h}{3.72 + h}\right)^2}. \tag{21}$$

* Bazin's values were plotted for (h) and (p) in metres, inconvenient for use and interpolation by American readers. We have therefore replotted these curves from their tabulated coördinates (tables on pp. 413 and 433, *Ann. des Ponts et Chaussées*, 2d semestre, 1888) to an open scale and from these curves taken values of m corresponding to (h) and (p) in feet, further checking them by interpolation from the same original tables.

253. Tabulated computations of m and 8.02 m; μ and 8.02 μ Using the values of μ so computed, Bazin calculated values of m for various heads (h) and heights of weir (p) in metres by hi formula (19), and tabulated the results.*

The values of m thus computed, Bazin found to agree with hi experimental coefficients within $\frac{1}{100}$.

In Table, XXXIV, Bazin's coefficients in terms of feet ar given.

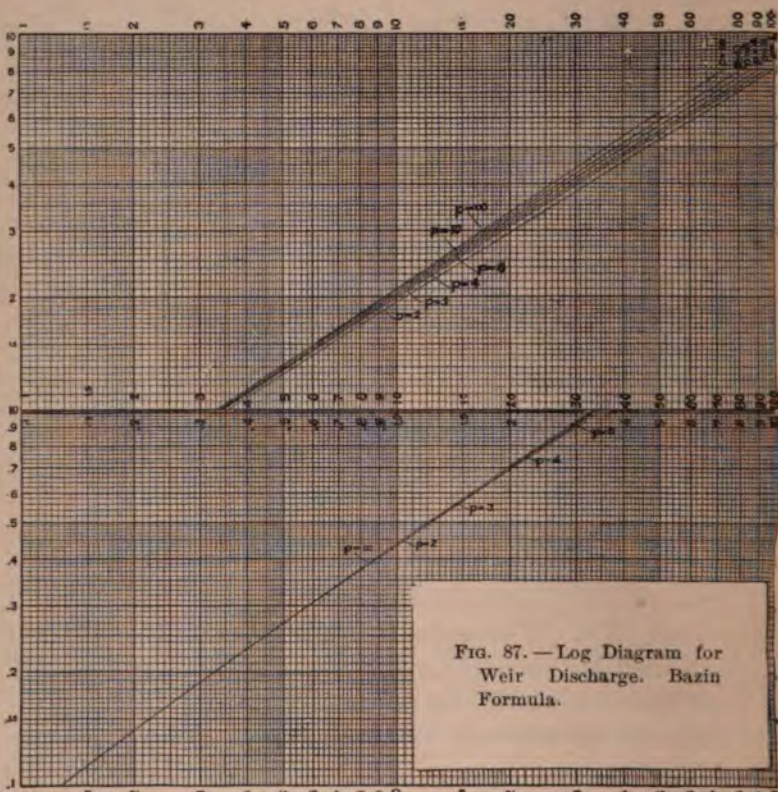


Fig. 87.—Log Diagram for Weir Discharge. Bazin Formula.

Discharge in *Cu. Ft. per Sec.* for One Foot of Crest.

Figure 87 is a diagram based on Table XXXIV for computin the discharge by Bazin's formula.

* *Annales des Ponts et Chaussées*, Vol. 16, 1888, p. 446.

TABLE XXXIV

VALUES OF m AND 8.02 m; μ AND 8.02 μ CORRESPONDING TO HEADS (h) AND HEIGHTS OF WEIR (p) IN FEET FOR USE IN BAZIN'S FORMULA $Q=mL(2g)^{\frac{1}{2}}h^{\frac{3}{2}}$ *

h	$p=$ 0.656 m 8.02 m	$p=$ 1.0 m 8.02 m	$p=$ 1.5 m 8.02 m	$p=$ 2 m 8.02 m	$p=$ 2.5 m 8.02 m	$p=$ 3 m 8.02 m	$p=$ 4 m 8.02 m	$p=$ 5 m 8.02 m	$p=$ 6 m 8.02 m	$p=$ 6.56 m 8.02 m	μ 8.02 μ
.2	.456	.449	.446	.444	.444	.443	.443	.443	.443	.443	.4423
	3.657	3.601	3.577	3.561	3.561	3.553	3.553	3.553	3.553	3.553	3.547
.3	.457	.446	.440	.438	.436	.436	.435	.435	.434	.434	.4337
	3.665	3.577	3.529	3.513	3.497	3.497	3.489	3.489	3.481	3.481	3.478
.4	.463	.448	.439	.435	.433	.432	.431	.430	.430	.430	.4288
	3.713	3.593	3.521	3.489	3.473	3.465	3.457	3.449	3.449	3.449	3.439
.5	.469	.451	.440	.435	.432	.430	.428	.427	.427	.427	.4254
	3.761	3.617	3.529	3.489	3.465	3.449	3.433	3.425	3.425	3.425	3.412
.6	.476	.455	.442	.435	.431	.429	.427	.425	.425	.424	.4227
	3.818	3.649	3.545	3.489	3.457	3.441	3.425	3.409	3.409	3.400	3.390
.7	.482	.460	.444	.436	.432	.429	.426	.424	.423	.423	.4207
	3.866	3.689	3.561	3.497	3.465	3.441	3.417	3.400	3.392	3.392	3.374
.8	.489	.465	.447	.438	.433	.430	.426	.424	.423	.422	.4193
	3.922	3.729	3.585	3.513	3.473	3.449	3.417	3.400	3.392	3.384	3.363
.9	.495	.470	.451	.440	.434	.430	.426	.424	.422	.422	.4182
	3.970	3.769	3.617	3.529	3.481	3.449	3.417	3.400	3.384	3.384	3.354
1.0	.501	.475	.454	.443	.436	.432	.426	.424	.422	.421	.4172
	4.018	3.810	3.641	3.553	3.497	3.465	3.417	3.400	3.384	3.376	3.346
1.1		.479	.457	.445	.438	.433	.427	.424	.422	.421	.4164
		3.842	3.665	3.569	3.513	3.473	3.425	3.400	3.384	3.376	3.340
1.2		.483	.461	.448	.439	.434	.428	.424	.422	.421	.4154
		3.874	3.697	3.593	3.521	3.481	3.433	3.400	3.384	3.376	3.332
1.3		.487	.464	.450	.441	.435	.428	.424	.422	.421	.4146
		3.906	3.721	3.609	3.537	3.489	3.433	3.400	3.384	3.376	3.325
1.4		.491	.467	.452	.443	.437	.429	.425	.422	.421	.4137
		3.938	3.745	3.625	3.553	3.505	3.441	3.409	3.384	3.376	3.318
1.5		.495	.470	.455	.445	.438	.430	.425	.422	.421	.4129
		3.970	3.769	3.649	3.569	3.513	3.449	3.409	3.384	3.376	3.311
1.6			.473	.457	.447	.440	.431	.425	.422	.421	.4121
			3.793	3.665	3.585	3.529	3.457	3.409	3.384	3.376	3.305
1.7			.475	.459	.448	.441	.431	.426	.422	.421	.4112
			3.810	3.681	3.593	3.537	3.457	3.417	3.384	3.376	3.298
1.8			.478	.461	.450	.442	.432	.426	.422	.421	.4104
			3.834	3.697	3.609	3.545	3.465	3.417	3.384	3.376	3.291
1.9			.480	.463	.452	.444	.433	.427	.423	.421	.4096
			3.850	3.713	3.625	3.561	3.473	3.425	3.392	3.376	3.285
2.0			.483	.465	.453	.445	.434	.427	.423	.421	.4090
			3.874	3.729	3.633	3.569	3.481	3.425	3.392	3.376	3.280

* This table was constructed by the authors as follows: Values of Bazin's μ were plotted to an open scale against the value of head in metres expressed as feet; through the plotted points a curve was drawn, and from this curve values of μ corresponding to $\frac{1}{10}$ ft. were taken, the values being further checked by proportional differences from Bazin's table. Based on these values of μ this table was computed by Bazin's formula exactly as his own original table, with which by interpolation the values were checked.

254. An approximate formula for discharge, agreeing table within $\frac{1}{100}$. Bazin computed an approximate

$$\mu = .405 + \frac{.00984}{h}.$$

$$Q = \left(.405 + \frac{.00984}{h}\right)\left[1 + 0.55\left(\frac{h}{p+h}\right)^2\right]L(2g)^{\frac{1}{2}}h^{\frac{3}{2}}.$$

255. A rougher approximate formula, usable for heads 1 foot when errors of 3 per cent in the formula are admis

$$m = .425\left[1 + \tfrac{1}{2}\left(\frac{h}{p+h}\right)^2\right] = .425 + .21\left(\frac{h}{h+p}\right)^2.$$

$$Q = \left[.425 + .21\left(\frac{h}{p+h}\right)^2\right]L(2g)^{\frac{1}{2}}h^{\frac{3}{2}}.$$

NOTE. All Bazin's formulas given in this book are in terms of feet.

Example. Given, a suppressed weir 10 feet long, with 2 feet high. Compute the discharge in cubic feet per secc the observed head is .9 foot.

By formula (20) and Table XXXIV,

$$Q = .4182\left[1 + .55\left(\frac{.9}{2.0+.9}\right)^2\right]10 \times .9 \times 8.02 \times .9^{\frac{1}{2}},$$

or $Q = 3.529 \times 10 \times .9^{\frac{3}{2}} = 30.13.$

By formula (22),

$$Q = \left(.405 + \frac{.00984}{.9}\right)\left[1 + .55\left(\frac{.9}{2.0+.9}\right)^2\right]10 \times 8.02 \times .9^{\frac{3}{2}}$$

By formula (23),

$$Q = \left[.425 + .21\left(\frac{.9}{2.0+.9}\right)^2\right]10 \times 8.02 \times .9^{\frac{3}{2}} = 30.48.$$

UNITED STATES DEEP WATERWAYS EXPERIMENTS* ON CRESTED WEIRS WITH FREE OVERFALL

256. A series of experiments was carried out at the University Hydraulic Laboratory in 1899 to determine t of C in the Francis formula for heads above 2 feet. charge was measured on a weir 16 feet long and 13.13 f

* G. W. Rafter and others, at the Cornell Hydraulic Laborator *Am. Soc. C. E.*, Vol. 44, p. 397. Recomputed by R. E. Horton, *U. S. G W. S. and I. Paper*, No. 200.

and computed by Bazin's formula; and the stream was allowed to flow unchanged in volume over a weir placed in tandem with it 6.56 feet long and 5.2 feet high. The results of these experiments, as recomputed by R. E. Horton, are given in Table XXXV.

TABLE XXXV

U. S. D. W. EXPERIMENTS ON SHARP-CRESTED WEIRS

STANDARD WEIR, 16 FEET LONG, 13.18 FEET HIGH	LOWER THIN-EDGED WEIR: $p = 5.2$, $L = 6.56$					
Q, Bazin Formula, in Cubic Feet Per Second	h in Feet	$\frac{h}{p+h}$	K, Hunking and Hart Formula	$H^{\frac{3}{2}}$	Q, Cubic Feet Per Second, Per Foot of Length (corrected)	$C = \frac{Q}{H^{\frac{3}{2}}}$
14.12	0.7462	0.1255	1.0041	0.6469	2.1066	3.256
19.42	0.9139	0.1495	1.0056	0.8787	2.9143	3.317
25.35	1.0885	0.1731	1.0075	1.1434	3.8183	3.331
32.24	1.2933	0.1992	1.0099	1.4849	4.8685	3.279
37.86	1.4436	0.2173	1.0122	1.7564	5.7252	3.260
45.13	1.6306	0.2387	1.0141	2.1116	6.8333	3.236
52.62	1.8115	0.2583	1.0166	2.4787	7.9750	3.218
52.77	1.8088	0.2581	1.0166	2.4730	7.9977	3.234
73.46	2.2389	0.3010	1.0225	3.4254	11.1516	3.226
92.79	2.6434	0.3370	1.0283	4.4193	14.0960	3.190
143.90	3.4660	0.4000	1.0398	6.6095	21.8902	3.312
202.37	4.2747	0.4512	1.0504	9.2867	30.8008	3.317
233.81	4.6773	0.4735	1.0557	10.6789	35.5933	3.333

CHOICE OF FORMULAS

257. When Francis weir settings can be duplicated or the velocity of approach is very low, one foot per second or less, there is general willingness on the part of both engineers and laymen to accept his formula for heads for from .5 to 2 feet, and the same is true of the Fteley and Stearns formula for heads of .07 to .5 foot. For higher heads the Cornell experiments, which are the only guides, indicate that the Francis formula may be used with reasonable accuracy up to heads of 5 feet.

Bazin's formula is the best where his conditions can be reproduced; and if the velocity of approach is high and the height of

weir low, his formula is the only one sufficiently flexible; for this reason it is the most useful.

Smith's coefficients are the result of the most thorough study; but are based upon experimental data of somewhat unequal accuracy. They do, however, furnish means for satisfactory interpolation to suit cases not covered precisely by the data which he used.

If possible, contracted weirs should be avoided, but are often necessary to insure atmospheric pressure underneath the nappe; if end contractions are unavoidable, the Francis formula should be used.

For rough measurements there has never appeared to be any good reason for departing from the Francis formula, which has the advantage of long usage and consequent familiarity, especially in legal cases; although it has often been used far beyond the limits laid by Mr. Francis himself. It should be borne in mind, however, that his formula applies only to sharp-crested weirs.

258. Precision obtainable in measurements with suppressed weirs. Where conditions match the original experiments and are otherwise favorable, measurements by weir and those made by other methods have repeatedly shown an agreement within 1 per cent. Higher precision can rarely be attained, even with the utmost care; if conditions are nearly duplicated, but the surroundings are unfavorable, errors of from 1 to 5 per cent may occur.

For contracted weirs duplicating Francis conditions, errors of about 2 per cent may occur; and in other cases the accuracy will vary.

Triangular and Trapezoidal Weirs

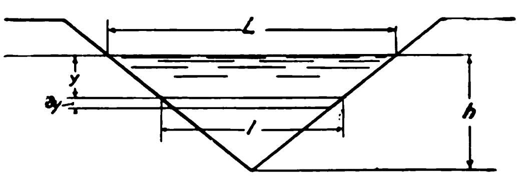


Fig. 88. — Triangular Weir.

259. Triangular weirs. For any orifice the discharge through any elementary lamina such as $l\delta y$ equals the area times the velocity due to the head on its center. See figure 88.

The theoretic discharge $\delta Q_t = l \delta y \, (2\,gy)^{\frac{1}{2}}$.

$$\text{But } \frac{l}{L} = \frac{h-y}{h}.$$

$$\text{Therefore } Q_t = \int_0^h (2\,g)^{\frac{1}{2}} \frac{L(h-y)}{h} y^{\frac{1}{2}} \delta y.$$

$$\text{Hence } Q_t = \tfrac{4}{15} L\,(2\,g)^{\frac{1}{2}} h^{\frac{3}{2}}. \qquad (24)$$

Professor Thompson (of Belfast) deduced experimentally a value of $C = .617$ for heads of .2 to .8 foot.

$$\text{Hence } Q = .617 \times \frac{4\,L}{15}(2\,g)^{\frac{1}{2}} h^{\frac{3}{2}} = 1.32\ L h^{\frac{3}{2}}. \qquad (25)$$

For right-angled notches $L = 2\,h$; and therefore

$$Q = 2.64\,h^{\frac{5}{2}}. \qquad (26)$$

260. Trapezoidal weirs. As will be seen by figure 89, the discharge is equivalent to the sum of the discharges of a sup-

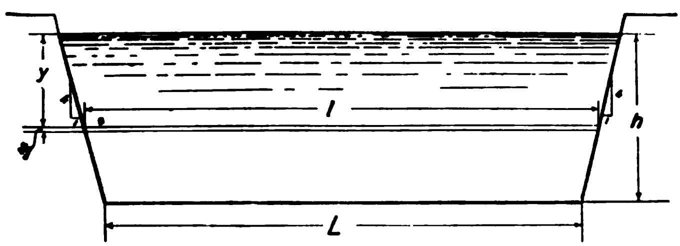


FIG. 89. — Trapezoidal Weir.

pressed weir and of one triangular weir of height (h) and each side having a batter z; hence neglecting contractions,

$$Q_t = \tfrac{2}{3}(2\,g)^{\frac{1}{2}} L h^{\frac{3}{2}} + \tfrac{4}{15}\, 2\,z\,(2\,g)^{\frac{1}{2}} h^{\frac{5}{2}}. \qquad (27)$$

When the slope $z = \frac{1}{4}$, which, as has been shown by Horton,* is just about sufficient to offset the effect of end contractions, the weir is called a Cippoletti weir, after an Italian engineer, who first described it and derived from the Francis experiments a coefficient which (or its equivalent 3.367) is now used. Hence

$$Q = 3.367\ L h^{\frac{3}{2}}. \qquad (28)$$

Correction for velocity of approach with the Cippoletti weir is made as for the Francis formula. The advantage of this weir

* *W. S. & I. Paper*, 200.

lies in that if a suppressed weir can not be used, with this form of weir a constant value of L may be taken, thus obviating the tedious computations required to compute the effect of end contractions. For continuous use where many varying heads must be registered and computed, it is convenient; and it is much used in irrigation practice in the western part of this country.

By adding 1 per cent to the values of Q in the weir table (LXIV), for Francis's formula, the discharge for a Cippoletti weir may be computed.

Weirs with Crest not Level

261. When the weir crest is not about level, accurate measurement of discharge can not be made; but unless the crest is too uneven, the average depth on it may be used. With the form of weir shown in figure 90 the method suggested by R. E. Horton is probably as good as any.

The flow through the elementary width δl is

$$\delta Q = Ch^{\frac{3}{2}}\,\delta l.$$

$$h = h_1 + \frac{h_2 - h_1}{L}\,l.$$

Total discharge, $$Q = \int_0 Ch^{\frac{3}{2}}\,\delta l = C\int_0^L \left(h_1 + \frac{h_2 - h_1}{L}\,l\right)^{\frac{3}{2}} \delta l.$$

Integrating, $$Q = \frac{2\,CL}{5\,(h_2 - h_1)}\left(h_2^{\frac{5}{2}} - h_1^{\frac{5}{2}}\right). \qquad (29)$$

In this formula, if for a sharp-crested weir, either the mean coefficient deduced by Thompson for a triangular weir, or that of Francis, may be used. If there are end contractions, reduce the length by

$$.2\left(\frac{h_1 + h_2}{2}\right).$$

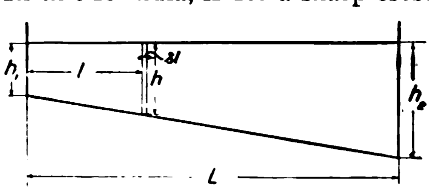


Fig. 90.

If the crest is not sharp, use an experimental value of C which most nearly fits the conditions.

Satisfactory results can not be expected in any case where the crest is not nearly level.

SUBMERGED WEIRS

262. If the water on the downstream face of a weir is higher than the crest, the weir is said to be submerged. See figure 91. With our present knowledge of coefficients such an apparatus is not to be selected for water measurements if other types of weirs can be installed. Not infrequently, however, the existence of a submerged weir in a stream provides a very valuable element in getting a permanent record of flow. Moreover, the effect of a submerged dam in raising the water may be determined approximately by computing the head needed to discharge the required volume of water, and offers a means of determining the effect of back water.

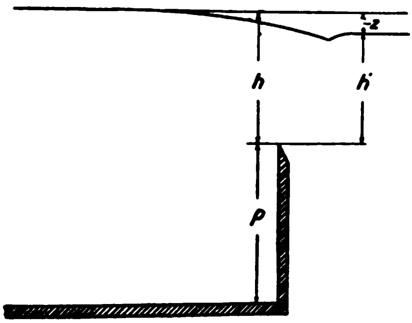


FIG. 91. — Submerged Weir.

The available experimental information is chiefly derived from a few experiments by Francis, some by Fteley and Stearns, and experiments of considerable range by Bazin.

h = depth of water measured to the crest level on the upstream side.

h' = depth of water measured to the crest level on the downstream side.

$z = h - h'$.

p = height of weir.

263. **Francis and Fteley and Stearns experiments.** The Francis experiments* (1848), six in number, included values of h from .85 to .97 foot, and of h' from .02 to .49 foot. The Fteley and Stearns experiments† (1877), twenty-two in number, included values of h from .33 to .81 foot, and of h' from .01 to .79 foot. In Fteley and Stearns's experiments end contractions were suppressed; and the heads h and h' were measured by piezometers each 6 feet from the crest. The method of conduct-

* *Lowell Hydraulic Experiments*, p. 102.

† *Trans. Am. Soc. C. E.*, Vol. 12, pp. 101-108.

ing the experiments was to submerge the weir after a st known flow had been established and note the effect of mergence upon the head. The Francis formula was used Fteley and Stearns in computing the flow passing the measu weir. From their own and Francis's experiments, Fteley Stearns derived the following formula and coefficients.

Fteley and Stearns formula for sharp-crested submerged w having no end contractions.

$$Q = CL\left(h + \frac{h'}{2}\right)(z)^{\frac{1}{2}}. \qquad ($$

TABLE XXXVI

COEFFICIENTS C FOR THE FTELEY AND STEARNS SUBMERGED-WEIR FORM

$\frac{h'}{h}$	0	1	2	3	4	5	6	7	8	
.0	—	3.33	3.33	3.34	3.34	3.36	3.37	3.37	3.37	[illegible]
0.1	3.37	3.36	3.35	3.34	3.34	3.33	3.32	3.31	3.30	[illegible]
0.2	3.29	3.28	3.27	3.26	3.26	3.25	3.24	3.23	3.23	[illegible]
0.3	3.21	3.21	3.20	3.19	3.19	3.18	3.18	3.17	3.17	[illegible]
0.4	3.16	3.15	3.15	3.14	3.14	3.13	3.13	3.12	3.12	[illegible]
0.5	3.11	3.11	3.11	3.10	3.10	3.10	3.10	3.10	3.10	[illegible]
0.6	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	[illegible]
0.7	3.09	3.09	3.10	3.10	3.10	3.10	3.11	3.11	3.11	3
0.8	3.12	3.13	3.13	3.14	3.14	3.15	3.16	3.16	3.17	3
0.9	3.19	3.20	3.21	3.22	3.23	3.25	3.26	3.28	3.30	3
1.0	3.36									

The correction for the effect of velocity of approach ma made by the Francis method, or simply by adding to h, $\frac{V_A^2}{2g}$.

The Fteley and Stearns formula is not applicable for valu $\frac{h'}{h}$ less than .08 unless air has free access under the nappe; is probably applicable beyond the limits of the experiments.

264. Herschel formula for sharp-crested submerged weirs, der from a recomputation of the Francis and the Fteley and Ste experiments, may be expressed as follows: *

* R. E. Horton, *W. S. & I. Paper*, 200, p. 140.

$$Q = 3.33\,L(nh)^{\frac{3}{2}}. \tag{31}$$

n = a factor depending on the ratio, $\frac{h'}{h}$.

Table XXXVII contains values of n.

TABLE XXXVII

COEFFICIENT n, HERSCHEL'S SUBMERGED-WEIR FORMULA.*

$\frac{h'}{h}$	0	1	2	3	4	5	6	7	8	9
0.0	1.000	1.004	1.006	1.006	1.007	1.007	1.007	1.006	1.006	1.005
0.1	1.005	1.003	1.002	1.000	0.998	0.996	0.994	0.992	0.989	0.987
0.2	0.985	0.982	0.980	0.977	0.975	0.972	0.970	0.967	0.964	0.961
0.3	0.959	0.956	0.953	0.950	0.947	0.944	0.941	0.938	0.935	0.932
0.4	0.929	0.926	0.922	0.919	0.915	0.912	0.908	0.904	0.900	0.896
0.5	0.892	0.888	0.884	0.880	0.875	0.871	0.866	0.861	0.856	0.851
0.6	0.846	0.841	0.836	0.830	0.824	0.818	0.813	0.806	0.800	0.794
0.7	0.787	0.780	0.773	0.766	0.758	0.750	0.742	0.732	0.723	0.714
0.8	0.703	0.692	0.681	0.669	0.656	0.644	0.631	0.618	0.604	0.590
0.9	0.574	0.557	0.539	0.520	0.498	0.471	0.441	0.402	0.352	0.275

265. Bazin formula † for sharp-crested submerged weirs. Bazin proposed as a general formula for submerged weirs the following expression:

$$Q = m_s L h (2\,gh)^{\frac{1}{2}}. \tag{32}$$

Where $m_s = m \times 1.05\left(1.0 + .2\,\frac{h'}{p}\right)\left(\frac{z}{h}\right)^{\frac{1}{3}} = m\left(1.05 + .21\,\frac{h'}{p}\right)\left(\frac{z}{h}\right)^{\frac{1}{3}}$.

m = the coefficient for a similar sharp-crested weir with free overfall and full crest contraction, having the same value of h and p.

Bazin gave also two formulas more limited and more precise; but it is probable that the formula given is as precise as the ordinary use of submerged weirs will warrant.

Example. Given a submerged sharp-crest suppressed weir 10 feet long; the water upstream stands .49 foot above the crest, and downstream .16 foot; the height $(p) = 1.5$ feet. Compute the discharge by (*a*) the Fteley and Stearns, (*b*) the Herschel, (*c*) the Bazin formulas.

* *Van Nostrand's Magazine*, Vol. 34, p. 176.

† *Annales des Ponts et Chaussées*, Mem. et Doc., 2d trimestre, 1898, p. 235.

(a) $\frac{h'}{h} = \frac{.16}{.49} = .326$, then C (by table) $= 3.194$.

$Q = 3.194 \times 10\left(.49 + \frac{.16}{2}\right)(.49 - .16)^{\frac{1}{2}} = 10.45$ cubic feet second.

To allow for velocity of approach, $V_A = \frac{10.45}{10(1.5 + .49)} =$ foot per second.

H the corrected head $= .49 + \frac{.52^2}{64.32} = .4942.$

Then $Q = 3.196 \times 10\left(.4942 + \frac{.16}{2}\right)(.4942 - .16)^{\frac{1}{2}} = 10.61$ c feet per second.

(b) By table, $n = .9512$.

$Q = 3.33 \times 10(.9512 \times .49)^{\frac{3}{2}} = 10.60$ cubic feet per second.

To allow for velocity of approach, $V_A = \frac{10.60}{10(1.5 + .49)} = .53$

H the corrected head $= .49 + \frac{.533^2}{64.32} = .4944.$

$Q = 3.33 \times 10(.952 \times .4944)^{\frac{3}{2}} = 10.74$ cubic feet per second

(c) m, for $h = .49$, and for $p = 1.50$, $= .440$.

$Q = .440(1.05 + .21\frac{.16}{1.50})\left(\frac{.33}{.49}\right)^{\frac{1}{2}} \times 10 \times 8.02 \times .49^{\frac{3}{2}} = 11.38$ c feet per second.

Discharge of a Weir under a Varying Head

266. No inflow, varying outflow, from a reservoir of prism form. Given a reservoir of prismatic form of area F filled height h_0 above the crest weir; required the time essary to lower the water face to a height h_t above crest of the weir by disch ing over it. See figure 92

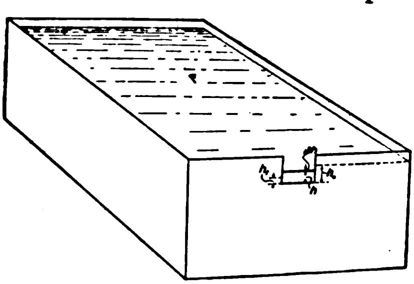


FIG. 92. — Weir with Varying Head.

Let h be the head at the ti

Let $F\delta h$ be the volume charged from the reservoi time δt.

The rate of flow at the time t will then be $\delta Q = CLh^{\frac{3}{2}}\delta t$.

Also $\delta Q = -F\delta h$ (minus, since Q decreases with h).

Then $CLh^{\frac{3}{2}}\delta t = -F\delta h.$

Whence $\delta t = -\dfrac{F\delta h}{CLh^{\frac{3}{2}}},$

and
$$\int_{t=0}^{t=T}\delta t = -\frac{F}{CL}\int_{h_0}^{h_t} h^{-\frac{3}{2}}\delta h. \qquad (33)$$

Integrating (33) between the limits h_t and h_o corresponding to $t = T$ and $t = 0$

The total time required to lower the surface from h_o to h_t is found to be
$$T = \frac{2F}{CL}\left(\frac{1}{h_t^{\frac{1}{2}}} - \frac{1}{h_o^{\frac{1}{2}}}\right) \text{ seconds.} \qquad (34)$$

h_t must have a finite value, else the problem is indeterminate. C must be selected both to suit the particular form of the weir and its setting, and have an average value which will represent varying conditions of head during the drawing off.

Example. Given a prismatic reservoir 100,000 square feet in area. Required the time necessary to lower the water by flow over a sharp-crested suppressed weir 20 feet long, from a head of 3.24 feet to a head of .36 foot.

Let C for the weir be 3.33.

The time required,
$$T = \frac{2 \times 100000}{3.33 \times 20}\left(\frac{1}{(.36)^{\frac{1}{2}}} - \frac{1}{(3.24)^{\frac{1}{2}}}\right) = 3333 \text{ seconds.}$$

267. No inflow, varying outflow, from a reservoir of irregular volume. Given a reservoir in which the water surface varies in area as it varies in elevation; the time required to lower the water surface is to be computed.

Determine the volume contained between successive differences in elevation (as small as may seem necessary).

Divide the volume between two successive elevations by the discharge through a weir for the average head due to the average elevation of the water surface between two successive differences, and the approximate time required for this depletion will result.

The total time is the summation of the times required to make each step.

. A reservoir is filled to elevation 103; what time will be required to lower its surface to the crest of a weir at elevation 100? The overflow weir is a sharp-crested suppressed weir 20 feet long. The required computations in tabular form are as follows. The results deduced are only approximately correct; but quite as accurate as practice would usually require.

Elevation of Water Surface	F Area of Water Surface, Square Feet	S Volume of Each .5-Ft. Lowering	h Average Head at Each .5-Ft. Lowering	Q Average Each .5-Ft. Lowering by 20-Ft. Weir	$t=\frac{S}{Q}$ For Each .5-Ft. Lowering, Seconds
103.0	400,000				
		190,000	2.75	303.7	626
102.5	360,000				
		170,000	2.25	224.8	756
102.0	320,000				
		150,000	1.75	154.2	973
101.5	280,000				
		130,000	1.25	93.1	1,396
101.0	240,000				
		110,000	0.75	43.3	2,540
100.5	200,000				
		90,000	0.25	8.3	10,843
100.0	160,000			Total time,	17,134 seconds

268. Uniform inflow, varying outflow.

Given a reservoir with a uniform inflow from a stream, and variable outflow through a weir.*

Compute the difference in the rate of inflow and outflow.

Divide the volume between successive elevations of the surface by this difference; the result will be the time for each interval of rise.

The total time is the summation of the time required for each step.

Example. A reservoir is filled to the elevation of its weir crest (100). Water is flowing in at the rate of 300 cubic feet per second; and begins to overflow a sharp-crested suppressed weir

* The same kind of computations will apply to orifices or sluices combined with weirs or alone.

20 feet long. To what elevation will the water rise and in what time? The required computations in tabular form are as follows. The results, while only approximate in any case, are sufficiently accurate for such problems; and a greater accuracy can be obtained by increasing the number of steps.

Elevation of Water Surface	F Area of Water Surface, Sq. Ft.	S Volume of Each .5-Ft. Rise	h Average Head at Each .5-Ft. Rise	Q_0 Outflow Average for Each .5-Ft. Rise	Q_1 Inflow Rate	$Q_1 - Q_0$	$t = \frac{S}{Q_1 - Q_0}$ Seconds
100.0	160,000						
		90,000	0.25	8.3	300	292	308
100.5	200,000						
		110,000	0.75	43.3	300	257	428
101.0	240,000						
		130,000	1.25	93.1	300	207	628
101.5	280,000						
		150,000	1.75	154.2	300	146	1,027
102.0	320,000						
		170,000	2.25	224.8	300	75.0	2,267
102.5	360,000						
		(92,500)*	(2.63)*	(283)*	300	(17)*	5,441
		190,000	2.75	303.7			
103.0	400,000						
							10,099 seconds

In the same general way and on a diagram made especially for this purpose, the rise or fall of a reservoir by the discharge over a weir through a given interval of time can be calculated where both the inflow and outflow are irregular.

Weirs of Irregular Section

269. Previous to Bazin's experiments the knowledge of the discharge over weirs of irregular section was not extensive; and in Bazin's experiments the heads were relatively low. Several series of experiments made at the Cornell University laboratory extended Bazin's work, especially in the matter of coefficients for higher heads.

*For rise 102.50 to 102.75. The water will rise to about 102.75 because at that elevation the rate of inflow and outflow about balance.

Inasmuch as it is necessary to reproduce absolutely the condition obtaining during any set of experiments in order to use the coeffi cients at all satisfactorily; and because it is almost never possibl to reproduce in practice the exact form of each weir crest, it is n thought advisable to go into the reasons for obtaining the resul which were found in the experiments referred to; but simply gi the results in figures showing the weirs and the important cond tions governing them; and in diagrams showing the relation b tween observed head and discharge.

R. E. Horton made a careful analysis of all the good availab experiments, including Bazin's; the latter he reduced to Engli measures, and certain others he recomputed. Horton also reduc all data to terms of the Francis formula ($CLH^{\frac{3}{2}}$), and comput values of C for use in this formula. These values of C, which a based upon the Francis method of making correction for the veloci of approach, together with all the necessary data, are given in t *Water Supply and Irrrigation Paper No.* 200.

270. Table XXXVIII is made up of selected types from th paper. The discharge curves are drawn with observed head (h) as ordinates and the discharge Q in cubic feet per second pe foot of crest as abscissæ; values of h and Q being experimenta determinations. This table will be found in the Appendix.

Problems

1. A weir 6 feet long is set in a channel 6 feet wide, and the crest is 6 fe high. Compute the discharge by the Francis formula when the observed hea is 1.916 feet.

2. A weir 6 feet long is set in a channel 16 feet wide, and the crest is 6 fe high. Compute the discharge by the Francis formula when the observed hea is 1.916 feet.

3. A weir 1.5 feet long is set in a channel 8 feet wide, and the crest 4 feet high. Compute the discharge when the observed head is .69 fo Use (a) Francis, and (b) Fteley and Stearns formulas.

4. A weir 5 feet long is set in a channel 10 feet wide and the crest is 4. feet high. If the observed head is .64 foot, compute the discharge (a) by t Francis, (b) by the Fteley and Stearns, (c) by the Smith formula.

5. It is desired to build a sharp-crested weir so that head on its crest sha not exceed 2 feet and which shall deliver 420 cubic feet per second. Th stream is 60 feet wide, and when the weir is in place, is not to be more th 7 feet deep, upstream. Determine length of weir by the Francis formula.

6. A weir 10 feet long with one end contraction is set in a channel 13 feet wide, and the crest is 4.5 feet above the bottom. The observed head is 1.36 feet. Compute the discharge by the Francis, and the Fteley and Stearns formulas: (a) no velocity of approach; (b) with velocity of approach.

7. A weir 19 feet long is set in a rectangular canal 19 feet wide. The crest of the weir is 9 feet above the bottom of canal. The hook gauge reading showed a head of .83 foot. Compute (a) the discharge, (b) the velocity of approach, and (c) the head due to same. Use Hamilton-Smith coefficients and compare with results computed by Fteley and Stearns formula made especially for a 19-foot weir.

8. Length of weir, 8 feet, width of channel, 10 feet, observed head, .36 foot. Height of crest, 1.0 foot. Compute the discharge (a) by the Francis, and (b) by the Fteley and Stearns formulas.

9. Same weir as in problem 8, but with an observed head of 1.542 feet. Compute discharge (a) by the Smith formula, and (b) by the Francis formula.

10. A weir 15 feet long is set in a channel 15 feet wide, the crest is 3.28 feet high, and the observed head is 1.181 feet. Compute the discharge by Bazin's formulas.

(a) The more precise formula.
(b) The approximate formula.
(c) The rougher approximate formula.

11. Given a suppressed weir 7 feet long, with the crest 4.5 feet above the bottom of the channel. Observed head is 1.36 feet. Compute discharge by the Francis, Fteley and Stearns, and Bazin formulas: (a) not correcting for the velocity of approach; (b) correcting for the velocity of approach.

12. If a weir has all the conditions the same as in problem 11, except the height, which is 2.0 feet, compute the discharge by the three formulas mentioned, (a) not correcting for the velocity of approach; (b) correcting for the velocity of approach.

13. Compute the discharge through standard triangular weir under a head of .81 foot.

14. Compute the discharge through a Cippoletti weir under a head of .9 foot; length of the crest, 6 feet.

15. Compute the discharge through a triangular notch, the sides of which meet at 90 degrees, when the head is .64 foot.

16. Given a submerged weir with crest 9 feet long, h is 2.4 feet, and h' is .64 foot. Compute the discharge (a) by Fteley and Stearns formula; (b) by the Herschel formula; (c) by the Bazin formula.

17. How high should a submerged sharp-crested weir be built to deepen by 25 per cent on the upstream side, the water of a stream 4 feet deep by 20 feet wide in original section, if the velocity in original section equals 3 feet per second?

18. A dam is to be built of the full width of a stream which is 100 wide and 2 feet deep, and the mean velocity of flow is 3 feet per second. C pute the probable observed head on the crest for the following types of dam

(*a*) Bazin's No. 86
(*b*) Bazin's No. 115
(*c*) Bazin's No. 137
(*d*) Bazin's No. 164
(*e*) U. S. D. W. No. 12
(*f*) U. S. D. W. No. 7
(*g*) U. S. G. S. No. 40
(*h*) U. S. G. S. No. 44
(*i*) U. S. G. S. No. 38

CHAPTER XIII

FLOAT MEASUREMENTS

271. The types of floats used for measuring the velocity of running water are surface floats, subsurface floats, or combinations of the two, and rod floats. The theory of float measurements is the same no matter what kind of floats are used, but the effect of external conditions may be very different upon the various kinds of floats that may be used.

272. Theory of float measurements. As a fundamental theorem it may be stated that the time required by a body floating in running water, if not influenced by wind, eddies, and cross currents, and not retarded by obstructions, to pass from one section of a stream to another section, is a measure of the velocity of the water which is acting upon the float. If the velocity of the water were uniform at every point in the successive cross sections through which the body is floating, a single run of the float would indicate the mean velocity of flow; and this velocity multiplied by the average cross-sectional area would give the discharge. Since, however, the velocity of the water at different points in a stream cross section is not uniform, and since the motion of the float is a resultant only of such forces as are acting upon it, the velocity of the float is a measure of the velocity of only a limited portion of the stream, equal to the area of the vertical projection of the float itself on a plane perpendicular to the direction of flow; and is not a measure of the velocities in other portions. While the movement of a float is usually influenced by eddy motions in the water, yet it may be safely stated that the forward or translatory motion of the water is usually modified in like manner. Therefore it may be assumed that the movement of the float running with the current between two parallel cross sections is a measure of the forward velocity of the water in the path of the float, and in no other part.

lighter float on the surface, where it serves as a marker. (to the relatively greater volume of the submerged float, the r of the combination is assumed to be the resultant of the acting on the larger float. See figure 94.

FIG. 94.—Subsurface Float.

The procedure in measuring velocity is lar to that of surface float measurements.

277. Application of subsurface float me ments. The subsurface float is designed t the velocity of the water in the different underneath the surface of the water. float maintains a uniform depth during th the time of passage should be a measure average velocity at the depth of submer Owing to the effect of eddy currents th merged float, if but slightly heavier than is moved about in an unpredictable path ; and it is difficult just what strata of the water any particular run of the floa resents. If the float is made heavy enough to resist the inf of eddy currents, the signal float must be made so large th combination becomes practically a twin float. In fact it i difficult to study the velocities at different depths with flo this character.

TWIN FLOATS

278. Twin floats. Two floats of similar shape and nearly weight, fastened together by a thin wire, and designed in manner that one floats on the surface and the other below the surface, are called twin floats. Experience has shown that the velocity of the pair is about the mean of the velocities of the two paths of the floats.

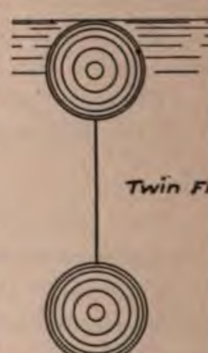


FIG. 95.—Twin Fl

279. Application. This type of float may give better results than either surface or subsurface floats. In determining the discharge of running streams, subsurface floats of all kinds will probably be very little used in the future, because

the rod float can ordinarily be used in all cases where the twin float is used, and with better results.

Rod Float Measurements

280. Rod floats. The only floats for measuring the discharge in open channels which have been used continuously for a great many years on a large scale are the so-called rod floats, which are cylinders, generally of metal, loaded at the bottom with sufficient heavy metal like lead so that they will float vertically. In theory the velocity of the rod should be very nearly the mean velocity of the water of its path; and comparative measurements confirm this assumption. The most careful study of the theory and practice of rod float measurements was made by the late James B. Francis of Lowell, Massachusetts, which study preceded the general use of these floats for gauging the water used for water power by the different mills at that place. With some modifications the methods followed and described in this book are those published in Francis's *Lowell Hydraulic Experiments.** This method of measurement has been used continuously for nearly seventy years with increasing accuracy.

281. Location of measuring flume. For the most accurate measurements with rod floats a straight stretch of channel, called a flume, is required, which should be located far enough from bends and other obstructions to avoid any disturbances of flow.

282. Shape of cross section of the flume. The cross section of the flume should be rectangular; or at least should be of nearly uniform cross section for its entire length and have the sides parallel. The sides and bottom of the flume should be lined with perfectly smooth planks or concrete, so that the floats can be made to run close up to the sides, and within a very few inches of the bottom, without touching. There is little advantage in attempting to make a measurement with rod floats whose submerged depth is not very nearly equal to the depth of the water.

283. The length of the flume. The total length of the flume including approaches will be governed by the average velocity of the stream, but should be sufficient to allow the rod floats to be

*See *Lowell Hydraulic Experiments*, pp. 156–208, Plates XVII and XIX.

put into the water far enough above the upper cross section or transit, as it is called, to attain the velocity of the water; run with the current far enough so that the error of observation with the stop watch shall be less than one per cent; and allow the float to run by the lower transit without changing its speed. Ordinarily with velocities of two to five feet a second, a hundred feet of clear length between transits is a good length, but less than this may be used where the velocity is low.

284. Arrangement of flume. The arrangements on the ground for a satisfactory flume measurement are as follows; the dimensions will vary to suit the conditions:

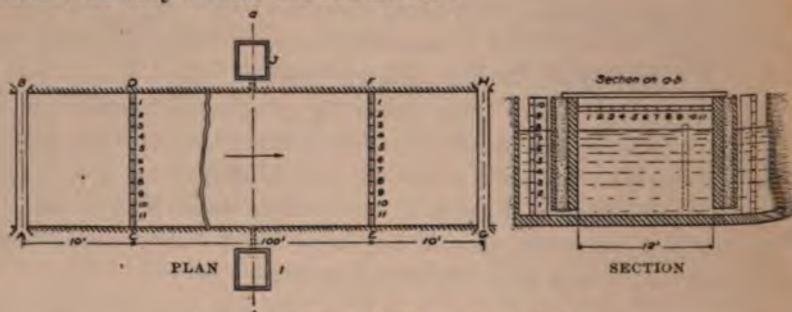


FIG. 96.—A Measuring Flume for Rod Floats.

AB (see figure 96) is a footbridge from which the floats are r leased. *GH* is a corresponding footbridge, perhaps 120 fe downstream for catching the floats and taking them from t water. *CD* and *EF* are timbers or I-beams used as markers s perpendicular to *DF* and *CE* and parallel to each other, which the distance in feet from the left bank looking downstrea is marked in figures easily read from the footbridge *AB* by t man who puts in the tubes. *I* and *J* are still boxes or pits co nected with the flume by pipes whose orifices are flush with t sides of the flume. In the still boxes are gauges or scales grad ated to feet, tenths, and hundredths, their zeros level with average bottom of the flume, or to agree with some datu These gauges should be read as often as necessary to get average height of the water during a measurement.

285. Apparatus. The floats may be of wood with lead weight fastened at one end; tin tubes made of tin plates soldered to

gether, with a section of lead of the proper weight at one end and a water-tight removable cover or stopper at the other; or of seamless drawn brass tubing $2\frac{1}{2}$ inches outside diameter with the lead weight cast into one end and a screw top on the other. The brass tube makes the best float, as it can be most carefully adjusted, is practically indestructible, and can be easily handled. The tubes are adjusted to float with the top a definite distance, perhaps 6 inches, above the water; and on the part which shows is marked plainly in feet and tenths the floating length of the tube. The other instruments necessary are a chronometer, preferably a stop watch, to read $\frac{1}{5}$ seconds, which will start and stop by the pressure of the finger; a good level and accurate tape to get the elevation and measure all the dimensions of the flume within the run of the float; and the necessary special notebooks.

286. Procedure in rod float measurements. Calibration of the flume and gauges. A profile of the cross sections of the flume should be made on the lines *CD*, *EF*, and as many intermediate points as will be necessary to give an average cross section with a degree of accuracy somewhat higher than can be attained by the measurements of velocity. If the flume is a permanent structure, these measurements need only be made occasionally, perhaps once a year, provided the flume is kept free from obstructions in the meantime. A drawing to scale of the average cross section should be made directly upon the computation sheet.

The setting of the gauge or gauges should be checked at least as often as the flume is cross-sectioned and the support to which the gauge is fastened should be of the most permanent kind. The distance between *CD* and *EF*, which is the exact run of the floats, and the setting of the scale boards should be determined whenever necessary, but are usually fixed in construction.

Duplication of runs. A complete gauging usually takes from one-half to three-quarters of an hour and includes a run of a float for every foot to two feet in width of the flume. On account of changes from time to time in the use of water from the canal, or of discharge into it, and also to get rid as far as possible of the effects of personal errors in the observations of the velocity and location of the floats, the gaugings should be made simultaneously in duplicate by two parties who work independently, one starting

at the left side of the flume and the other at the right. The tw measurements should be entirely distinct; but any break in tl regular sequence of running the floats should be filled in befo the two parties leave the ground.

Starting the float. Each float should be put into the water fro the bridge AB with such care that when it reaches CD it w be running evenly with the current, with its axis vertical and wit out bumping on the bottom.

Noting the path of the float. As the float passes under t markers of the upper station CD, the starter notes its distar from the left side of the flume and calls out this distance to t recorder; and again as it passes under the lower transit stati EF. If there are no cross currents, the float should maint nearly the same relative distance from the left side of the flur If the tube does not run in a path nearly parallel to the sides the flume, but runs wild, this particular run should be repeatec

Time of a run. The usual method of timing the run of the fl is to start an ordinary stop watch set at zero as the rod pas under the upper transit at CD and stop the watch wher reaches the lower transit EF, the distance between being trave by the observer, who walks or runs as the case may requ The time required for the run of the float is noted directly seconds.

Elevation of water surface. The elevation of the water in still box should be read at least once during every run of the flo

Returning the floats. The floats are taken out of the wate the bridge GH and carried to the bridge AB for further u and there should be enough floats to make the measureme continuous.

Form of notes for record. The form of notes shown has b found to be convenient for keeping all the notes necessary fo flume measurement. The velocities are usually given in feet second, the distance from the left side in feet and tenths, and gauge heights in feet, tenths, and hundredths above the botto The notes give all the observations for two simultaneous measu ments of the same flow. See pages 245 and 246.

NOTE — The flume was of trapezoidal section, not rectangular. The zer the flume gauge coincided with the lowest part of the bottom.

Measurement made with Loaded Floats of Quantity of Water used by Lowell Machine Shop Wheels Nos. 1 and 2 (82¼-Inch Boyden Wheels). From 8.40 to 9.08 a.m., May 27, 1903

FIRST PARTY

1			2			3	4	5	6	7	8	9	10		11
Time of Transit by Stop Watch						Time occupied in passing from the Upstream Station to the Downstream Station, a Distance of 50 Feet	Velocity of the Tube	Intended Distance of the Tube from the Left Side of the Flume, looking Downstream	Length of the Immersed Part of the Tube	Distance from Left Side at			Reading of Flume Gauge		
At the Upstream Station			At the Downstream Station							Upper Station	Lower Station	Mean	Time		Height
Hr.	Min.	Sec.	Hr.	Min.	Sec.	Sec.	Ft. per Sec.	Ft.	Ft.	Ft.	Ft.	Ft.	Hr.	Min.	Ft.
						21.8	2.294	0	3.67	0.8	0.5	0.65	8	40	5.13
Stop watch used						22.0	2.273	1	3.67	0.9	1.3	1.10			.14
No. 138,266						21.3	2.347	2	4.00	2.5	1.5	2.00			.16
						20.6	2.427	3	4.25	3.0	2.2	2.60			.14
						21.8	2.294	3	4.00	2.6	1.5	2.05			.16
						18.0	2.778	4	4.25	4.3	3.7	4.00			.15
						19.0	2.632	5	4.67	4.8	5.5	5.15			.16
						16.8	2.974	6	5.00	5.5	7.1	6.30			.15
						17.2	2.907	7	5.00	7.3	7.0	7.15			.16
						17.6	2.841	8	5.00	7.7	8.8	8.25			.17
						18.0	2.778	9	5.00	8.6	9.5	9.05			.14
						20.4	2.451	10	5.00	10.7	12.0	11.35			.14
						19.3	2.591	10	5.00	10.0	10.0	10.00			.13
						20.8	2.404	12	5.00	11.8	10.7	11.25			.14
						20.0	2.500	13	5.00	12.6	12.4	12.50			.14
						22.0	2.273	14	5.00	13.5	14.3	13.90			.14
						21.0	2.381	14	4.67	14.0	12.8	13.40			.14
						22.8	2.193	15	4.67	14.8	14.3	14.55			.13
						22.8	2.193	16	4.25	16.0	17.5	16.75			.13
						21.2	2.358	17	3.67	16.8	14.0	15.4			.11
						25.8	1.938	18	3.67	18.1	18.0	18.05			.11
						23.0	2.174	18	3.67	18.4	16.5	17.45			.11
						22.0	2.273	19	3.67	18 6	15.5	17.05			.11
						25.2	1.984	19	3.67	18.7	17.8	18.25	9	08	.12
															5.138

Length of flume = 50.0 feet.

Measurement made with Loaded Floats of Quantity of Water used by Lowell Machine Shop Wheels Nos. 1 and 2 (82½-Inch Boyden Wheels). From 8.40 to 9.08 a.m., May 27, 1903

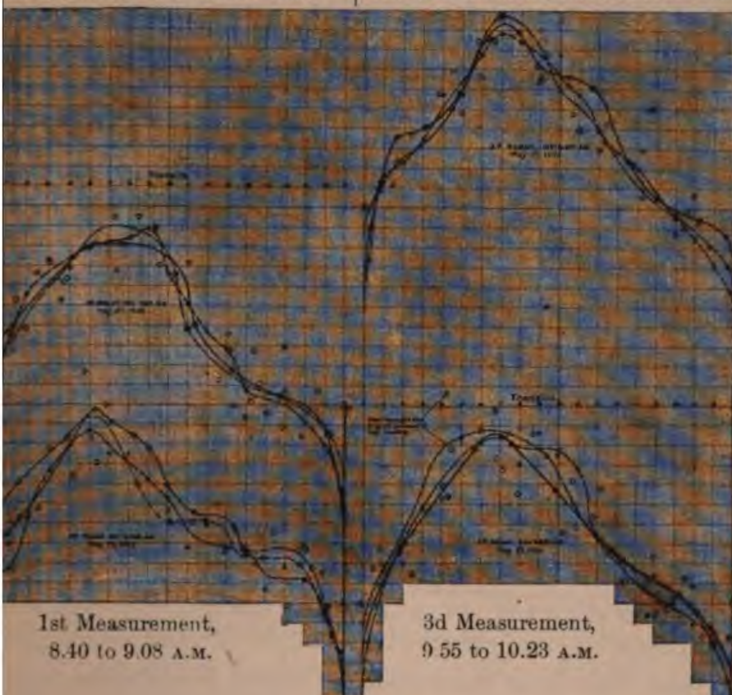
SECOND PARTY

1			2			3	4	5	6	7	8	9	10		11
Time of Transit by Stop Watch						Time occupied in passing from the Upstream Station to the Downstream Station, a Distance of 50 Feet	Velocity of the Tube	Intended Distance of the Tube from the Left Side of the Flume, looking Downstream.	Length of the Immersed Part of the Tube	Distance from Left Side at			Readings of Flume Gauge		
At the Upstream Station			At the Downstream Station							Upper Station	Lower Station	Mean	Time		Height
Hr.	Min.	Sec.	Hr.	Min.	Sec.	Sec.	Ft. per Sec.	Ft.	Ft.	Ft.	Ft.	Ft.	Hr.	Min.	Ft.
Stop watch used No. 14,232						23.2	2.155	19	3.67	18.3	17.5	17.9	8	40	5.14
						21.8	2.294	19	3.67	18.4	15.5	16.95			.14
						22.1	2.262	18	3.67	18.4	15.0	16.7			.14
						27.3	1.832	18	3.67	18.0	18.8	18.4			.14
						25.2	1.984	17	4.0	16.7	15.5	16.1			.15
						23.7	2.110	16	4.0	16.0	16.2	16.1			.15
						22.5	2.222	15	4.25	15.0	14.8	14.9			.14
						21.1	2.370	14	5.00	13.5	13.5	13.5			.15
						22.2	2.252	13	5.00	12.9	13.0	12.95			.16
						20.7	2.415	12	5.00	11.8	12.0	11.9			.16
						20.8	2.404	11	5.00	10.7	9.5	10.1		Read continuously at each run of tube. Depth of water above end 62.945.	.16
						19.3	2.591	10	5.00	9.5	7.5	8.5			.14
						20.3	2.463	10	5.00	10.0	10.0	10.0			.13
						20.9	2.392	9	5.00	8.7	11.5	10.1			.13
						16.7	2.994	8	5.00	7.7	6.0	6.85			.13
						18.5	2.703	7	5.00	6.8	6.0	6.4			.13
						17.7	2.825	6	4.67	6.2	6.0	6.1			.12
						15.8	3.165	5	4.67	5.5	6.0	5.75			.13
						17.7	2.825	4	4.25	4.2	5.5	4.85			.13
						20.2	2.475	3	4.0	3.0	4.0	3.5			.12
						19.2	2.604	2	3.67	2.2	3.0	2.6			.12
						21.2	2.358	1	3.67	1.1	2.2	1.65			.13
						19.3	2.591	0.5	3.67	0.5	3.0	1.75			.1[illegible]
						18.7	2.674	0.0+	3.67	0.6	2.5	1.55			.1[illegible]
						23.1	2.165	0.0+	3.67	0.8	0.6	0.7	9	08	.1[illegible]
													Mean		5.1[illegible]

Length of flume = 50.0 feet.

PUTATIONS OF ROD FLOAT AND CURRENT METER MEASUREMENTS.
MADE MAY 27, 1903

Ieasurement, 9.15 to 9.47 A.M. | 4th Measurement, 10.30 to 11.00 A.M.



- Rod float measurement (First Party).
- Rod float measurement (Second Party).
- Current meter measurement.

.—Diagram showing the Velocities in a Flume. Determined by Duplicate Runs Rod Floats, and a Simultaneous Current Meter Measurement (Integrating in tions).

COMPUTATION OF 1ST MEASUREMENT

8.40 to 9.08 A.M.

mean velocities from the diagram for each foot in width; in feet per second.

mean depth for each foot in width; in feet. Section was trapezoidal.

discharge in cubic feet per second.

correction (see § 289).

(A) Rod Floats			(B) Rod Floats			(C) Current Meter		
V	Δ	Q	V	Δ	Q	V	Δ	Q
2.213 ×	3.857 =	8.536	2.110 ×	3.856 =	8.136	2.005 ×	3.856 =	7.731
2.303 ×	4.115 =	9.477	2.448 ×	4.114 =	10.071	2.152 ×	4.414 =	8.853
2.408 ×	4.374 =	10.533	2.592 ×	4.373 =	11.335	2.258 ×	4.373 =	9.874
2.545 ×	4.633 =	11.791	2.700 ×	4.632 =	12.506	2.572 ×	4.632 =	11.914
2.705 ×	4.892 =	13.233	2.796 ×	4.891 =	13.675	2.731 ×	4.891 =	13.357
2.880 ×	5.112 =	14.723	2.905 ×	5.111 =	14.847	2.840 ×	5.111 =	14.515
2.969			2.930			2.847		
2.889			2.798			2.744		
2.827			2 590			2.667		
2.709	18.811		2.441	17.983		2.575	17.900	
2.526	×		2.410	×		2.448	×	
2.450	5.138 =	96.651	2.414	5.137 =	92.379	2.327	5.137 =	91.95
2.441			2.400			2.292		
2.363 ×	5.113 =	12.082	2.257 ×	5.212 =	11.538	2.270 ×	5.112 =	11.60
2.260 ×	4.898 =	11.069	2.229 ×	4.897 =	10.915	2.219 ×	4.897 =	10.86
2.278 ×	4.643 =	10.577	2.228 ×	4.642 =	10.342	2.127 ×	4.642 =	9.87
2.261 ×	4.388 =	9.921	2.211 ×	4.387 =	9.700	2.121 ×	4.387 =	9.30
2.150 ×	4.133 =	8.886	2.079 ×	4.132 =	8.590	2.074 ×	4.132 =	8.570
1.987 ×	3.865 =	7.680	1.933 ×	3.864 =	7.469	1.976 ×	3.864 =	7.63
		225.159			221.503			216.05
	$K =$	-3.733		$K =$	-4.652			
Total $Q = 221.43$			Total $Q = 216.85$			Total $Q = 216.05$		

The three other measurements (stated in the same order) gav[e] the following discharges, in cubic feet per second:

2d measurement: *A*, 213.18; *B*, 212.41; *C*. 215.92.
3d measurement: *A*, 211.86; *B*, 215.09; *C*. 218,92.
4th measurement: *A*, 215.58; *B*, 211.15; *C*. 215.28.

287. Computations. Every run of the float requires a separa[te] computation. In the set of measurements given, the consta[nt] length, 50 feet, divided by the number of seconds given i[n] column 3 gives the values for column 4, the velocities of the flo[at] for each run. The location of the average position of the flo[at] in the flume during the run is given by column 9. The dep[th] of water during each run is given by column 11.

Graphical representation of the velocities of the floats. If th[e] flume were exactly an even number of feet wide and rectangul[ar] in cross section, and if the floats could be made to run at the mi[d-]

dle of each one-foot space and parallel to the sides of the flume for the entire distance, and if the water surface remained unchanged throughout a gauging, the arithmetical mean of the velocities of the floats would represent the mean velocity of the water. This mean velocity multiplied by the area or the sum of these mean velocities multiplied by the depth would equal the total quantity of water in cubic feet per second passing the flume. It would be, however, extremely difficult to secure just these conditions; and it is only necessary to get enough observations to bring out the transverse velocity curve. The usual practice in working up the measurements is to plot on cross section paper observations of the velocity of each float as ordinates against the mean distances from the left side as abscissas (see figure 97); the sheet representing, except for the depth, all the details of the measurement. Through these plotted velocities the mean or balanced velocity curves may be drawn for the two complete measurements; and, if necessary, additional characteristic curves for each flume taken at other times may be added to the sheets as a guide. From the balanced lines the velocity for each foot may be read off; the sum of these, including a proportional amount for the velocity in the odd distance beyond the last even foot, multiplied by the arithmetical mean of the depths of the water during the measurement, will give the discharge in cubic feet per second very nearly. There are two conditions, accumulation of water during measurements and correction for depth of flotation, which require a further consideration, and which may affect more or less the final comparative discharge.

288. Accumulation. If the flow through the flume is constant and the water is being drawn out below as fast as it is coming in, the quantity measured may represent the flow in cubic feet per second for the entire canal. If, however, the water is running into a basin or other canal which is rising or falling, the amount actually drawn out from this basin may differ from the discharge through the flume by the amount of this filling or drawing down. There are a great many cases where this may have an important bearing upon the use of water by waterwheels or the discharge of a gate, and the results will be vitiated if the effect of this accumulation is not taken into account.

289. Correction formula. A float can not usually be run wi its lower end nearer the bottom than one to two inches, and mo often it is not so near. Since the velocity of the rod float is n influenced by this lower and generally slower layer of water, slight correction must be made to get the true velocity. Jan B. Francis, by comparing the rod floats and weir measuremen deduced the following formula for correcting rod float measu ments.

$$C = 1 - .116(\sqrt{D} - .1).$$

C = a coefficient of correction which multiplied by the o served velocity will give the corrected velocity;

D = difference between the depth of water in flume an length of the immersed part of the tube, divided b the depth of water.

The values of the coefficient C for different values of D ar given in the following table, abbreviated from the *Lowell Hy draulic Experiments;* an inspection of these figures shows tha a difference of .5 foot between the length of the tube and th depth of the water, where the flume is 10 feet deep, gives a co rection of only 1.5 per cent from the observed velocities. Corre tions less than 1 per cent are certainly closer than the accurac of the float measurements will warrant.

TABLE XXXIX

VALUES OF THE COEFFICIENT $[1 - .116(\sqrt{D} - .1)]$.

D	0	1	2	3	4	5	6	7	8	9
.0		1.000	.995	.992	.988	.986	.983	.981	.979	.97[illegible]
.1	.975	.973	.971	.970	.968	.967	.965	.964	.962	.96[illegible]
.2	.960	.958	.957	.956	.955	.954	.952	.951	.950	.9[illegible]
.3	.948									

Direct interpolation may be made for intermediate values.

290. Alternative method of computation. The computation of rectangular channels may be made, after plotting, by using planimeter to determine the sum of the mean ordinates, or velo

ties, for each independent set of runs; and the mean of these two determinations divided by the width should give the mean velocity. This method is somewhat easier and usually is sufficiently accurate.

291. The plotting and computations for the float measurements, given on pages 245 and 246, are shown in figure 97 and pages 247 and 248. In addition to these measurements, others were made on the same day; and also a current meter measurement simultaneously with each pair of float measurements. All these other measurements are also shown. The two series especially mentioned may be identified on the drawing by their periods of time of measurement.

292. Application of rod float measurements. The sphere of usefulness of rod float measurements is somewhat limited, and the expense of making them is relatively great. Their regular use in the future will probably be limited to straight, deep canals or flumes where a high degree of accuracy is required, where a sufficient force of men is regularly employed for this and other purposes, and where it is very necessary to gauge all the water used for power and other purposes, without interfering with the operation of the mills. Ordinarily, the difficulty of getting good results from the sum of individual measurements, or readings of waterwheels, is due to the fact that the total discharge, which is simply the sum of the individual waterwheels, often does not include the leakage of the water used for manufacturing purposes other than power; but the flume measurements of the total quantity passing to each mill will cover everything. There is very little opportunity to make such measurements in rivers or canals which do not have a regular cross section; and for such conditions there is no question that measurements by current meter will take the place of those formerly made by rod floats.

The most notable published gaugings by rod floats are those by Humphreys and Abbott of the Mississippi River, those described by James B. Francis in the *Lowell Hydraulic Experiments*, Darcy and Bazin's gaugings, and the gaugings of certain rivers in India.

293. Limits of accuracy. With a straight, smooth flume of great depth, and velocities ranging from 2 to 5 feet per second, quantities of water from a few hundred to 4000 cubic feet per

second have been repeatedly measured with a probable error of 1 to 2 per cent. This form of measurement, which in its successive steps gives the product of the cross section and the velocity of the water as indicated by the rod floats, is a perfectly natural one; and its simplicity appeals to the non-technical man.

Problems

1. A stream of rectangular section is 20 feet wide and was flowing 10.5 feet deep when a set of velocity measurements was made by rod floats. The velocities thus determined in sections 2.5 feet wide were: (in feet per second) 1.0, 1.1, 1.2, 1.3, 1.4, 1.3, 1.2, 1.1. Find the discharge in cubic feet per second. The average depth of flotation of the rods was 10.25 feet.

2. From the following notes of a measurement of discharge made with rod floats compute the discharge in cubic feet per second.

The measuring flume was of rectangular cross section; the mean width was 41.2 feet; the depth of water in the flume was 8.4 feet; depth of flotation of rods was 8.0 feet.

No.	Distance from Left Side of Flume	Velocity	No.	Distance from Left Side of Flume	Velocity
	Feet	Feet per Second		Feet	Feet per Second
1	.75	3.005	12	19.90	4.375
2	2.10	3.045	13	22.60	4.762
3	4.10	3.645	14	24.20	4.605
4	5.90	3.685	15	25.70	4.667
5	8.20	4.000	16	28.98	4.930
6	10.50	4.320	17	29.20	5.000
7	10.70	4.168	18	30.80	5.070
8	13.85	4.323	19	32.42	5.289
9	14.10	4.375	20	36.35	5.600
10	16.28	4.243	21	37.15	5.425
11	17.79	4.489	22	38.40	5.556
			23	41.00	4.930

3. From the following notes of a duplicate simultaneous measurement of discharge made with rod floats compute the discharge. Cross section of the measuring flume was rectangular, mean width was 41.2 feet; depth of water in the flume was 8.2 feet; depth of flotation was 7.8 feet.

No.	Distance from Left Side of Flume	Velocity	No.	Distance from Left Side of Flume	Velocity
	Feet	Feet per Second		Feet	Feet per Second
1	.45	3.334	12	17.75	4.862
2	.60	3.101	13	20.30	4.930
3	.95	3.098	14	20.75	4.895
4	2.60	3.888	15	21.60	4.795
5	5.05	3.955	16	25.25	5.300
6	6.90	4.025	17	27.62	5.300
7	8.75	4.145	18	29.90	5.600
8	11.05	4.378	19	32.35	5.931
9	12.10	4.295	20	35.00	6.088
10	14.65	4.405	21	39.95	5.690
11	16.40	4.488	22	40.40	5.285

No.	Distance from Left Side of Flume	Velocity	No.	Distance from Left Side of Flume	Velocity
	Feet	Feet per Second		Feet	Feet per Second
1	.70	3.185	16	21.82	5.000
2	1.28	3.070	17	22.55	5.000
3	2.20	3.608	18	23.00	5.073
4	3.80	3.645	19	23.85	5.225
5	5.85	4.007	20	24.96	5.229
6	8.67	4.268	21	26.50	5.301
7	10.00	4.323	22	27.96	5.469
8	11.76	4.321	23	29.11	5.831
9	15.74	4.488	24	30.67	5.832
10	16.10	4.665	25	33.60	5.832
11	19.20	4.665	26	35.00	5.833
12	19.20	4.864	27	35.56	5.833
13	19.25	4.795	28	40.15	5.487
14	19.45	5.001	29	40.42	4.930
15	20.28	4.730			

CHAPTER XIV

CURRENT METER MEASUREMENTS

294. The current meter. If a small screw, like a propeller or like a waterwheel with vanes or buckets, free to revolve between two bearings, is held in the path of moving water, it will acquire an angular velocity depending upon the velocity of the water in the neighborhood of the point of immersion. Such a wheel, if fitted with a recording apparatus to indicate the number of revolutions, is called a current meter.

295. Varieties of current meters. There are, perhaps, twenty types of current meters, of which several, illustrating types commonly used in the United States, are shown in figures 98, 99, 100, 101, and 102. Meters may be roughly classified as those which revolve about (*a*) a vertical axis, or (*b*) a horizontal axis, the essential differences being in the wheels, the manner of holding the meters in the water, and the type of recording device. Meters are usually designated by the name of the designer or maker. The important considerations in choosing a meter are the strength of the wheels and frame to resist deformation, the absence of friction in the bearings, and the certainty that the number of revolutions will be properly recorded. All kinds of meters attached to rods or poles can be used in streams of depths less than about 12 feet, and in velocities less than 6 feet per second; but for greater depths and higher velocities extra precautions must be taken or special devices used for controlling the meter.

296. Fteley-Stearns meter. The Fteley-Stearns meter, shown in figure 98, is similar to the one used by Fteley and Stearns in the Sudbury River experiments; and is suited to velocities less than 6 feet per second and streams fairly free of floating débris. It is of the screw type with helicoidal blades. A band encircling the blades protects them from distortion, and also helps to minimize the turning effort due to eddy currents or to moving the

meter vertically up and down when taking the so-called integrating measurements. The wheel is supported in a metal frame by means of a horizontal axle which is suspended by two bearings

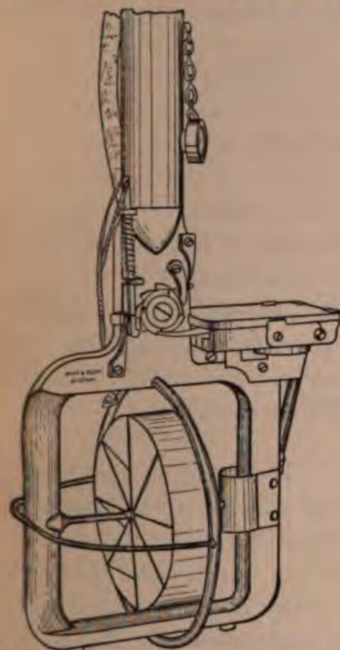


FIG. 98.—Fteley-Stearns Current Meter.

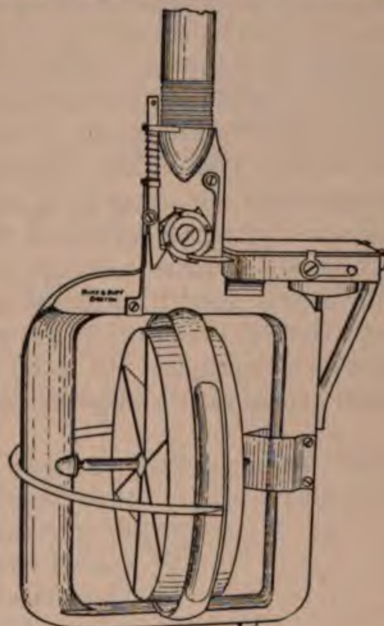


FIG. 99.—Improved Fteley-Stearns Current Meter.

set into the frame. This frame has a circular guard around the wheel in the middle of and at right angles to the frame to protect the meter as far as possible from floating substances and the force of the current.

This meter as built may be held so that the center of the wheel will be within 3 inches of the bottom or sides of the stream if the cross section is at all regular.

The meter is coupled to a brass tube in one piece or sections, graduated from the extreme bottom of the meter in feet and tenths, in order to provide the observer with a gauge to show the depth of the meter below the point of immersion. A special contrivance clamped to the rod will indicate the direction of the axis of the meter when submerged. Where the depth of water is much over 6 feet or the velocity of the current more than 5 feet a sec-

ond, the meter had better be attached by means of a coupling to a piece of iron pipe of suitable stiffness and length.

The horizontal axis of the meter meshes through a pair of gears into another shaft, to which is attached a toothed counter wheel gearing into two others, the three recording successively 10, 100, and 1000 revolutions. The meter may also be fitted with the electric recording device or telephone circuit common to other meters, which enables the operator to count the revolutions of the wheel. The ordinary method, however, of determining the velocity with this meter is to observe the reading of the wheels before and after the gears have been thrown in and out of mesh, by means of a ratchet to which is attached a stout string.

In depths more than 12 feet and velocities much more than 6 feet a second, the meter is not easily handled; and special care must be taken to hold the meter rod vertical and have the horizontal axis of the wheel parallel with the current.

297. Improved Fteley-Stearns meter. It has been found that the ordinary type of Fteley-Stearns meter was not strong enough to operate successfully in velocities greater than 6 feet a second, and to stand severe usage in a great many streams under difficult conditions of measurement. For these reasons it has been modified and further strengthened by Safford; a cut of the improved meter is given in figure 99. The wheel is heavier, is 5 inches in diameter instead of $3\frac{1}{2}$, and has 6 vanes instead of 8. The axle is much heavier where it passes through the wheel and tapers down to two cone bearings at each end. While the wheel is larger and stiffer and strong enough

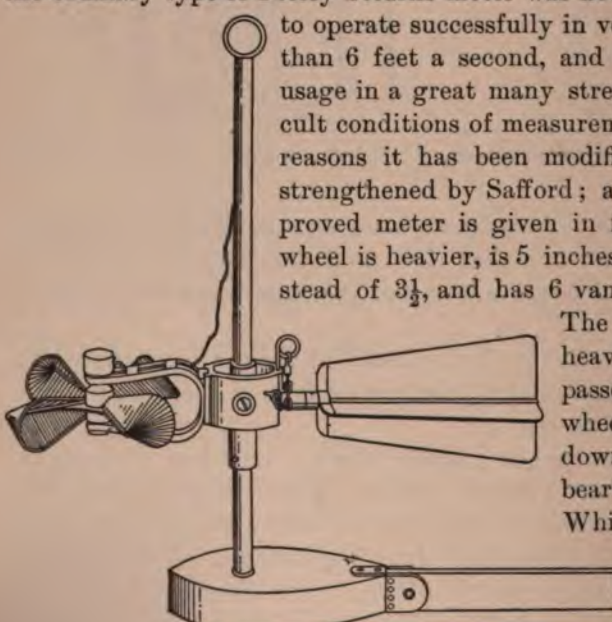


Fig. 100. — Price Current Meter.

to withstand velocities as high as 13 feet per second, it revolves as easily at the low velocities as any other type of meter. The diagram, figure 104, showing successive ratings covering four years, indicates the small change in rating during this time.

298. The Price meter. The Price meter, shown in figure 100, consists of 5 conical buckets fixed to a vertical axis and revolving between two bearings supported by a frame. To the other end of the frame may be attached a rudder which helps to keep the axis of the meter parallel to the current. The frame and rudder are pivoted to a brass rod about 2 feet long, the upper end terminating in an eye to which is attached a wire or other cable to hold the meter; to the other end may be attached a lead weight, also fitted with a rudder, which serves to keep the meter in position and the axis vertical. In shallow water the meter is attached to a graduated brass rod like that used with the Fteley-Stearns meter and handled without the rudder and lead weight. This meter is always provided with some sort of electric indicator, either an automatic recording device or a telephone buzzer. The former gives simply total revolutions; the latter will also keep an observer informed whether the meter is working properly; but either may be used independently.

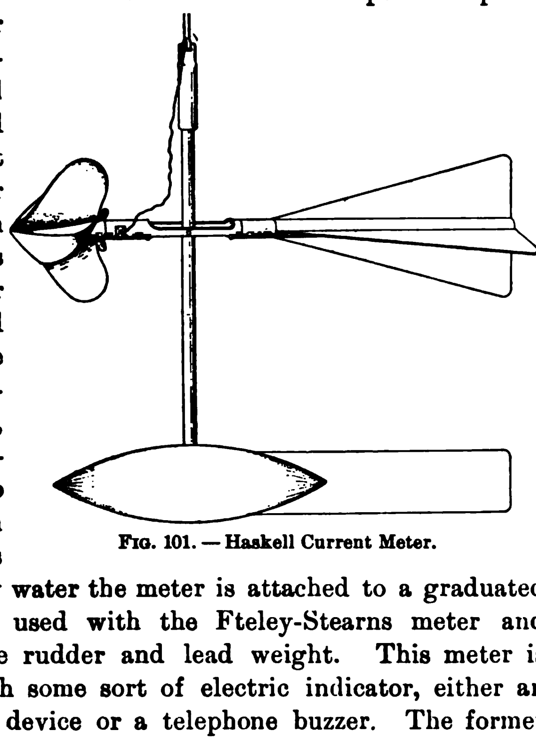


FIG. 101. — Haskell Current Meter.

299. The Haskell meter. The Haskell meter, shown by figure 101, is of the screw propeller type. It is operated in a manner similar to the Price meter.

300. A German type is shown in figure 102.

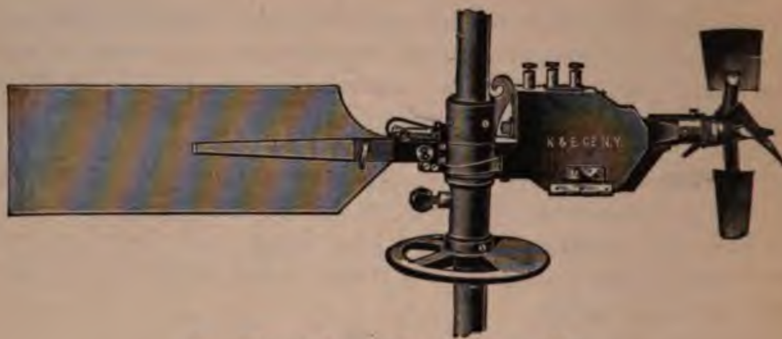


FIG. 102. — A German Type of Current Meter.

301. Comparison of meters. Where the section of a stream or canal is fairly regular and the current positive, any good meter, if rated correctly and properly used, will give satisfactory results. Where the cross section at the place selected for measurement is not regular, where cross currents exist, and rocks and other obstructions in the bottom are unavoidable, the result may be fair if the meter selected for use is not affected unduly by these unfavorable conditions. The Price meter usually registers more revolutions for a given mean velocity than the Fteley-Stearns type, unless the conditions are particularly good, due to the fact that any motion of the water, whether positive or negative, may be recorded as positive by the Price meter; while the Fteley-Stearns meter simply records the algebraic sum of all the revolutions in one direction. The Fteley-Stearns meters may actually be rated to run backward. Small meters have an advantage over large meters in that they can be set closer to the bottom and sides of a stream; but larger meters have advantages where only a few readings near midstream are required.

Measurements made by holding the meter with a rod, where possible, are much to be preferred to those made with a meter suspended from a cable. There is also a certain advantage if the observer can hear the number of revolutions during the measurement, but the accuracy of the measurements does not depend so much upon these conditions as upon the regularity of the section and velocity, and the care with which all unusual difficulties are overcome. The Fteley-Stearns meter, for instance, can not be taken out of the water in winter without freezing over,

and it is necessary to arrange either to thaw it out, or to use an electric recording device. The Price meter is probably used more generally in the United States than any other type on account of its successful use for measuring stream flow in the work of the United States Geological Survey; but for measuring in water power canals the Fteley-Stearns type is to be preferred.

302. Necessity of calibration. Current meters can be constructed in such a manner that the revolutions per second very nearly correspond to velocities in feet per second, which is sometimes a great convenience; also experience has shown that duplication in construction often assures similarity in performance. Precise duplication is not feasible; and the exact behavior of a new meter can not be predicted in advance. Use or abuse will cause variations in the performance of the best meter, and tightening or loosening a screw will greatly change the rating. Delicacy of adjustment in the meter and a thorough knowledge of its strong and weak points on the part of the observer are indispensable to continued good results. Meters must ordinarily not only be calibrated or rated when new to determine the relation between revolutions and velocities, but should also be rerated as often as any accident or repairs to the meter make it necessary.

303. Methods of rating. A meter may be rated either (*a*), by holding it stationary in water running at a known velocity, or (*b*) by moving the meter at a known velocity through perfectly quiet water. The effect of the wind, if there is any, should be neutralized by moving the meter in opposite directions through the water.

304. Rating in moving water. To hold the meter in its proper recording position in running water would seem to be the simplest and most natural method of rating, but variations in the velocities at different points in the cross section of a stream, however, make this method usually impracticable. A meter might be rated by holding it in a stream just below the surface and finding the velocity of flow by floats submerged to the same depth as the wheel. This and similar methods are liable to the inaccuracies and disadvantages inevitable in float measurements, as well as to the difficulty in finding in any one section and under similar conditions a range of velocities wide enough for conditions actually

needed in the use of the meter. Such a rating would natura give to the meter all the error of the float velocities. A compa son of the mean velocity in a vertical stream section a few f wide determined by rod floats, with the mean velocity determin at the same time by integrating the current meter through t section, is a good check upon the accuracy of the rating; but itself it is not a good way to rate a meter.

A Pitot tube is probably the best device for rating a meter running water.

305. Rating in still water. A meter may be rated in still wat (*a*) by propelling the meter at uniform rates of speed over a straig measured course, or (*b*) by moving the meter in a circular pa attached to a rotating arm of known length, which can be mov

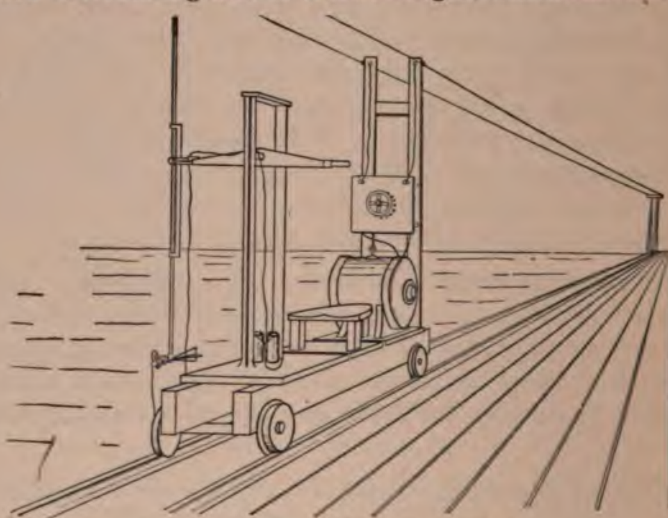


Fig. 103. — A Current Meter Rating Station.

at a uniform rate of angular velocity. The straight course is t more common; but excellent results have been gotten at a circ lar rating station at Worcester, Mass.

The meter may be fastened by a wooden beam to the bow of rowboat, which can be rowed at a uniform rate, and kept in li by ranges on the shore; and the time of the run noted by obse ers sighting across ranges or through engineers' transits at t ends of the course. This is a field method, however, perh

good to cover velocities from one half to six feet per second. A small power boat is even better than a rowboat, as it can be run at a more uniform speed, and give a wider range of velocities. A more satisfactory method is to have the rating course along the edge of a still body of water, and attach the meter to a small car propelled by hand, or by an electric motor, on a straight level track. The meter should be submerged from one to three feet; and if there is the slightest chance of any surface currents, runs and observations should be made in both directions. Figure 103 shows the arrangement for propelling the meter at one of the United States Geological Survey current meter rating stations. Any suitable form of carriage may be used. The general method of the rating from a straight, level track gives better results than any other kind of rating.

306. Notes of a current meter rating. The following is the record of a rating of a Fteley-Stearns meter fastened to a car moved by hand on a straight, level track in one of the locks at Lowell, Mass.; meter held two feet below the surface of the water.

1 Length of Course in Feet	2	3 Time in Seconds	4 Average Time Up and Down in Seconds	5 Meter Reading	6 Total Revolutions of Meter	7 Average Revolutions of Meter Up and Down	8 Revolutions of Meter per Second Col. 7 / Col. 4	9 Velocity Col. 1 / Col. 4
Buff & Berger meter, April 4, 1907				184				Feet per Second
80	Down	192.2		208	24			
80	Up	198.8	195.5	231	23	23.5	0.120	0.409
80	Down	160.2		268	37			
80	Up	160.8	160.5	308	40	38.5	0.240	0.498
80	Down	120.5		368	60			
80	Up	121.2	120.8	427	59	59.5	0.492	0.662
80	Down	80.0		499	72			
80	Up	80.1	80.05	570	71	71.5	0.893	0.999
80	Down	39.8		651	81			
80	Up	40.8	40.3	731	80	80.5	2.000	1.985
80	Down	24.1		813	82			
80	Up	24.9	24.5	895	82	82.0	3.347	3.265
80	Down	13.5		977	82			
80	Up	14.2	13.85	61	84	83.0	5.992	5.776
80	Down	10.5		145	84			
80	Up	11.0	10.75	229	84	84.0	7.814	7.442

307. Rating curves. The figures in columns 8 and 9 are plo on cross section paper to a natural scale; the revolutions of meter per second as ordinates and the velocity of the ca

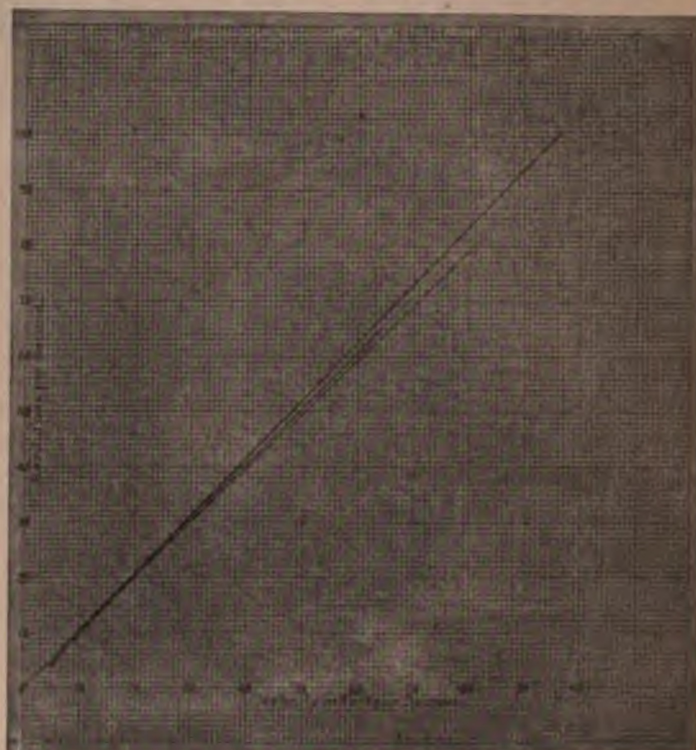


FIG. 104. — Rating Curves for Four Years of an Improved Fteley-Stearns Current M

abscissas. Through these observations, a mean line is dr which will serve as a rating curve for the meter, as long as meter is in the same mechanical condition as it was at the tim rating. See figure 104.

Ratings for low velocities. Owing to frictional resistan meters will not begin to revolve until the water has a suffic minimum velocity, which will vary with the type of meter, also, from day to day, with the same meter. The necessary n mum velocity to start the wheel is from .2 to .5 of a foot second, varying with the type. With velocities under less .5 of a foot per second, many current meters can not be used

confidence, unless the number of such low velocities in the cross section to be gauged is small, or the meter designed and rated particularly for low velocities.

308. Form of rating curves. The rating curves for most meters are very nearly straight excepting for the low and very high velocities. Some ratings start with a curve and not with a straight line. In any case the two ends of the line, that is, for the very slow and the very swift velocities, are always open to suspicion, and should not be used with the same confidence as for moderate velocities.

309. Current meter stations—location. A current meter station, which means a place for measuring flow, should be located to fulfill as nearly as possible the following conditions: the channel should be straight for at least 100 feet; the bed of the stream should not be subject to scour; the cross section should be uniform and free from obstructions and projections such as bowlders, bridge piers, drift, or growths. The flow of the stream at the place of measurement should not be directly affected by rapids, the discharge from other streams, disturbances caused by gates, spillways, water or back water from dams, ponds, or other streams.

310. Equipment of stations. The station equipment for current meter gaugings consists of (1) a place from which to operate the meter, which may be a bridge or cable and car, or a boat guyed to a cable; (2) some reference line to locate soundings and measurements, which may be a tag line stretched across the stream, or a base line measured on the bridge; (3) a gauge or gauges for reading the elevations of the water surface either above the bottom of the stream or some known datum; (4) permanent bench marks for obtaining elevations and reference points for location; and (5) in swift water, stay lines to keep the meter in position.

311. Bridge stations. Where the channel is suitable, a bridge, already built, having but few piers, and those not greatly disturbing the flow, and a floor convenient to the water surface, is the best place from which to make measurements on account of the low cost of preparation, accessibility, and the ease and rapidity of making measurements. Where, however, the bridges are high

above the water, have many piers, and where the current is swift, measurements are likely to be unsatisfactory.

312. Cable stations. Where no other method is possible and the span not over 500 feet, a car suspended and running on a

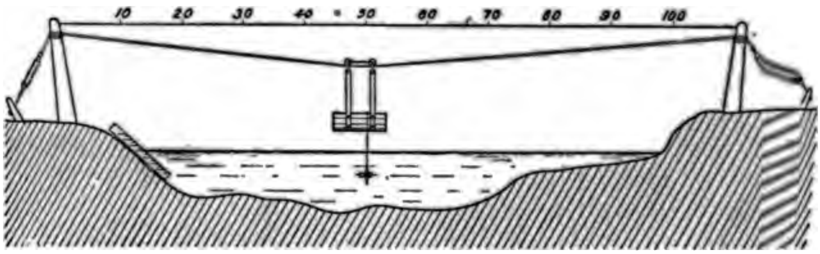


FIG. 105.—Cable Station for Current Meter Measurements.

cable stretched across the stream may be used to carry the observer. See figure 105.

313. Boat stations. At stations where no bridge is available or the span too great for a cable, or the water is too deep for wading, the meter may be fastened to an arm extending from the

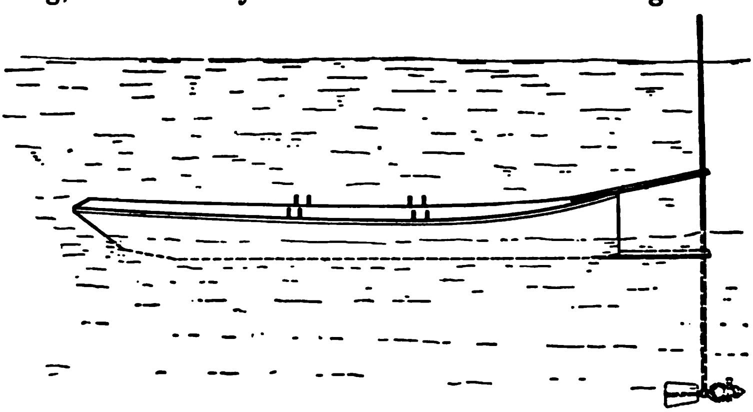


FIG. 106.—Current Meter attached to a Boat for Rating, or Measurement.

bow of a boat. The boat should be held by stay lines from the bow to a fastening on each shore. See figure 106.

314. Wading stations. In water up to about two and one half feet in depth, where the velocity is not too swift, satisfactory measurements may be made by an observer standing in the

stream and holding the meter upstream and to one side of his person. This method often is the only one by which gaugings can be made during the very low stages of a stream; and even then it may be necessary to make the gauging at other than the usual station.

315. Gauges. Gauges for observing the elevation of the water surface may be vertical or inclined boards with black graduations painted on a white background, or one of the many forms of float or chain gauges. If they are to become permanent gauges for any station, they should be placed so as to cover all stages from the highest to the lowest and be protected as far as possible from ice and drift; and in northern climates they may often to advantage be placed opposite the mouths of small streams, where the current is sufficient to keep open water during the winter.

316. Discharge measurements. The discharge in cubic feet per second is the product of two factors, — the cross-sectional area of the stream in square feet and the mean velocity of flow in feet per second. The velocity is determined from current meter observations properly corrected, the cross-sectional area from soundings or elevations of the bottom at suitable intervals, either equal or of sufficient number to show all marked changes in the profile of the bottom.

317. Soundings. The profile of the bed of the stream at the gauging station should be made by sounding, or obtaining the elevation of the bottom, at intervals of 1, 2, 4, 5, 10, 15, or 20 feet, depending upon the width of the stream, local conditions, and the degree of precision required. Distances or intervals should be measured from some permanent initial point located on the shore, if possible beyond the reach of the highest stage of the stream. The profile should include the bed and banks of the stream up to the elevation of greatest discharge. Soundings or elevations may be made from a bridge, a car on a cable, a boat, or by wading. Soundings up to 10 or 15 feet may be made with a graduated rod; for great depths with a sounding lead, which is a suitable weight attached to a rope, a steel tape, or graduated cable. Measurements must, of course, be vertical; and suitable precautions should be taken to prevent the lead or rod burying itself in soft bottoms.

318. Determining the cross sections. The distances acro stream should be plotted as abscissas and the depths as ordi on cross section paper to an open natural scale; and on horizontal line should be drawn to represent the water surf the time at which gaugings were made. From this scale dra

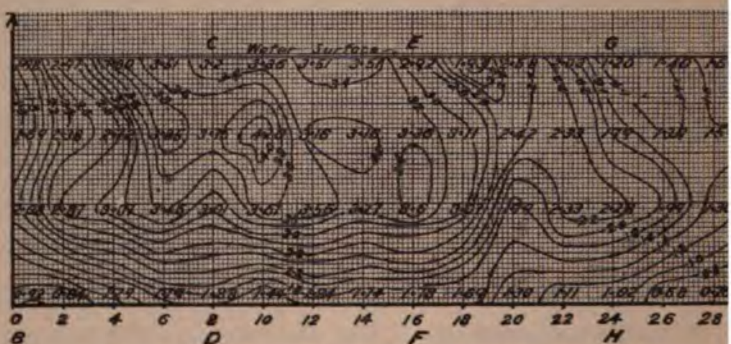


FIG. 107. —Multiple Point Measurements with a Current Meter.

required areas for any stage of the water surface or any p of the cross section may be obtained, either by dividing the into portions and computing or by planimeter. See figure 1

319. Determination of velocity. The velocity is determin placing the meter against the flow with its axis exactly v or horizontal according to its type. The number of revol of the meter in an interval of time is obtained by getting th ference between the readings of the register dials at the ning and end of the measurement, or by counting the num revolutions of the wheel in a fixed interval of time. The n of revolutions of the wheel per second being known, the ve is found by comparison with the rating curve. Velocitie determined are only for that part of the stream where the is held, or through which it is moved; and a precise dete tion of the mean velocity of the stream requires either many independent readings at different places in the cro tion, called the *point method;* or a determination of the n of revolutions of the wheel while passing at a uniform r speed through the required area. This latter method, call *integrating method*, may be applied to a part or the whole cross section.

320. The multiple point method. The multiple point method consists of many and successive, but independent, observations continuing a fixed number of seconds, perhaps from 25 to 50 (depending upon the velocity), at different points in the cross section. For convenience in computation the points should be so spaced in the whole section or a portion of it that the arithmetical mean of all the measured velocities may be taken as the approximate mean velocity of flow. The number of observations of velocity will depend upon the size of the stream, the velocity of the current, the precision required, the necessity of making a complete traverse without appreciable fluctuations in the elevation of the water surface, and the amount or interest at stake. No specific rules can be given, but a few preliminary observations of the velocities will often answer most of these questions in advance.

The gauge for reading the elevation of the water should be observed at frequent intervals and should be studied in advance to fix the period to be covered by each gauging. Periods of rapid fluctuations of the water surface are not favorable to good results; but if the measurements must be made at such a time, the points should be less numerous. A complete traverse of relatively few points is sometimes to be preferred to a more detailed traverse covering a longer time; and a repetition of the measurements is more valuable than a single long one. During a rapid change in the elevation of the water surface there may be a still greater change in the velocities at different points in the cross section.

In irregular cross sections to secure good results it is usually necessary to treat vertical sections of equal width as separate flumes, taking readings on the vertical center line at points vertically equidistant. The arithmetical mean of all the velocity determinations multiplied by the area of the section equals the discharge through that section. The sum of the discharges in all the vertical sections equals the discharge of the stream.

Form of Notes.

Time	Station	Depth	Gauge	Reading of Meter	Revs. of Meter	Revs. per Second	Velocity of Water

Computation of discharge. The velocities should be plotted on the cross section drawing. See figure 107. If the points are

so spaced as to give equal weight to each (usually possible only in regular sections), the arithmetical mean of the velocities in feet per second multiplied by the area of the section in square feet equals the discharge in cubic feet per second. Or equal velocity curves may be drawn as contours from which the discharge may be determined by using a planimeter to determine the area of each equal velocity contour and by computing the result by the average end area or prismoidal formula as if it were a volume of a solid. This is an easy and instructive method for students, is one of the very best methods of computing a point measurement, and serves admirably as a basis for the study of variations in the velocity of a given cross section.

321. Approximate point methods. The .2 and .8 method. The United States Geological Survey in its hydrographic work frequently measures the velocity at .2 and .8 of the depth of any section bounded by vertical lines, and uses the mean of the two velocities as the mean velocity of the section without further correction.

Single point methods. In the single point method the meter is held at .6 of the depth of the section, which is said to be about the depth of mean velocity; or at some point for which the coefficient of mean velocity has been determined, commonly at 1 foot below the surface, using then a coefficient of .90.

Double and single point readings are but rough approximations but serve a useful purpose in securing information of stream flow at low cost.

It is clear from experiences that these approximate methods are not applicable to water power channels either above or below gates, or below waterwheels, or wherever the relation of the velocities depends not only upon the friction of the bottom and the sides, but also upon the location and construction of the openings at which the water is admitted or drawn from the channel.

322. Integrating in one operation. The average velocity may be gotten by lowering and raising the meter at a uniform rate from the top to the bottom and back again, the operator also carrying the meter at a uniform rate from one side of the stream to the other. If skillfully and accurately done, this operation will give the arithmetical mean velocity by mechanical or manual

integration. Its use is limited, by the endurance of the operator, to small streams which can be traversed in about 10 minutes or less; but in many cases is the most satisfactory form of measurement for such streams, because it permits of a great many measurements under different conditions and avoids many disturbing conditions.

323. Integrating in sections. If the stream cross section is divided by verticals into areas of equal width, through the center line of which the meter is lowered and raised at uniform speed, the mean velocity in feet per second for each section may be gotten in one observation. This method is on the whole to be preferred for determinations of discharge. It requires less time in the field and office than a complete traverse by the multiple point method, and produces as good results, if not better. While it takes more time than the one operation integration method, less endurance is required on the part of the operator, and the chances for error are reduced. In flumes where rod floats are usable it furnishes the best possible check on the running of floats, because the meter can be integrated vertically through the section covered by the float. The results can be computed arithmetically, or partly graphically by a method similar to that used with rod floats. Pages 247 and 248 show the results of four sets of measurements with rod floats and also simultaneous measurements with a current meter.

324. Corrections. In every stream and in every stage of any stream the velocities at different parts vary. Velocities in open channels are usually lower at the sides and bottom than elsewhere. As the meter can never be placed nearer than 2 to 3 inches from the perimeter of the flume, oftener not so near, it follows that the neglect of this considerable area of relatively low velocity may give excessive discharge. The correction for low velocities on the sides may usually be obtained by continuing the velocity curve beyond actual readings to the sides. In a similar manner a few plotted point readings in verticals will give a curve for the velocity in a vertical section, which if continued to the bottom will give some reasonable value for the bottom velocity. If the channel is more than 5 feet deep, the correction used for rod floats may be used (Table XXXIX).

325. Limits of accuracy obtainable with current meters. With equal skill and accuracy the precision obtained by multiple point or integrating measurements will vary with the conditions at the place of measurement and the methods used; for well-lined rectangular channels and with velocities of 1 to 5 feet per second errors of more than 3 per cent are unnecessary, but somewhat higher precision may often be obtained. For wide or deep rivers with very low or very high velocities errors of 5 per cent to 10 per cent may be unavoidable. In rough raceways with many eddy currents, errors of 10 per cent or more are not unusual.

The .6, or *the .2 and .8*, methods may be seriously in error, and their degree of precision can not be well defined.

326. Application. Current meters may be used in any open streams or channels which have a width and depth sufficient to submerge the meter; and from this up to depths of 30 feet, and widths of perhaps 1000. Current meter measurements are used more than any other method for rivers or other open channels with steadily increasing use; and, except in cases where weirs or flumes are already in place, this is the cheapest and most satisfactory available method for measuring stream flow.

Problem

1. The rating curve of a Fteley-Stearns current meter gives the equation: $V = 0.8\ R + 0.12$ foot, where V is velocity of water, R is revolutions per second of meter. By integrating in vertical sections the current meter showed the revolutions tabulated. Find the velocity and discharge in each section, and the total discharge in cubic feet per second. Arrange results in tabular form.

(*a*) MEAN WIDTH OF (RECTANGULAR) MEASURING FLUME, 6.8 FEET—DEPTH OF WATER IN THE FLUME, 3.5 FEET

DISTANCE FROM LEFT SIDE OF FLUME	REVOLUTIONS PER SECOND		
Feet	*a*	*b*	*c*
.5	3.57	3.67	3.84
1.5	3.50	3.70	3.90
2.5	3.83	3.70	3.90
3.5	3.70	3.79	3.90
4.5	3.75	3.95	3.92
5.5	3.69	3.83	3.69
6.35	2.88	3.40	3.00

) MEAN WIDTH OF (RECTANGULAR) MEASURING FLUME, 6.8 FEET. DEPTH OF WATER IN THE FLUME, 3.6 FEET

TANCE FROM SIDE OF FLUME	REVOLUTIONS PER SECOND		
Feet	*a*	*b*	*c*
.5	3.45	3.11	3.49
1.5	3.75	3.24	3.91
2.5	3.75	3.66	3.86
3.5	3.85	3.85	3.92
4.5	3.95	3.70	3.62
5.5	3.53	3.42	3.62
6.35	3.24	2.91	3.11

CHAPTER XV

FLOW OF WATER IN CHANNELS

Flow in Pipes under Pressure

327. The determination of the volume of flow in ex
channels may be made by any suitable method of measure
but the predetermination of the probable volume of fl
channels which are to be constructed must be based upo
knowledge of the laws of flow as has been derived fron
vious experiments upon channels of similar material, forn
dimensions.

Problems concerning the flow in channels are complicat
many influencing factors of which the precise effects ar
quently not predictable. As formulas used in calculatin
charge depend upon coefficients based upon published experi
of which there are many, accuracy from the use of such for
therefore, depends upon the selection of suitable coefficie
comparing the known or assumed conditions of any problen
the supposedly similar conditions under which the flow was
ured in determining the coefficients. Such comparisons m
in error because of the difficulties in duplicating the interi
faces of new channels, in predicting the effects of *wear* and
old channels and because of uncertainties either in original i
gations on which coefficients are based, or of errors in interp
data. The effects of bends, of changes in cross-sectional
and of obstructions are known only from a limited num
experiments. Moreover, in a majority of cases in desi
channels, not only is the actual volume of flow which the cl
must deliver subject to temporary fluctuations, but the conc
of use may be entirely changed.

328. The volume of flow, the discharge. The volume o
commonly called the discharge, is the quantity of water p
through any stream cross section in a unit of time, usu

second. The discharge (Q) equals the cross section area (A) of the stream multiplied by the mean velocity of flow (V). That is,

$$Q = AV.$$

Unless otherwise stated, the area A will be assumed to be in square feet, and the mean velocity V in feet per second; hence, Q will be in cubic feet per second.

In addition to the area and velocity, two other factors which depend upon the dimensions are commonly used in formulas for stream flow, *i.e.* the wetted perimeter and mean hydraulic radius.

329. **The wetted perimeter** (*w.p.*) is the linear dimension of that part of the boundary line of the cross section of a channel which is in contact with the water.

330. **The mean hydraulic radius** (R) may be found by dividing the cross-sectional area (A) of a stream by its wetted perimeter (*w.p.*).

Examples. (*a*) Compute the mean hydraulic radius of a stream in a rectangular canal 50 feet wide if the water is 10 feet deep.

$$A = 50 \times 10 = 500 \text{ square feet.}$$
$$w.p. = 10 + 50 + 10 = 70 \text{ feet.}$$
$$R = \frac{500}{70} = 7.14 \text{ feet.}$$

(*b*) Compute the mean hydraulic radius of:

(1) A stream flowing in a circular pipe entirely full;

$$A = \frac{\pi D^2}{4};\ w.p. = \pi D;\ R = \frac{A}{w.p.} = \frac{\frac{\pi D^2}{4}}{\pi D} = \frac{D}{4}.$$

(2) A stream flowing in a circular pipe half full;

$$A = \frac{\pi D^2}{8};\ w.p. = \frac{\pi D}{2};\ R = \frac{A}{w.p.} = \frac{\frac{\pi D^2}{8}}{\frac{\pi D}{2}} = \frac{D}{4}.$$

331. **For streams in circular channels** flowing at any depth, the areas, wetted perimeters, and the mean hydraulic radii may be taken from Table LXV in Appendix, which is based on a diameter of 1, and may be readily converted to any other diameter.

332. Steady flow. If the discharge is constant for successiv equal intervals of time, the flow is said to be steady. Computa tions of discharge, unless otherwise specified, are based on an actua or assumed steady flow.

333. Uniform flow. For successive stream sections in whic the discharge is steady and the velocity is constant, the flow i said to be uniform.

334. Open and closed channels. Channels for conveying wat may be classified as (1) closed channels; or (2) open channels.

(1) **Closed channels** or channels under pressure are those which the flowing water has no continuous free surface, and which the intensity of pressure at every point in the stream in excess of that due to the depth of water; that is, the flowi water entirely fills the channel and exerts an upward pressure the entire top cover. Such channels may be of any cross-sectio form, but are usually circular.

Supply and distribution pipes in water supply systems, pum ing mains, hot-water heating pipes, penstocks, draft tubes of tu bines, and fire hose are common examples of closed channels.

(2) **Open channels** are those in which the upper surface of th flowing water is exposed to atmospheric pressure, and in whic the intensity of pressure at every point in the stream is due alon to the depth of the point below the water surface.

Rivers, canals, and, ordinarily, aqueducts and sewers are ope channels; but a river or canal may become a closed channel i covered with ice, and an aqueduct or sewer may become a close channel if the water flows under pressure. See Chapter XVI fo flow in open channels.

The fundamental laws applying to flow in closed and open cha nels are probably identical; and the same general formulas ar with proper modifications, equally applicable to open or close channels. Nevertheless, whether the water in a channel is t flow under pressure or open, must be determined in advance o construction for other reasons. Any wrong assumption may r sult in failure to deliver the required volume of flow, or in delive ing the water at undesired pressures, or in actual failure of th structure itself, due to excessive internal intensity of pressure.

335. Relation of area to velocity. A change in the cross-sectional area of either closed or open channels is accompanied by a change in the mean velocity; but there is a difference between closed and open channels in the manner in which adjustments of velocity to changes in cross-sectional area take place. In open channels, although a stream must conform to the shape of its bottom and sides, yet the elevation of its free surface, the cross-sectional area, and the velocity will vary in a manner that can only be accurately determined by observations. Knowledge of the behavior of a stream in an open channel with variable flow can rarely be accurately used for another channel with variable flow. In closed channels, on the other hand, excepting short stretches near points where abrupt changes in sections occur, the areas of the channels and the areas of the streams are identical; and changes in the velocity conform to known changes in area; and comparison with other channels of similar size, form, and nature is easier and more trustworthy.

336. The total energy available at a given point in a stream is the sum of the kinetic energy due to velocity of flow, the energy due to the intensity of pressure in the water, and the energy due to elevation of the given point with reference to some other point within or without the stream. Hence,

$E = wQ\left(\frac{V^2}{2g} + \frac{p}{\gamma} + \frac{p_a}{\gamma} + h_e\right) =$ the total available energy in foot pounds per second. (1)

$p =$ the intensity of pressure measured from atmospheric pressure; if more than atmospheric, p is $+$, if less, p is $-$.

$p_a =$ atmospheric pressure.

$h_e =$ difference in elevation between the given point and the point of reference.

The total head at any given point, therefore, is

$$H = \frac{V^2}{2g} + \frac{p}{\gamma} + \frac{p_a}{\gamma} + h_e. \tag{2}$$

337. Lost head. The movement of a body of water sets up various resistances, in the overcoming of which energy is expended, just as energy is expended on a locomotive in overcoming

track and other resistances. Energy so used is mechanical irrecoverable; it can not reappear in any form, or do furth work. The work done on a moving body in overcoming a resis-ance may be found by multiplying the resistance, if known, b the distance through which the body moves against the resistanc It follows, then, whether the resistance is constant or variabl that if the stream supplies the energy to overcome resistance every successive downstream section will have less energy or les head than those which precede it. **The difference between th total head at one section in a stream, and the total head in anothe section, is lost head.** By Bernoulli's theorem :

$$\frac{V_0^2}{2g}+\frac{p_0}{\gamma}+\frac{p_a}{\gamma}+h_0=\frac{V_2^2}{2g}+\frac{p_2}{\gamma}+\frac{p_a}{\gamma}+h_2+h_\lambda.$$

Hence
$$h_\lambda=\frac{V_0^2-V_2^2}{2g}+\frac{p_0-p_2}{\gamma}+h_0-h_2. \qquad (\_$$

For example, between A and C, figure 108, by equation (3) t lost head is 2 feet; between A and E the lost head is 60 feet.

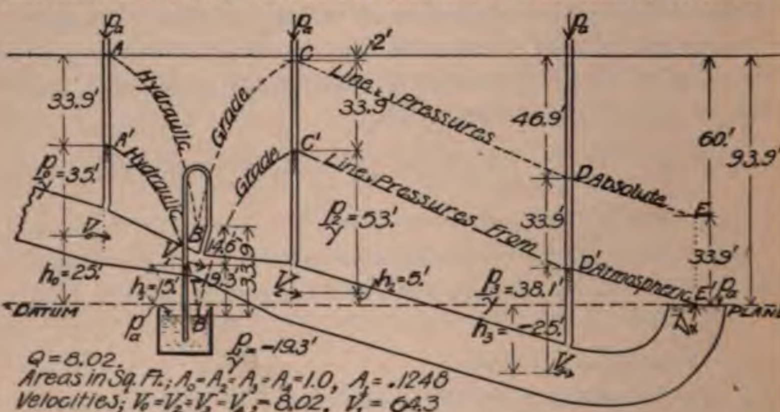


FIG. 108. — A Pipe with Steady but Variable Flow under Pressure.

Mere changes in velocity, intensity of pressure, or elevatio must not be confused with lost head. While at one point in stream nearly all the energy may be due to velocity, and in an other section of the same stream the energy may be nearly a due to pressure, or to elevation, yet the total energy or head ma be nearly the same; and the only loss of head would be that du

to the energy necessarily consumed in overcoming resistances and in bringing about the transformation of energy.

At section *A*, figure 108, the velocity head is but 1.0 foot, the total pressure head is 68.9 feet, and the head due to elevation 25 feet; but at section *B* the velocity head is 64.3 feet, the total pressure head is 19.3 feet less than atmospheric pressure, and the head due to elevation 15 feet; the head lost between *A* and *B* is only 1.0 foot, as the total heads are 94.9 feet at *A*, and 93.9 feet at *B*.

338. **The hydraulic grade line** is a profile on the axis of a stream, so drawn that a vertical distance from any point on the profile to any point in the stream is a measure of the intensity of pressure at the latter point.

The hydraulic grade line of a channel under pressure is a prôfile of the water surfaces in a series of actual or imaginary piezometers set in the channel. The line *ABCDE*, figure 108, is the true hydraulic grade line, if intensity of pressure reads from zero; if read from atmospheric pressure, the line *A'B'C''D'E'* is the hydraulic grade line.

The hydraulic grade line of an open channel is a profile of the water surface.

An imaginary line drawn from the water surface in a reservoir or piezometer at the source of flow to the water surface at its destination (see *A'E'*, figure 108), may be designated the hydraulic grade line between these points. If a channel is to flow under pressure, it must be so built that no part of the stream lies above this line, otherwise the channel will not flow entirely full, air will collect in the summit, and the flow will be modified. Where such an arrangement is necessary, an air pump may be installed to pump out the air; but this is neither a permanent nor satisfactory remedy.

If a pipe is laid entirely below the hydraulic grade line, its depth is immaterial so far as the discharge is affected; but increased depth means increased internal intensity of pressure, and this requires a stronger channel.

Graphical representation of the head due to elevation. If on the profile of the hydraulic grade line a horizontal line is drawn representing the elevation of the point to which the total head is

referred, the length of the vertical from any point in the stream to this horizontal line (or datum plane) represents the head due to the elevation of that point.

The length of the vertical from the datum plane to the hydraulic grade line is obviously a measure of the sum of the heads due to intensity of pressure, and to elevation at any point in the stream which is located on this vertical.

The neglect of atmospheric pressure in drawing the hydraulic grade line will cause no practical error, if the intensity of pressure is everywhere in a stream equal to or in excess of atmospheric; for example, $C'D'E'$ is, for all practical purposes, the equivalent of the true hydraulic grade line, CDE. If, however, the pressure anywhere drops below atmospheric, the grade line must, in order to show the actual intensities of pressure, include atmospheric pressure; for example, see ABC and $A'B'C'$.

339. The slope of the hydraulic grade line, or its inclination, is commonly measured by the sine of the angle of inclination; viz.:

$$S = \frac{h}{L}. \qquad (4$$

$S =$ the sine of the angle of inclination measured from the horizontal, and commonly called *the slope.*

$h =$ the difference in elevation between two successive points on the hydraulic grade line.

$L =$ the distance between the two successive points measured along the axis of the channel.

If the flow is uniform, the slope of the hydraulic grade line is constant, always downward in the direction of flow, and equals the loss of head per unit distance of channel run, viz.:

$$S = \frac{h_\lambda}{L}.$$

If the flow is variable, the difference (h) in elevation between successive points in the hydraulic grade line, even if referred to a datum plane, does not always measure the lost head, because it does not take into account the heads due to velocity. The slope in open and closed channels has a general downward tendency except at places where there is a marked change in velocity. In closed channels, transformations of energy from one form to

another may cause sudden rises and falls of the grade line, which in themselves are neither gains nor losses of head. In figure 108, between A and B, where the pressure and elevation are changing into velocity, there is an abrupt drop; and between B and C, where velocity is changing back again, there is an equally abrupt rise, but the actual lost head is slight. With variable flow the hydraulic grade line is a succession of different slopes.

Illustration. Consider the following case of steady flow, as shown in figure 108. Let $Q = 8.02$ cubic feet per second.

The total heads at sections A, B, C, D, and E are found by formula (2) to be as follows:

$H_0 = 94.9$ feet; $H_1 = 93.9$; $H_2 = 92.9$; $H_3 = 48.0$; $H_4 = 34.9$.

The total heads in excess of atmospheric pressure are as follows:

$$H_0 - 33.9 = 61; \quad H_1 - 33.9 = 60; \quad H_2 - 33.9 = 59;$$
$$H_3 - 33.9 = 14.1; \quad H_4 - 33.9 = 1.0.$$

The lost heads between successive points may be computed by (3) as follows:

$H_0 - H_1 = 94.9 - 93.9 = 1$; or eliminating atmospheric pressure $(61 - 60)$.

$H_1 - H_2 = 93.9 - 92.9 = 1$; or eliminating atmospheric pressure $(60 - 59)$.

$H_2 - H_3 = 92.9 - 48.0 = 44.9$; or eliminating atmospheric pressure $(59 - 14.1)$.

$H_3 - H_4 = 48.0 - 34.9 = 13.1$; or eliminating atmospheric pressure $(14.1 - 1.0)$.

The total loss, $H_0 - H_4 = 60.0$

340. The determination of the most economical size of channel to use involves many practical considerations. A channel may be so designed that practically all the total head, or only a small part of it, is lost. If the cross-sectional area is large, the head lost in delivering a given volume of flow will be relatively small; but the cost of construction will be relatively large. If the cross-sectional area is small, the head lost will be relatively great; and the loss of head may often be more costly than to increase the size of channels. The lost head in a water power plant reduces the

output of energy from the wheels; in a water supply system it reduces the pressure, which may be needed to deliver water at high points. The problem is usually one of deciding whether an increased fixed charge and maintenance charge due to larger channels is more or less than the value of the loss of head or the cost of reproducing the required head by pumping. A thorough knowledge of the factors which affect resistances to flow is essential in the design of channels.

Conditions affecting resistances to flow. Resistance is independent of the intensity of pressure in the flowing water.

Resistance increases with an increase in the area of the rubbing surface; with increased roughness of the lining of the channel; with the square of the velocity (within 15%), for velocities with which the engineer has to deal (for critical velocity see Chapter VI); with abrupt changes in the cross-sectional area of the channel; at bends or at junctions with other channels; with decrease in depth in certain open channels, without an increase in the rubbing surface; with an increase in suspended matter in the water.

Resistance decreases with rise of temperature.

The various resistances will assume different proportions in every different problem; and may have to be considered, or may be neglected, according to their relative magnitude. Variations due to suspended matter and to temperature changes, except in the case of very small pipes, are commonly not considered.

341. Formulas for flow in channels. It is extremely doubtful whether any single formula which is satisfactory and generally applicable can be evolved even after a considerable increase in existing data; and the utmost that can be expected is a set of formulas each strictly applicable to its own set of conditions, and generally applicable, but with less precision, to a wider range. Two formulas are widely used in American and English practice, virtually two statements of the same formula, and based upon the assumption that the resistances vary as some function of the square of the mean velocity of flow. They have wide use in spite of several well-known inconsistencies, because they are convenient and have given as high precision as many practical problems appear to warrant.

342. The Chezy formula is probably more used than any other formula. It may be used either for open or closed channels; and for steady uniform flow is usually stated as follows:

$$V = C(RS)^{\frac{1}{2}}. \tag{5}$$

C = coefficient increasing with the mean hydraulic radius or with increasing diameter, and for new clean channels usually increasing with the mean velocity of flow, and decreasing with the roughness of the channel.

R = mean hydraulic radius.

S = sine of the slope of hydraulic grade line.

Another form of the Chezy formula. The common text-book formula which, as nearly as may be determined, was proposed by Weisbach,* and which is used only for pipes under pressure, is as follows:

For steady uniform flow under pressure in circular pipes,

$$h_f = f\frac{L V^2}{D\,2g}. \tag{6}$$

h_f = the head lost in friction (in feet) in a pipe of length L, and of diameter D.

f = the friction factor or coefficient of friction, decreasing with an increase in the diameter of the pipe and commonly with an increase of velocity of flow; and increasing with the age and roughness of the surface in contact with the water.

L = the length (in feet) of the pipe measured on its axis.

D = the internal diameter (in feet) of the pipe.

V = the mean velocity of flow in feet per second.

g = the acceleration due to gravity, taken here as 32.16.

Equations (5) and (6) are merely different expressions for the same formula. Either may be used to suit convenience.

343. Exponential formulas. Several formulas are in use which recognize the fact that resistances to flow do not vary exactly as the square of the mean velocity, but according to some other

* *Mechanics*, Coxe's translation, p. 866. The formula is not uncommonly designated as Weisbach's, Darcy's, Weston's, or Chezy's.

exponential value. Of these formulas, which are commonl
ignated exponential formulas, the best in use in this cou
Williams and Hazen's, which was worked out from the be
lished data; it is as follows:

$$V = C_w R^{.63} S^{.54} .001^{-.04}.$$

This coefficient C_w is based chiefly upon the roughness
channel lining and should not be confused with C in the
formula. The inconvenience in computing by this formul
be obviated by the use of Williams and Hazen's *Hydraulic*
or their slide rule.

The plotting of velocities or discharges against friction h
logarithmic paper shows that for a constant hydraulic radi
a given condition of lining, the slope of the log curve is
constant for any given channel. A comparision of a num
such plottings shows graphically why it is difficult to m
general formula fit such a widely varying range of conditi
are found in practice, especially in dealing with channels of
the surface has been roughened by use or by growths of v
kinds. See Table XLIX.

Though it seems evident that we shall never have one fo
to fit accurately all kinds of channels, it appears probabl
we may have a small group of formulas each of which w
some particular class of channels.

Flow of Water in Pipes under Pressure—Unif Flow

344. Pipes. A pipe is a hollow cylindrical tube of su
length to show during flow an appreciable loss of head in fr
in addition to the head lost in entrance and in acquiring ve

The materials used in constructing pipes are cast iron, w
iron, steel, lead, brass, rubber, wood, glass, concrete, v
clay, and various other substances.

345. Uniform flow under pressure requires the followin
ditions: a steady volume of flow which entirely fills a
of straight pipe, which has a constant cross-sectional area an
uniform roughness.

Under the conditions of steady, uniform flow in a pipe

pressure, the cross-sectional area, the mean velocity, and the discharge are all constant in every section of the pipe.

Let figure 109 represent a straight length of pipe with steady uniform flow under pressure.

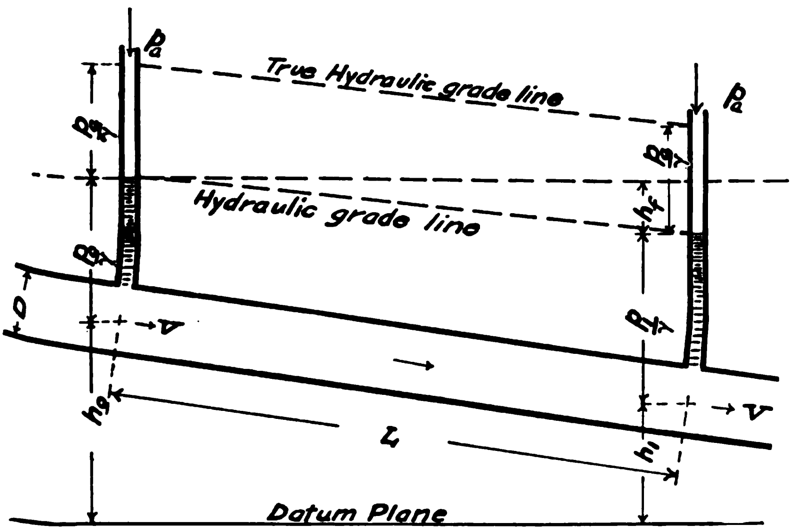


FIG. 109. — Uniform Flow in a Pipe under Pressure.

Let D = the internal diameter of the pipe in feet.

Then $A = \frac{\pi D^2}{4}$ = cross-sectional area of stream in square feet. (8)

$V = \frac{Q}{A} = \frac{4\,Q}{\pi D^2}$ = mean velocity of flow in feet per second. (9)

$Q = A\,V = \frac{\pi D^2}{4}\,V$ = discharge in cubic feet per second. (10)

346. The friction head. Between any two consecutive sections, as 0 and 1, the water will part with some of its energy; and the head at 1 will be less than at 0. As the pipe is straight and the flow both uniform and steady, such loss of head as may occur will be due to the resistance to motion, which is in part rubbing friction between the pipe surface and the water, and in part is internal resistances in the water itself due to disturbances which are incident to flow. This resistance, which is a complicated phenomenon, not thoroughly understood, is designated "friction."

The loss of head due to resistances in a straight pipe with uniform flow is called **friction head**, and may be designated h_f.

The friction head may be actually determined in an existing pipe by measuring the pressure heads and elevations at two consecutive sections of a pipe and making the following computations:

Let

$\frac{p_0}{\gamma}$ = the pressure head at the center of the pipe at 0.

h_0 = the elevation of the center of the pipe above the datum plane at 0.

$\frac{p_1}{\gamma}$ = the pressure head at the center of the pipe at 1.

h_1 = the elevation of the center of the pipe at 1.

Then by (2) the total head at 0,

$$H_0 = \frac{V^2}{2g} + \frac{p_0}{\gamma} + \frac{p_a}{\gamma} + h_0;$$

and the total head at 1,

$$H_1 = \frac{V^2}{2g} + \frac{p_1}{\gamma} + \frac{p_a}{\gamma} + h_1.$$

The head lost in friction,

$$h_f = H_o - H_1 = \frac{p_0}{\gamma} + h_0 - \frac{p_1}{\gamma} - h_1 = \text{here } h_\lambda. \tag{11}$$

347. Formulas for uniform flow in pipes under pressure. If at the same time the volume of flow and the area of the pipe are also determined, the relation between the friction head and the velocity of flow may be computed; and from a series of observations covering a sufficient range of conditions, the law of flow for a particular pipe may be deduced. From experimental data obtained in this way, the probable flow in other pipes may be predicted. For many particular cases fairly complete information is available and reliable predictions may be made; but for a large number of cases dependence has to be placed upon empirical formulas.

Problems concerning the flow in pipes arise in determining either the head required, the mean velocity, the discharge, or the diameter, if the length and the other essential factors are given.

The Chezy formula in two of its common forms given in equations (5) and (6) will here be used, viz.:

$$V = C(RS)^{\frac{1}{2}}; \text{ or } h_f = f\frac{LV^2}{D\,2g}.$$

For uniform steady flow in circular pipes:

The mean hydraulic radius, $R = \frac{D}{4}$; (12)

The slope of the hydraulic grade line, $S = \frac{h_f}{L}$; (13)

The area of the stream, $A = \frac{\pi D^2}{4}$. Then,

The friction head, $h_f = \frac{4\,V^2 L}{C^2 D}$; or $h_f = f\frac{LV^2}{D\,2g}$. (14)

The mean velocity of flow in feet per second,

$$V = \frac{C}{2}\left(\frac{h_f D}{L}\right)^{\frac{1}{2}}; \text{ or } V = 8.02\left(\frac{h_f D}{fL}\right)^{\frac{1}{2}}. \qquad (15)$$

The discharge in cubic feet per second, $Q = A\,V = \frac{\pi D^2 V}{4}$;

$$Q = .3927\,C\left(\frac{h_f D^5}{L}\right)^{\frac{1}{2}}; \text{ or } Q = 6.3\left(\frac{h_f D^5}{fL}\right)^{\frac{1}{2}}. \qquad (16)$$

The diameter in feet required to deliver a given discharge,

$$D = 1.453\left(\frac{LQ^2}{h_f C^2}\right)^{\frac{1}{5}}; \text{ or } D = .479\left(\frac{fLQ^2}{h_f}\right)^{\frac{1}{5}}. \qquad (17)$$

Comparison of coefficients C and f,

$$C = \left(\frac{8g}{f}\right)^{\frac{1}{2}} = \frac{16.04}{(f)^{\frac{1}{2}}}; \text{ and } f = \frac{8g}{C^2} = \frac{257.28}{C^2}. \qquad (18)$$

Both of the above sets of equations are used as may be convenient; coefficients have in some instances been derived in terms of f, in others in terms of C, but they are strictly convertible. In the tables of coefficients in common use both coefficients C and f are here given; in Table XLIX of experimental results only values of C are given.

348. Relative discharging capacities of pipes. Pipe computations may also be facilitated by considering the following relations:

Let L and D be constant; then by (15),

$$Q \text{ varies as } \left(\frac{h_f}{f}\right)^{\frac{1}{2}}; \text{ or if } f \text{ is constant, as } h_f^{\frac{1}{2}}.$$

Let L and h_f be constant; then by (16),

$$Q \text{ varies as } \frac{D^{\frac{5}{2}}}{f^{\frac{1}{2}}}; \text{ or if } f \text{ is constant, as } D^{\frac{5}{2}}.$$

Let L and h_f be constant; then by (17),

The required diameter varies as $(fQ^2)^{\frac{1}{5}}$; or if f is consta as $Q^{\frac{2}{5}}$.

349. The Darcy-Weston friction factor. The values of f largely used in American practice, converted from Darcy' formula by Edmund B. Weston,† are computed from the follo ing equation (19). Darcy's experiments were on pipes from inch to 20 inches in diameter.

For pipes having interior surfaces similar to new cast-iron pip for velocities of flow greater than .33 foot per second:

$$f = .01989 + \frac{.001666}{D}. \quad \text{(See Table L.)} \qquad (1$$

The values of f computed by formula (19) do not take in account variations in f with changes in velocity, and do not p cisely agree with well-authenticated experiments made sin Darcy, nor for pipes beyond 20 inches diameter, which was t largest size used by Darcy. For the small pipes of the ordina waterworks practice, they give results as precise as either t assumptions as to capacity may ordinarily be made, or the eff of increasing roughness can be predicted.

350. For wrought-iron and steel riveted pipes, the discharge less than for cast-iron pipes of the same age and diameter, cause the overlapping plates and projecting rivet heads at t joints usually offer more resistance than the joints in cast-ir pipes. By countersinking the rivets and using taper points w beveled edges a much greater capacity can be obtained. F coefficients see Table XLIX.

* Henri Darcy, *Recherches Expérimentales Relatives au Mouvement de l'Eau d les Tuyaux.* Paris, 1857.

† E. B. Weston. *Tables showing Loss of Head due to Friction of Water in Pi* D. Van Nostrand Co., New York.

351. For old cast-iron and steel pipes. As iron or steel pipes age with service, not only will their interior surfaces usually become roughened by the destruction of the preservative coating, and the formation of vegetable and other growths increase the friction head; but also these growths may increase with time to such an extent as to actually diminish the actual discharge area of the pipe. See figure 110.

There is at the present time sufficient experimental data to forecast the depreciation of iron and steel pipes sufficiently accurately for ordinary waterworks and water power purposes, but hardly enough to formulate satisfactorily these results; the increasing number of large and small pipes and aqueducts which are being measured periodically will probably make this possible in the near future. **Empirical formulas for predicting the deterioration of pipes should be cautiously used.**

In Table XLIX are given examples of actual measurements of flow in old and new pipes which show the effect of depreciation in use due to the tuberculation and vegetable growths.

352. For pipes having interior surfaces similar to brass and lead, and for diameters from $\frac{1}{2}$ inch to $3\frac{1}{2}$ inches, Weston derived from the results of two hundred and thirty-eight experiments on twenty-seven different pipes the following expression for computing values of friction coefficient:

$$f = .0126 + \frac{.0315 - .06\,D}{(V)^{\frac{1}{2}}}. \tag{20}$$

For small brass pipes the best available data are from Saph and Schoder's experiments given in Table XLIX.

Little or no allowance for roughening need be made for pipes made of lead, brass, or any material which does not ordinarily pit or gather rough tubercles on the interior; but there may be some increase in friction loss in lead pipes due to deposits with some kinds of water.

353. Values of coefficients for use in the Chezy formula are given as follows:

Experimental values. Table XLIX gives the results of a number of the best published experiments on pipes. For pipes larger than 3 inches in diameter, the table contains a large portion of the

data on which the coefficients in present use are based. Many of the experimental coefficients may be directly used; if so, they are in general to be preferred; if not directly usable, they will serve as a guide in the use of empirically determined values.

Darcy-Weston coefficients. Table L gives values of the friction factor f computed by Weston's formula (19), for Darcy's experiments together with equivalent values of C.

Smith coefficients. Table LI gives values of C derived by Hamilton Smith, Jr., together with the equivalent value of f and the friction head for 1000 feet of straight pipe. These values should be used only for very smooth channels.

Fanning coefficients. Table XLII gives values of C and f corresponding to Fanning's coefficient, which was equal to $\frac{f}{4}$. They are applicable only to very smooth channels.

Weston coefficients for brass and lead pipes. Table LIII gives values of f computed by formula (20), equivalent values of C, and the friction head for 1000 feet of pipe.

354. The Kutter formula, although intended and largely used for pipes as well as for open channels, is not recommended for pipe calculations. To estimate directly a suitable value of C is simpler, and probably quite as accurate as to estimate a value of n.

The Kutter formula for use in open channels is taken up in Chapter XVI.

355. *Examples.* The following examples will illustrate the computation of the flow in pipes.

Given a new straight, smooth cast-iron pipe 1000 feet long, laid as shown in figure 112. The intensity of pressure on the center of the pipe at O is 60 pounds per square inch. The center of the pipe at O is 100 feet higher, and at 1 it is 70 feet higher than an arbitrary reference (datum) plane.

A. *Given:* Diameter = 2 feet; discharge = 9.42 cubic feet per second; and length = 1000 feet.

Compute the head lost in friction, and the intensity of pressure at 1.

$$V = \frac{Q}{A} = \frac{9.425}{3.1416} = 3 \text{ feet per second}; \; f = .01989 + \frac{.001666}{2} = .0207.$$

0. — A Cast-iron (water) Pipe cut apart to show the Accumulation of Tubercles in 54 Years of Service.

By (14), $h_f = \frac{4 \times 3^2 \times 1000}{111^2 \times 2} = 1.45$ feet; or $h_f = .0207 \frac{1000 \times 3^2}{2 \times 64.32}$ $= 1.45$ feet.

By (3), $\frac{p_a}{\gamma} + \frac{V^2}{2g} + \frac{60}{.433} + 100 = \frac{p_a}{\gamma} + \frac{V^2}{2g} + \frac{p_1}{.433} + 70 + 1.45.$

Whence $\frac{p_1}{.433} = 167$ feet; and $p_1 = 72.3$ pounds per square inch.

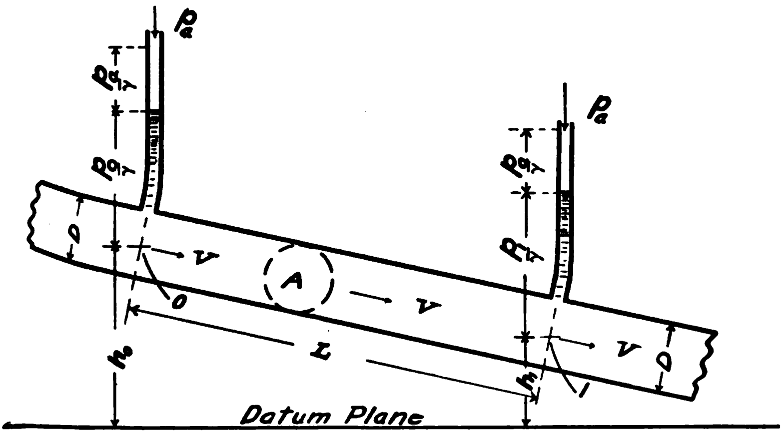


FIG. 112. — Uniform Flow in a Pipe under Pressure.

B. *Given:* Friction head = 1.45 feet; diameter = 2 feet; and length = 1000 feet; compute mean velocity.

By (15) $V = \frac{111}{2}\left(\frac{1.45 \times 2}{1000}\right)^{\frac{1}{2}} = 3;$

or $$V = 8.02\left(\frac{1.45 \times 2}{.0207 \times 1000}\right)^{\frac{1}{2}} = 3 \text{ feet per second.}$$

C. *Given:* Friction head = 1.45 feet; diameter = 2 feet; $L = 1000$ feet. Compute the discharge.

By (16), $Q = .3927 \times 111\left(\frac{1.45 \times 2^5}{1000}\right)^{\frac{1}{2}} = 9.42;$

or $$Q = 6.3\left(\frac{1.45 \times 2^5}{.0207 \times 1000}\right)^{\frac{1}{2}} = 9.42 \text{ cubic feet per second.}$$

D. *Given:* Length = 1000 feet; $Q = 9.42$ cubic feet per second; friction head = 1.45. Compute D.

Here f is unknown, but a trial solution may be made, using the mean value of $f = .02$, $C = 113$.

By (17), $$D = 1.453\left(\frac{1000 \times 9.42^2}{1.45 \times 113^2}\right)^{\frac{1}{5}} = 1.98 \text{ feet};$$

or $$D = .479\left(\frac{.02 \times 1000 \times 9.42^2}{1.45}\right)^{\frac{1}{5}} = 1.98 \text{ feet.}$$

The nearest commercial size is 2.0 feet (see Table VI); and f for this size is .207, hence no further computation is required.

If the value of f can not be selected by means of the data given, some value should be assumed and successive trial solutions should be made until all the factors of the problem agree. Rarely more than two solutions are required to select commercial sized pipes, because they vary by considerable differences in diameter. With tables of discharge such as Weston's and Williams and Hazen's, selections may be made with little computation.

NOTE. Certain experiments on a 24-inch pipe which can not at present be published gave the following friction heads for 1000 feet at a velocity of 3 feet a second; after 3 months' service about 1.2 feet; after 1 year's service about 1.5 feet, after 2 years about 1.6 foot.

The effect of age on certain cast-iron pipes in Table XLIX should be studied.

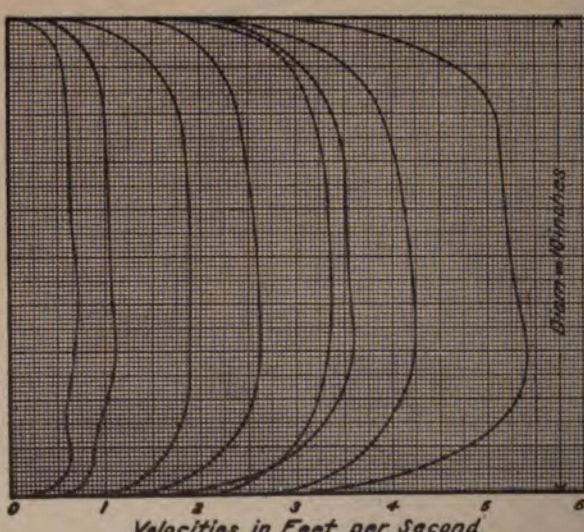


FIG. 113. — Velocity Curves in a Pipe.

356. Distribution of velocities in the cross section of a pipe. The velocity of the water in a smooth-lined pipe under pressure varies from a minimum at the perimeter to a maximum at or near the center. Late investigations by Bazin (1897); by Cole (1897); and by Williams, Hubbell, and Fenkell (1901–1902), indicate that in a general way the distribution of

s in a new pipe are fairly represented by the surface of an of revolution with its axis coinciding with the axis of The mean velocity is from .8 to .9 of the center ve- ie mean ratio deduced by Williams being .84. The at the perimeter is about .5 the maximum velocity. And t of mean velocity is at about .75 of the radius of the a its center. With rough or incrusted linings the distri- velocities will vary greatly. In any case, the distribution consequent ratios can only be determined satisfactorily by e with a Pitot tube (see Chapter VII, figure 38, for a f velocity curves). Figure 113 shows a series of velocity ade by pitometer measurements in 10-inch diameter old pipe.

Variable Flow in Pipes under Pressure

osses of head in variable flow. If a pipe is not straight, niform cross-sectional area, in addition to the friction re will be other losses due to changes in velocity or of flow. These losses vary in magnitude, and should lered in any problem if their magnitude is sufficient to e degree of accuracy required in the result. These y be conveniently expressed as a function of the velocity directly as lost head in feet. The usual classification due to change in velocity or direction, which are called sses, is as follows:

t at entrance to the pipe, $h_o = \xi_o \frac{V^2}{2g}$.

t because of enlargement of the cross-al area, $h_e = \xi_e \frac{V^2}{2g}$.

t because of contraction of the cross-al area, $h_\kappa = \xi_\kappa \frac{V^2}{2g}$.

t at bends, $h_b = \xi_b \frac{V^2}{2g}$.

t at valves and gates, $h_s = \xi_s \frac{V^2}{2g}$.

'ormulas to include all losses of head. The total head roducing flow may in addition to the head due to veloc-

ity (h_v) and the friction head (h_f) include one or more of the above losses. A complete equation covering all these losses of head may be written as follows:

Let H = the total head available for, or required to produce, a given flow of water.

Then $$H + \frac{p_a}{\gamma} = h_v + h_f + h_o + h_e + h_x + h_b + h_c + \frac{p_a}{\gamma}$$

Or in terms of $\frac{V^2}{2g}$,

$$H = \frac{V^2}{2g}\left(1 + \frac{fL}{D} + \xi_o + \xi_e + \xi_x + \xi_b + \xi_c\right). \tag{21}$$

Hence, $$V = 8.02\left[\frac{H}{1 + \frac{fL}{D} + \xi_o + \xi_e + \xi_x + \xi_b + \xi_c}\right]^{\frac{1}{2}};$$

or simply $$V = 8.02\left(\frac{H}{1 + h_\lambda}\right)^{\frac{1}{2}}. \tag{22}$$

h_λ = sum of heads lost between given sections in any pipe,

and $$Q = AV = \frac{\pi D^2 V}{4}.$$

359. In a **uniform straight pipe**, or one which is laid with easy curves and may be considered practically straight, leading from a reservoir or from another very large pipe, only the **head required to get up velocity**, the **head lost at entrance**, and the **friction head need be considered.**

Then $$V = 8.02\left[\frac{H}{1 + \xi_o + \frac{fL}{D}}\right]^{\frac{1}{2}}. \tag{23}$$

In many such pipes the velocity head and the entrance head are relatively so small in comparison with the friction head that their omission will not affect the accuracy of the result. For example, if a pipe is 7500 feet long and $f = .01$, or if the pipe is 2500 feet long and $f = .03$, the neglect of $(1 + \xi_0)$ in formula (23) will cause an error of only 1 %, which implies a degree of precision rarely obtained in pipe problems.

360. **If minor losses are negligible**, all the available head is said to be used in friction, that is, the total available head H is assumed to equal the friction head h_f, and (23) becomes

$$V = 8.02\left(\frac{HD}{fL}\right)^{\frac{1}{2}}. \qquad (24)$$

This formula is obviously different from (15) only in using H for h_f; consequently formulas (14) for head, (16) for discharge, and (17) for required diameter may be used by a mere change of h_f to H.

The minor losses may be approximately determined with more or less accuracy by the following formulas and methods.

361. Head lost at entrance. For a short distance at the end where a pipe takes out of a reservoir, experiments have shown that the stream first contracts and then enlarges, and finally fills the pipe as in a short tube of similar form. The losses in a short tube depend on the relation between velocity in the contracted section and the velocity in the pipe where it is flowing full; and this relation is largely determined by the form of the opening. It is fair to assume that the losses in the short distance at the entrance to a pipe may be computed as for a short tube. Therefore

The head lost at entrance, $h_o = \left(\frac{1}{C^2} - 1\right)\frac{V^2}{2g} = \xi_o\frac{V^2}{2g}$. (25)

$V =$ the mean velocity of flow in the pipe.

$C =$ the coefficient of discharge of a short tube having a form similar to the entrance end of the pipe.

$\xi_o =$ factor for entrance loss $= \left(\frac{1}{C^2} - 1\right)$.

The common forms of entrance are as follows:

A. The end of the pipe set flush in a flat surface. (See figure

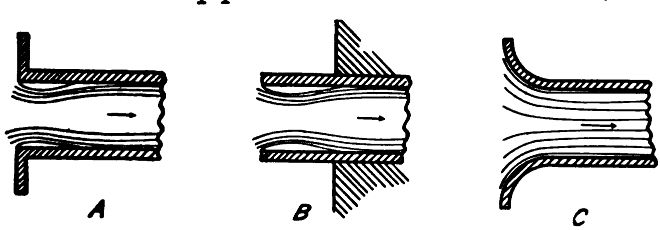


FIG. 114. — Three Common Forms of Pipe Inlets.

114 A.) This form corresponds to a standard short tube. Hence $C = .82$; and $\xi_o = .5$. (See Chapter X, § 188.)

B. The end of the pipe projecting inward. (See figure
This form corresponds to a reëntrant short tube. $C = .72$ and $\xi_o = .93$. (See Chapter X, § 187.)

C. The end of the pipe shaped like a bell mouthpiece figure 114 C.) This corresponds to a rounded orifice. $C = .95$ to $.99$; and $\xi_o = .11$ to $.02$. (See Chapter X, § 1

362. Loss because of enlargement of section. Abrupt ment. Figure 115 represents a section of a pipe throug a steady discharge ing under pressu sections OB and pipe is entirely fille the following r exist:

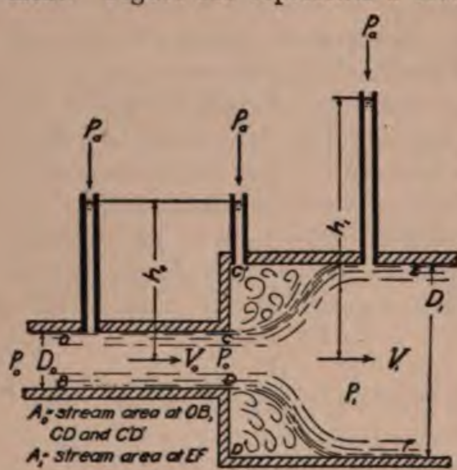


FIG. 115.—Abrupt Enlargement in a Pipe.

$$Q = A_0 V_0 = A_1 V$$

$$\frac{V_0}{V_1} = \frac{A_1}{A_0}; \quad V_0 = \frac{A}{}$$

$$V_1 = \frac{A_0 V_0}{A_1}.$$

The change in from V_0 to V_1 is panied by a loss of which causes the sum total of energy in the stream at E less than at OB.

Let W = weight of volume of flow in pounds per secon

h_e = the head lost because of enlargement.

$h_e = \xi_e \dfrac{V^2}{2g}$.

ξ_e = factor for enlargement loss.

By Bernoulli's theorem;

$$W\left(\frac{p_a}{\gamma} + \frac{V_0^2}{2g} + h_0\right) = W\left(\frac{p_a}{\gamma} + \frac{V_1^2}{2g} + h_1 + h_e\right).$$

Therefore the head lost,

$$h_e = \frac{V_0^2 - V_1^2}{2g} + h_0 - h_1.$$

The force required to change W pounds of water per second from V_0 to V_1 (see § 97) is

$$F = \frac{W}{g}(V_1 - V_0) = \frac{wA_1V_1}{g}(V_1 - V_0). \tag{D}$$

Theoretically the intensity of pressure at CD and at $C'D'$ (neglecting the space filled by eddies) should be equal to the intensity of pressure at OB because the sectional areas of the stream are equal. There should be therefore at the entrance to the large pipe a total pressure in the direction of flow equal to $+h_0wA_1$ (the plus sign indicating direction of motion). At EF there is a total pressure opposed to flow equal to $-h_1wA_1$ (the minus sign indicating direction opposed to motion). The algebraic sum of the total pressures at these two sections should theoretically be the force causing change in velocity; therefore,

$$F = +h_0wA_1 - h_1wA_1 = wA_1(h_0 - h_1). \tag{E}$$

From (D) and (E), $wA_1(h_0 - h_1) = \dfrac{wA_1V_1}{g}(V_1 - V_0)$. (G)

From (G), $h_0 - h_1 = \dfrac{V_1^2 - V_0V_1}{g} = \dfrac{2V_1^2 - 2V_0V_1}{2g}$. (H)

Substitute from (H) into (C); then

$$h_e = \frac{V_0^2 - V_1^2 + 2V_1^2 - 2V_0V_1}{2g} = \frac{(V_0 - V_1)^2}{2g}. \tag{26}$$

This loss is in addition to the friction head.

Since by (A), $V_0 = \dfrac{A_1V_1}{A_0}$,

$$h_e = \left(\frac{A_1}{A_0} - 1\right)^2 \frac{V_1^2}{2g} = \left(\frac{D_1^2}{D_0^2} - 1\right)^2 \frac{V_1^2}{2g} = \xi_e \frac{V_1^2}{2g}. \tag{27}$$

TABLE XL

TABLE OF VALUES OF $\xi_e = \left[\left(\frac{D_1}{D_0}\right)^2 - 1\right]^2$

$\left(\frac{D_1}{D_0}\right)^2$	10	9	8	7	6	5	4	3	2	1
$\xi_e = \left[\left(\frac{D_1}{D_0}\right)^2 - 1\right]^2$	81	64	49	36	25	16	9	4	1	0

But little experimental data confirming this formula are available. Weisbach states that he made experiments which confirm this theory of loss of head, and that Borda first found it. Gibson at a

lat lso made experiments the results of which tend to confirm e theory. See also Brightmore's experiments (p. 299).

radual enlargement. If the section is enlarged evenly in a s ent distance and the edges rounded, the loss due to enlargement be practically eliminated.

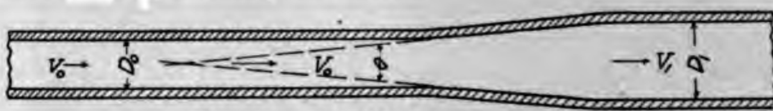


FIG. 116. — Gradual Enlargement in a Pipe.

Fliegner * showed that for gradual enlargement (see figure 116),

$$\xi_e = \left[\left(\frac{D_1}{D_0}\right)^2 - 1\right]^2 \sin \theta.$$

In ordinary practice in designing channels, it has been found that considerable loss can be avoided by making all necessary

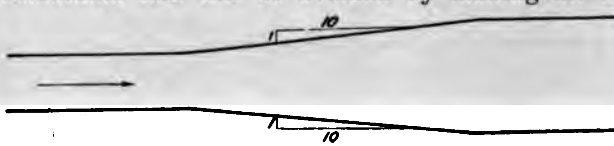


FIG. 117. — Gradual Enlargement in a Pipe.

changes as gradual as possible. A batter of 1 to 10 as shown in the above figure (117), for velocities less than six feet per second, is usually a favorable one.

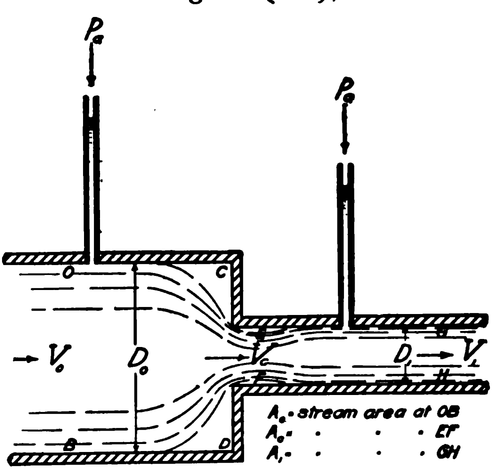


FIG. 118. — Abrupt Contraction in a Pipe.

364. Loss because of contraction of section. Abrupt contraction. Figure 118 represents a section of a pipe through which a steady discharge is flowing under pressure. At sections *OB* and *GH* the pipe is assumed to be entirely filled; at section *EF* the stream cross section

* Unwin's *Hydraulics*, p. 170.

is smaller than the area at the entrance edge of the smaller pipe, or at GH. The following relations exist between velocity and area at these three sections:

$$Q = A_0 V_0 = A_c V_c = A_1 V_1. \tag{J}$$

Therefore, $$V_c = \frac{A_1 V_1}{A_c}. \tag{K}$$

As the edge at entrance to the smaller pipe corresponds to a sharp-edged orifice, the area of the contracted section will, as for an orifice, depend on the ratio, $\frac{A_0}{A_1}$; from which C, the coefficient of contraction, may be determined. If C_c is known,

$$A_c = C_c A_1; \text{ and } \frac{A_1}{A_c} = \frac{1}{C_c}; \text{ hence } V_c = \frac{V_1}{C_c}. \tag{M}$$

The change in velocity from V_0 to V_1 is accompanied by a loss of energy and causes a loss of head.

Let h_κ = head lost because of contraction $= \xi_\kappa \frac{V^2}{2g}$.

ξ_κ = factor for contraction loss.

The head lost because of contraction may be divided into two parts: one loss between OB and EF; and the other between EF and GH. The first is similar to the loss in an orifice, is relatively small, and is frequently neglected; the second is the loss due to sudden enlargement. The sum of these losses,

$$h_\kappa = \frac{V_c^2}{2g}\left(\frac{1}{C_v^2} - 1\right) + \frac{(V_c - V_1)^2}{2g}. \tag{N}$$

Substituting the value of V_c from (M),

$$h_\kappa = \frac{V_1^2}{2g}\left[\frac{1}{C_c^2}\left(\frac{1}{C_v^2} - 1\right) + \left(\frac{1}{C_c} - 1\right)^2\right]; \tag{28}$$

and $$\xi_\kappa = \frac{1}{C_c^2}\left(\frac{1}{C_v^2} - 1\right) + \left(\frac{1}{C_c} - 1\right)^2. \tag{29}$$

Table of values of C_c and ξ_κ. The values of C_c on which ξ_κ is based are taken from a plotted curve of Freeman's coefficients * of discharge for ring nozzles, and deduced on the assumption that C_v=0.975 which is assumed by Freeman's curve. (See § 205, Table XXVI.)

* These seem the best available experimental data, and agree closely with Weisbach's coefficients.

TABLE XLI

$\frac{D_1^2}{D_0^2}$	.1	.2	.3	.4	.5	.6	.7	.8	.9
C_c	.632	.644	.659	.676	.696	.717	.744	.784	.890
$\frac{1}{C_c^2}\left(\frac{1}{C_v^2}-1\right)$	.130	.125	.120	.114	.107	.101	.094	.085	.064
$\left(\frac{1}{C_c}-1\right)^2$	.339	.306	.267	.229	.191	.156	.118	.076	.015
ξ_κ	.469	.431	.387	.343	.298	.257	.212	.161	.079
Weisbach's values of C_c*	.624	.632	.643	.659	.681	.712	.755	.813	.892

365. A diaphragm produces abrupt contraction, which has effect of greater magnitude than or nary contraction, because the sudd enlargement of the stream after pa ing the area A_d is greater than if t contraction is made into a pipe of t same bore as the opening in the diaphragm. See figure 119.

FIG. 119.—Diaphragm in a Pipe.

$$Q = C_c A_d V_c = AV; \text{ hence } V_c = \frac{AV}{A_d C_c}. \qquad ($$

$$h_\kappa = \frac{V_c^2}{2g}\left(\frac{1}{C_v^2}-1\right) + \frac{(V_c - V)^2}{2g} \qquad ($$

$$= \frac{A^2V^2}{2gA_d^2C_c^2}\left(\frac{1}{C_v^2}-1\right) + \frac{V^2}{2g}\left(\frac{A}{A_dC_c}-1\right)^2 \qquad ($$

$$= \frac{V^2}{2g}\left[\frac{A^2}{A_d^2C_c^2}\left(\frac{1}{C_v^2}-1\right) + \left(\frac{A}{A_dC_c}-1\right)^2\right]. \qquad ($$

The following table is computed by formula (30), using same coefficients of contraction (C_c) as in Table XLI :

TABLE XLII

$\frac{A_d}{A}$	.1	.2	.3	.4	.5	.6	.7	.8	.9
ξ_κ	233	49	18	8.0	4.0	2.0	1.0	0.4	0.2

366. A bushing of sufficient length to form a short tube on interior of the pipe, as in figure 120, will cause a loss head, equal to the sum of head lost in contraction into the tube + h lost in enlargement from tube to pipe; therefore

* Coefficients for imperfect contraction, Weisbach's *Mechanics*, p. 888.

$$h_x = \frac{V_c^2}{2g}\left(\frac{1}{C_v^2} - 1\right) + \frac{(V_c - V)^2}{2g} + \frac{(V_1 - V)^2}{2g}. \qquad (31)$$

h_x may then be computed by Tables XLI and XL. On the whole this loss for similar ratios will be less than for a diaphragm; and in making up pipe, hose, and nozzles, some form of bushing is unavoidable.

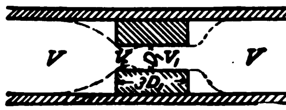


FIG. 120.—Bushing in a Pipe.

Brightmore's experiments on sudden enlargement and contraction showed for 6 to 3, or 3 to 6-inch diameters a close agreement with theoretic values; for 6 to 4, and 4 to 6, the loss was less than the theoretic values.

367. Gradual contraction. If the section is decreased evenly in a sufficient distance, loss due to the contraction is practically eliminated. Abrupt contractions, diaphragms, or bushings should if possible be avoided.

368. Head lost in bends. Between the beginning and end of a bent or curved portion of a pipe, the loss in head will be greater than for an equal length of a straight portion of the same pipe. The reasons are many, but important among them are the increase in maximum velocity and redistribution of velocities. The increased resistance due to the effect of the bend may be computed as a separate additional loss and added to the friction head.

Let h_b = the additional head lost due to the bend $= \xi_b \dfrac{V^2}{2g}$.

ξ_b = factor for additional head lost in bend.

The experimental data available for computing ξ_b are meager and somewhat discordant.

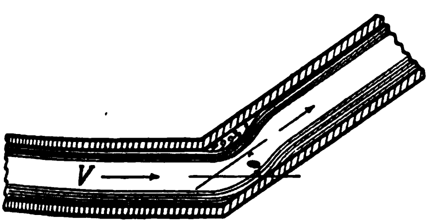


FIG. 121.—Sharp Bend.

369. For sharp bends or elbows. Weisbach* deduced from experiments on a smooth iron pipe 3 centimetres (1.2 inches) in diameter the following formula (see figure 121):

$$\xi_b = 0.95 \sin^2 \frac{\theta}{2} + 2.05 \sin^4 \frac{\theta}{2}. \qquad (32)$$

* *Mechanics*, Coxe's translation, p. 895.

By form (32) Weisbach calculated the following table:

TABLE XLIII

θ	20°	40°	60°	80°	90°	100°	120°	140°
ξ_b	.05	.14	.36	.74	.98	1.26	1.86	2.43

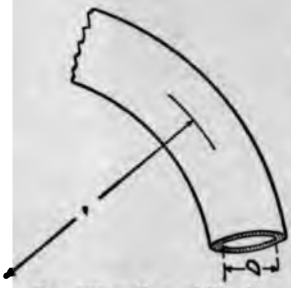


FIG. 122.—Curved Bend.

For smaller diameters he found ξ_b to be greater; for a diameter of 1 centimetre (.4 inch) and $\theta = 90°$, $\xi_b = 1.54$.

370. Curved bends offer less resistance than elb Weisbach* from his own and ibua experiments deduced the follo formula for circular pipes:

$$13 + 1.85\left(\frac{D}{2\rho}\right)^{\frac{7}{2}}. \qquad (33)$$

D = internal diameter the pipe.

ρ = radius of curva f the bend.

By formula (33) Weisbach calculated the following table:

TABLE XLIII A.

$\frac{\rho}{D}$	5	2.5	1.67	1.25	1.	.84	.72	.63	.56	.5
ξ_b	.13	.14	.16	.21	.29	.44	.66	.98	1.41	1.98

Weisbach's experiments were limited in range and made only on very small pipes; they probably can not be safely applied to a wide range of conditions.

371. Williams, Hubbell, and Fenkell experiments† on curves. In 1897–1901 Williams, Hubbell, and Fenkell made an elaborate set of experiments on the flow in new cast-iron pipes, coated with coal tar, of 12, 16, and 30 inches diameter. The results given below are taken from a curve plotted by them, and are based chiefly on the 30-inch pipe; they show the increase in resistance caused by curves in portions of the pipe 80 diameters long, which included the curve, in comparison with an equal length of all straight pipe. The curves all had 90° central angle.

* *Mechanics*, Coxe's translation, p. 897.

† *Trans. Am. Soc. C. E.*, Vol. 47, 1902, pp. 1–335.

TABLE XLIV

RELATIVE EXCESS LOSS OF HEAD DUE TO CURVES IN A LENGTH OF 80 DIAMETERS, INCLUDING THE CURVES

For the 30-inch pipe:

$\frac{\rho}{D}$	2	$2\frac{1}{2}$	3	4	5	10	15	20	25
Per cent increase due to curve:	14	13	14	18	24	50	67	80	93

For the 12-inch pipe the percentage increase due to curves for five experiments was as follows:

$\frac{\rho}{D}$	1.08	2	3	4
Per cent increase due to curve:	24.6	10.8	17.1 30.	19.1

There were only two experiments on the 16-inch pipe, a 5-foot radius showing an excess of 25 per cent, and a compound curve showing an excess of 28 per cent over an equal length of straight pipe.

The minimum increase observed was when $\frac{\rho}{D} = 2\frac{1}{2}$. For greater ratios the resistance due to curvature was observed to increase with the length of curve, that is, with increase of radius for a given central angle, which appeared contrary to all previous experiments; but Davis's experiments (§ 375), and Brightmore's (§ 376), indicate a similar increase though from different minima.

372. Freeman's experiments on fire hose, which are given in Table XXVIII (page 178), appear to confirm Weisbach's theorem that the resistances due to curvature increase with the sharpness of the curve.

373. Saph and Schoder's experiments. Saph and Schoder made experiments on a 2-inch seamless drawn brass pipe which were published in the *Transactions of the American Society of Civil Engineers*, vol. 47, pages 295 to 328: "The total excess losses of head due to curves showed no tendency to follow any simple law." The results are too voluminous to be reproduced.

They made another set of experiments* on 6-inch pipes, from which the following table is taken:

* *Trans. Am. Soc. C. E.*, Vol. 62, 1909, pp. 67–96.

TABLE XLV

EXCESS LOSSES OF HEAD DUE TO CURVES, EXPRESSED IN TERMS OF VELOCITY HEADS; $h_b = \xi_b \frac{V^2}{2g}$

No. of Curve	Material	Diameter, Mean of Four End Caliperings	Radius in Feet ρ	Radius in Pipe Diameters $\frac{\rho}{D}$	Length on Center Line in Feet	Values of ξ_b Velocity, in feet per second			
						3	5	10	16
1	Wrought Iron	6.09 in.	10.00	20	16.77	0.34	0.27	0.19	0.14
2	Wrought Iron	6.18 in.	7.50	15	12.84	0.15	0.09	0.05	0.02
3	Wrought Iron	6.16 in.	5.00	10	9.01	0.21	0.16	0.11	0.08
4	Wrought Iron	6.11 in.	4.00	8	7.34	0.28	0.22	0.17	0.13
5	Wrought Iron	6.11 in.	3.00	6	5.89	0.28	0.21	0.17	0.14
6	Wrought Iron	6.09 in.	2.50	5	5.08	0.14	0.12	0.11	0.11
7	Cast Iron	5.91 in.	2.00	4	3.64	0.24	0.18	0.16	0.12
8	Cast Iron	5.95 in.	1.50	3	2.86	0.21	0.18	0.16	0.12
9	Cast Iron	5.91 in.	1.08	2.16	2.54	0.22	0.19	0.17	0.13
10	Cast Iron	5.91 in.	0.95	1.90	1.75	0.25	0.21	0.19	0.16
11	Cast Iron	5.95 in.	0.88	1.76	3.62	0.24	0.24	0.23	0.22
12	Cast Iron	5.93 in.	0.67	1.34	1.05	0.39	0.33	0.29	0.26

The straight pipe experiments made at the same time will be found in Table XLIX.

374. Alexander's experiments* on varnished wooden pipes $1\frac{1}{4}$ inches internal diameter are expressed as follows:

Where $\frac{D}{2\rho}$ is not greater than 0.2,

$$h_b = 1.66\left(\frac{D}{2\rho}\right)^{0.83} L(.00498)\, V^{1.777}. \qquad (34)$$

Where $\frac{D}{2\rho}$ is from 0.2 to 0.5,

$$h_b = 25.0\left(\frac{D}{2\rho}\right)^{2.5} L(.00498)\, V^{1.777}. \qquad (35)$$

h_b is the loss of head due to bends in excess of the friction head for an equal length (L) of straight pipe. For the straight pipe Alexander determined the equation $S = .0407\, V^{1.77}$.

* C. W. L. Alexander, *Proc. Inst. of Civil Eng.*, Vol. 159, p. 363.

Alexander also derived the following equation from a computation of Saph and Schoder's 2-inch brass pipe experiments:

$$h_b = 3.27\left(\frac{D}{2\,\rho}\right)^{.83} L(.00262)\,V^{1.76}. \qquad (36)$$

375. Davis's experiments on bends and elbows. G. J. Davis reported* the results of experiments made at the University of Wisconsin, part of which are here given. They are of especial interest because they include experiments on commercial elbows and tees. See figure 123.

All the curves experimented on made a turn of 90°, and had radii of curvature as given in the following table, XLVI.

Nos. 1 to 6, inclusive, were standard fittings purchased from the Crane Company. No. 7 was a special casting. Nos. 8 and 9 were bent from 2-inch pipe.

Drawings of Nos. 1 to 6, inclusive, are shown in figure 123. No. 7 was similar to Nos. 2 and 3, the only difference being in the longer radius of curvature. Nos. 8 and 9 were fitted with ordinary flanges, no attempt being made to obtain a flush joint.

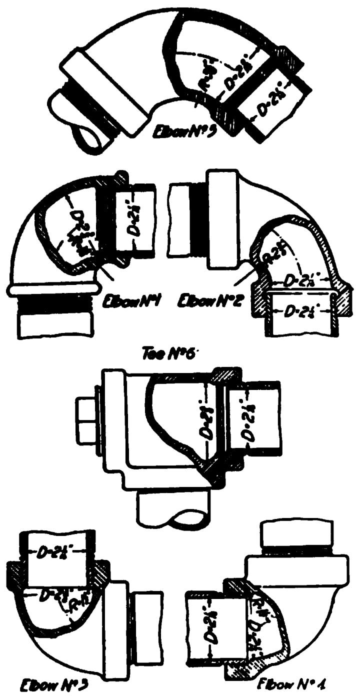


Fig. 123.—Elbows and Tee, referred to in Table XLVI.

Table XLVI is taken from Davis's paper.

The elbows were preceded and followed by straight pipe. The head was read at several piezometers; one (B) was located 1 foot upstream, another (E), 17 feet downsteam from the elbow.

The excess loss of head due to curvature, is taken as the difference between the total loss of head between B and E as shown by piezometers, and the head that would be obtained by multiplying

* *Trans. Am. Soc. C. E.*, Vol. 62, pp. 97–112.

TABLE XLVI

No.	Description	Internal Diameter of Elbow in Inches
1	Malleable iron elbow	$2\frac{5}{8}$
2	Cast iron, long-turn, drainage elbow . . .	$2\frac{1}{16}$
3	Cast iron drainage elbow	$2\frac{5}{8}$
4	Cast iron, short turn	$2\frac{3}{8}$
5	Cast iron, long sweep	$2\frac{9}{32}$
6	Cast iron tee	
7	Cast iron	$2\frac{1}{16}$
8	Wrought iron or steel	$2\frac{1}{16}$
9	Wrought iron or steel	$2\frac{1}{16}$

the actual center-line length on the curve by the frictic
per foot of straight pipe.

The straight pipe was a nominal 2-inch wrought iron;
mental results on the straight pipe are given in Table XLI

Velocity in Feet per Second	Excess Loss of Head in Feet, due to Curves					
	$\frac{\rho}{D} = 0.00$	$\frac{\rho}{D} = 0.728$	$\frac{\rho}{D} = 1.15$	$\frac{\rho}{D} = 2.50$	$\frac{\rho}{D} = 5.00$	$\frac{\rho}{D}$
2	0.101	0.040	0.022	0.0086	0.0043	
3	0.230	0.092	0.055	0.023	0.014	
4	0.405	0.165	0.104	0.048	0.032	
5	0.640	0.260	0.175	0.086	0.062	
6	0.860	0.370	0.260	0.140	0.105	
8	1.650	0.660	0.500	0.280	0.240	
10		1.020	0.840	0.500	0.470	
15		2.35	2.10	1.40	1.45	

Taken from a curve given by Davis.

376. Brightmore's experiments* on elbows and bends ir
4-inch pipes. The elbows and bends were all rusted by sal
niac, and also the 4-inch straight pipe which was coupled v
4-inch bends; but the 3-inch pipe was new and galvanized

* A. W. Brightmore, *Proc. Inst. of Civil Engrs.*, Vol. 169, pp. 315 e

loss due to the bend was found not to take place entirely in the bend, but also in the straight pipe following. A length of straight pipe 6⅔ feet long for the 3-inch, and 5 feet, or 6 feet 7¼ inches for the 4-inch, was, therefore, included with the bend in measuring the loss of head. From the loss of head in these combined bent and straight pieces, the friction head for an equal length of straight pipe was subtracted, giving the loss of head due to the bend. The following values were scaled from curves given by Brightmore:

TABLE XLVII

EXCESS LOSS OF HEAD IN FEET, DUE TO CURVES

3-INCH VELOCITIES	ELBOW	$\frac{\rho}{D}=2$	$\frac{\rho}{D}=4$	$\frac{\rho}{D}=6$	$\frac{\rho}{D}=8$	$\frac{\rho}{D}=10$	$\frac{\rho}{D}=14$
3	.140	.0667	.05	.05	.05	.025	.0167
5	.450	.167	.125	.142	.125	.075	.0500
7	.950	.317	.242	.275	.242	.142	.0917
10	1.825	.642	.475	.63	.55	.283	.217
4-INCH							
3	.167	.0667	.0417	.050	.0333	.0250	
5	.433	.158	.108	.133	.108	.0833	
7	.925	.275	.217	.250	.208	.167	
10	1.833	.600	.458	.508	.458	.358	

377. Losses in gates and valves. The following information as to losses of head due to gates and valves, though far from complete, is the best obtainable. See figure 124.

The lost head in gates and valves may be expressed as

$$h_v = \xi_v \frac{V^2}{2g},$$

where ξ_v = factor for the loss through gates and valves.

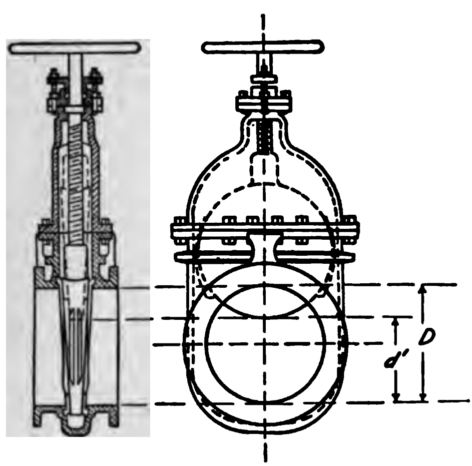


FIG. 124. — Gate Valve.

x

Ga s are constructed in such a manner that when enti the water way is of the same section as the pipe. Usua are irregularities and roughness due to the se tings, any case the recess for the gate will break up the uniformity flow.

Wi ubbell, and Fenkell found the resistance of a wide-open gate equal to about .0698 × .94 = .0656 feet.

TABLE XLVIII

V BACH'S * EXPERIMENTS ON SMALL GATE VALVES

Let D = the dian r e;
d' = ich the gate is raised;
a' = en gate is raised.

$\frac{d'}{D}$	$\frac{7}{8}$	[illegible]	$\frac{5}{8}$	[illegible]	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
$\frac{a'}{A}$	.95	.86	[illegible]	[illegible]	.47	.32	.16
ξ_v	.07	.26	[illegible]	[illegible]	5.5	17	98

KUICHLING'S † EXPERIMENTS ON A 24-INCH GATE VALVE

$\frac{d'}{D}$	$\frac{2}{3}$	$\frac{7}{12}$	$\frac{1}{2}$	$\frac{5}{12}$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{12}$
ξ_v	0.77	1.55	3.27	6.33	8.63	11.9	22.7	41.2

Cock valves. Weisbach's experiments on cock valves in small pipes.

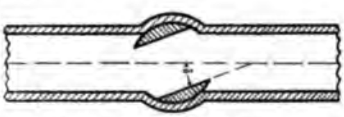


FIG. 125 *a*. — Cock Valve.

FIG. 125 *b*. — Throttle Valve.

Let δ = angle through which cock is turned. See figure 125 *a*.

δ	5°	10°	20°	30°	40°	45°	50°	55°	60°	65°	82½°
$\frac{a'}{A}$	.93	.85	.69	.54	.39	.32	.25	.19	.14	.09	closed.
ξ_v	.05	.29	1.6	5.5	17	31	53	106	206	486	

Throttle valves. Weisbach's experiments on throttle valves in small pipes.

* *Mechanics*, Coxe's translation, p. 901. † *Trans. Am. Soc. C. E.*, Vol. 26, p. 449.

Let δ = angle through which valve is turned. See figure 125 *b*.

δ	5°	10°	20°	30°	40°	45°	50°	55°	60°	65°	70°
$\frac{a'}{A}$	.91	.83	.66	.50	.36	.29	.23	.18	.13	.09	.06
ξ_v	.24	.52	1.5	3.9	11	19	33	59	118	256	751

Pipe Computations

The following are given to illustrate the application of the formulas of the flow in pipes to certain common problems.

378. A straight pipe leading from a reservoir and discharging freely into air, or discharging into another reservoir. See figures 126 *a* and *b*.

The losses are similar in both cases, namely, the head lost in entrance (h_o) and the friction head (h_f).

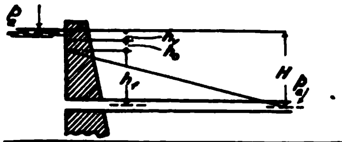


Fig. 126 *a*.

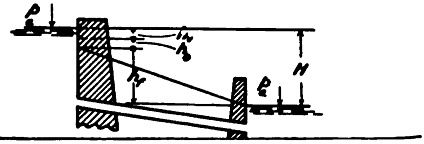


Fig. 126 *b*.

By Bernoulli's theorem, stating the factors in terms of $\frac{V^2}{2g}$. The total head,

$$H + \frac{p_a}{\gamma} = \frac{V^2}{2g} + \xi_o \frac{V^2}{2g} + f \frac{L V^2}{D\, 2g} + \frac{p_a}{\gamma}. \tag{37}$$

$$H = \frac{V^2}{2g}\left(1 + \xi_o + f\frac{L}{D}\right). \tag{38}$$

$$V = 8.02\left[\frac{H}{1 + \xi_o + f\frac{L}{D}}\right]^{\frac{1}{2}}. \qquad (23)\ (39)$$

$$Q = 6.3\, D^2\left[\frac{H}{1 + \xi_o + f\frac{L}{D}}\right]^{\frac{1}{2}}. \quad \text{See § 347.} \tag{40}$$

$$D = .479\left(\frac{[(1 + \xi_o)D + fL]\,Q^2}{H}\right)^{\frac{1}{5}}. \tag{41}$$

Example. *Given:* a straight new clean cast-iron pipe 1 foot in diameter, 1000 feet long, with square-edged entrance. Use Darcy's **friction factors,** Table L.

$$V_0 = \frac{50.27}{12.566} = 4; \; V_1 = \frac{50.27}{3.1416} = 16; \; V_2 = \frac{50.27}{7.0686} = 7.11$$

$\xi_o = .5$ (§ 361); ξ_κ when $\frac{A_1}{A_0}$ is $.25 = .41$ (see Table XI

ξ_e when $\frac{D_2^2}{D_1^2}$ is $\frac{9}{4} = 1.56$ (see § 362).

From the table of heads due to velocities

$$\frac{V_0^2}{2g} = 0.25; \; \frac{V_1^2}{2g} = 3.98; \; \frac{V_2^2}{2g} = 0.78.$$

Then

$$H = .13 + .25 + 2.53 + 1.63 + 3.73 + 82.39 + 1.22 - 3.20 + 10.$$
$$= 99.34 \text{ feet.}$$

If gradual reducers and enlargers are used as they should b and if the head lost at entrance and the velocity head may neglected, the total head may be said to be used in friction hea in which case it may be said that

$$H = f_0 \frac{L_0 V_0^2}{D_0 2g} + f_1 \frac{L_1 V_1^2}{D_1 2g} + f_2 \frac{L_2 V_2^2}{D_2 2g} = 95.58;$$

the total head would be 95.58 instead of 99.34.

Substituting in the preceding equation,

$$V_0 = \frac{4\,Q}{\pi D_0^2}; \; V_1 = \frac{4\,Q}{\pi D_1^2}; \text{ and } V_2 = \frac{4\,Q}{\pi D_2^2};$$

the following formulas for the total head and the discharge a derived:

The total head, $$H = \frac{8\,Q^2}{g\pi^2}\left(f_0 \frac{L_0}{D_0^5} + f_1 \frac{L_1}{D_1^5} + f_2 \frac{L_2}{D_2^5}\right). \quad (4$$

The discharge, $$Q = 6.3 \left\{ \frac{H}{f_0 \frac{L_0}{D_0^5} + f_1 \frac{L_1}{D_1^5} + f_2 \frac{L_2}{D_2^5}} \right\}^{\frac{1}{2}}. \quad (5$$

These last two formulas may be extended to any number of c ferent diameters, by adding the proper value $f \frac{L}{D^5}$ for each n diameter.

380. A pipe line with two (or more) **branches** connecting a reservoir with other reservoirs each of which is at a different e

vation (see figure 128) becomes a complicated problem which may be solved only by successive trials.

Let *0* be the supply reservoir, and *1* and *2* be the reservoirs into which the water is to flow; and at any instant let h_0, h_1, and h_2 be the elevations of the water surfaces in the three reservoirs,

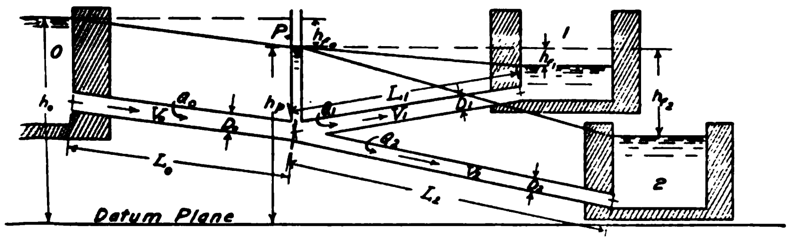


FIG. 128. — Branching Pipe Lines.

and h_p be the elevation of the water in a piezometer (P) at the junction of the two branches. If all pipes are flowing full, and the two branches are steadily discharging into *1* and *2*, h_p can have at any instant only one value that will satisfy the required conditions, which are:

that $Q_0 = Q_1 + Q_2$;

that h_{f_0} shall equal $f_0 \dfrac{L_0}{D_0}\dfrac{V_0^2}{2g}$, and also equal $h_0 - h_p$;

that h_{f_1} shall equal $f_1 \dfrac{L_1}{D_1}\dfrac{V_1^2}{2g}$, and also equal $h_p - h_1$;

that h_{f_2} shall equal $f_2 \dfrac{L_2}{D_2}\dfrac{V_2^2}{2g}$, and also equal $h_p - h_2$.

Such problems may involve either finding the discharges Q_0, Q_1, and Q_2, or the diameters D_0, D_1, and D_2; in either case the procedure is to find by trial the elevation h_p which shall satisfy the above conditions. This process, though tedious, may be simplified by the aid of discharge tables.

Example. $h_0 = 200$ feet; $h_1 = 155$ feet; $h_2 = 143$ feet. $D_0 = 4$ feet; $D_1 = 2.5$ feet; $D_2 = 3$ feet. $L_0 = 10{,}000$ feet; $L_1 = 5000$ feet; $L_2 = 8000$ feet. The discharge in each pipe is to be determined. By trial it will be found that if $h_p = 175$ feet, $Q_1 = 27.5$, $Q_2 = 43.4$, and $Q_0 = 70.9$ cubic feet per second; and all the other factors will agree.

If for any case the given conditions are essential, an equa reservoir must be constructed at P, having sufficient capa allow fluctations without changing the elevation of the surface at the junction. A slight change in conditions reverse the flow from *1* to *2*.

381. Compound pipes. Systems of pipes which branch point and reunite at another, forming several routes by whi water may be delivered not only to scattered consumers, b by which in case of a break in one line users may be suppl the other lines, are now considered essential in water supp tems, especially if they are to afford suitable fire protection.

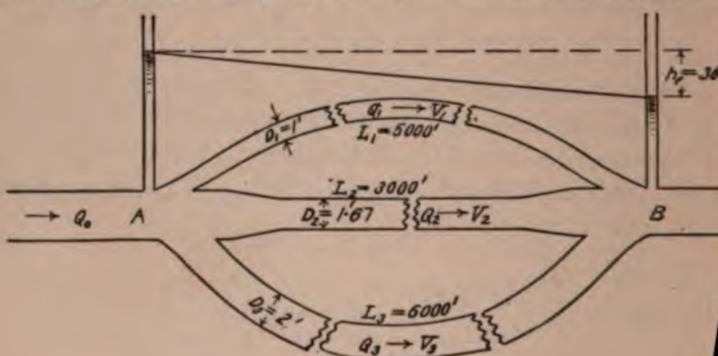


FIG. 129. — Compound Pipe Line.

Let it be assumed that piezometers are set at A and B 129; then during flow the total head H equals the diff water levels between the two piezometers.

Assuming that all the head (H) is used in friction, the through each pipe will be computed as through a simple pi

$$Q_1 = 6.3\left(\frac{30 \times 1^5}{.0216 \times 5000}\right)^{\frac{1}{2}} = 3.32 \text{ cubic feet pe}$$

$$Q_2 = 6.3\left(\frac{30 \times 1.67^5}{.021 \times 3000}\right)^{\frac{1}{2}} = 15.59 \quad " \quad "$$

$$Q_3 = 6.3\left(\frac{30 \times 2^5}{.0207 \times 6000}\right)^{\frac{1}{2}} = 17.92 \quad " \quad "$$

The total discharge through the main, $Q_0 = 3.32 + 15$ 36.83. If, on the other hand, the total discharge Q

head at A, the discharge of each individual pipe, and the head at B may be computed by trial by assuming various values of the friction head until the sum computed for the assumed head, $Q_1 + Q_2 + Q_3$, equals Q_0.

A method suggested by Freeman provides a convenient solution for this class of problems. Assuming all the head is to be used in friction, and that the friction factor may be taken as constant for all velocities, the procedure is as follows:

Take values of the heads required to discharge various volumes through each line, and for each line draw a discharge curve with heads in feet or pounds per square inch as ordinates against discharge in any convenient units as abscissas all on the same sheet. It is not essential that each line of pipe shall be of uniform diameter; it may be composed of any number of different diameters. At a series of convenient ordinates take off an abscissa from each curve, add the abscissas of each curve at any point, the sum will be the abscissa for the given ordinate for a discharge curve for the combined system. In this manner the combined discharge curve may be constructed. Take, for example,

G.	Line 1	Line 2	Line 3
	h_f	h_f	h_f
500	3.35	.15	.12
1000	13.50	.60	.48
2000	53.95	2.43	1.92
3000	121.40	5.49	4.38
4000		9.75	7.80
5000		15.24	12.18
6000		21.93	17.52
7000		29.88	23.83
8000			31.08

The plotted curves are shown in figure 130. G = gallons per minute.

From the combined curve it appears that if $h_f = 30$ feet, the total discharge is 16,350 G.; if $h_f = 10$ feet, the discharge is 9400 G. If, on the other hand, the discharge be 12,000 G. the head necessary between A and B is 16.3 feet.

By the aid of discharge tables such as Weston's or Williams and

Hazen's such curves may be rapidly constructed, and will gre
reduce the labor of planning intricate pipe layouts.

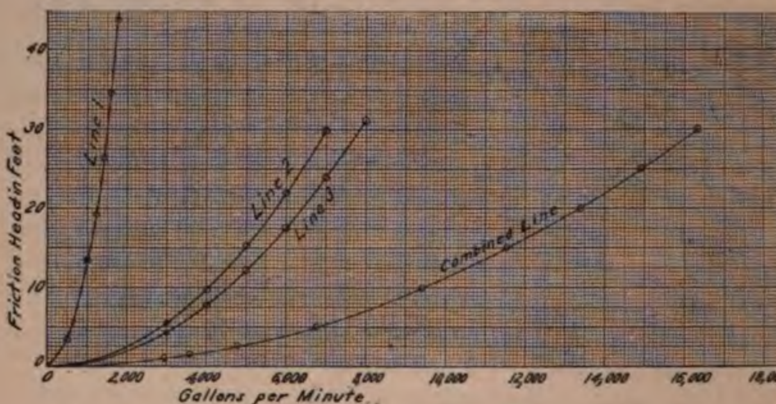


FIG. 130.

Any waterworks problem which requires that the total lo
of head be known will probably be approached from the standp
of new pipes; but the design must provide for additional lo
on account of the reduction in carrying capacity due to age.
addition to the head lost under the most severe requireme
there must always remain sufficient head to maintain the pres
required for sprinklers and hydrants.

TABLE XLIX

PIPE EXPERIMENTS

TABLE OF EXPERIMENTAL VALUES OF C IN THE FORMULA $V = C(R$
AND OF EXPERIMENTAL VALUES OF FRICTION HEAD (H) FOR 1000 F

For small brass and galvanized pipes Saph and Schoder's
periments are given. Many other good experiments on s
pipes have been made, but the list is too large to be reprodu
and Saph and Schoder's are the most consistent and accurate.

The table contains a large portion of the available reliabl
periments on cast-iron, riveted steel and sheet-iron, and wo
stave pipes, of greater diameter than three inches. They ar
varying degrees of accuracy, especially the rusty and tubercul
old pipes.

The data upon which the results given are based have

taken from original sources, a few exceptions noted. The table has been computed by a method which was suggested* some years ago by Professor Osborne Reynold, namely, by *logarithmic homologues.*

The relation between slope and velocity may for a constant hydraulic radius be expressed as follows:

$$S = m'V^n. \tag{44}$$

m' = an experimental coefficient.

n = an experimental exponent.

m' and n will be constant for a given pipe if the condition and nature of the interior surface do not change.

If the friction head for a given length be expressed by H_f, then

$$H_f = LS = Lm'V^n.$$

The length used as a basis of comparison in this table is 1000 feet; calling $Lm' = m$.

Then H_f, the friction head for 1000 feet $= mV^n$. (45)

From curves drawn through points plotted on logarithmic paper, with experimental values of V as abscissas, and of H_f as ordinates, and representing the variation of H_f with V, m and n may be determined graphically; m being the value of H_f when V is 1 foot per second, n being the tangent of the angle which the curve (which for ordinary velocities should be a straight line) makes with the horizontal.

m and n may be computed from the logs of the experimental values of H_f and V, as follows: Divide the sum of the logs of all the values of H_f by the number of observations; likewise the sum of all the logs of V; the results are the logs of the two coördinates of the center of gravity of plotted points. The center of gravity of the two groups above and below this point may be determined in a similar manner; thus giving three points which should lie in a straight line which fixes the curve desired.

Let the coördinates of these three points be, of the lower H_f' and V'; of the middle H_f'' and V''; of the upper H_f''' and V'''.

$$\text{Then, } n = \frac{\log H_f'' - \log H_f'}{\log V'' - \log V'} \text{ or } = \frac{\log H_f''' - \log H_f''}{\log V''' - \log V''}$$

$$\text{or } = \frac{\log H_f''' - \log H_f'}{\log V''' - \log V'}. \tag{46}$$

If n is determined, m may be computed;

$$\text{Log } m = \frac{\log H_f'}{n(\log V')} \text{ or } = \frac{\log H_f''}{n(\log V'')} \text{ or } = \frac{\log H_f'''}{n(\log V''')}. \qquad ([redacted]7)$$

The values of C to be used in the Chezy formula

$$V = C(RS)^{\frac{1}{2}}$$

may be found from m and n as follows:

Here $\qquad R = \frac{D}{4},\ S = \frac{H_f}{1000},$ and $H_f = mV^n.$

Then $\qquad C = \left(\frac{4000\ V^2}{DmV^n}\right)^{\frac{1}{2}}.$

$$\text{Log } C = \tfrac{1}{2}[\log 4000 + (2 - \quad g\ V - \log D - \log m]. \qquad ([redacted]8)$$

Values of C given in this table thus determined.

For each pipe, experimental va ues of mean velocity were plotted on logarithmic square against experimental values of friction head in feet for et of pipe. Through these points lines were drawn which, as nearly as could be estimated, represented the relation between velocity and friction head. By this trial curve it was possible to estimate very closely whether a set of experiments was consistent, and which observations, if any, were sufficiently discordant to be rejected. Very few results except discordant data for velocities less than one foot per second were omitted.

Values of m and n were then computed from the logs of the experimental values of H_f and V, if there were a sufficient number to fix a curve; in a very few discordant sets scaled values are given. Certain experimenters themselves have determined m and n or equivalents; in such cases their values have been used, and are marked with an asterisk (*).

Values C and H_f here given have all been newly recomputed; by the exponential formula, wherever values of m and n are given.

C and H_f were independently computed from the given data; not one from the other. If no calibrated diameter was determined by the experimenter, the nominal diameter was used in calculation.

The results are put down to the third significant figure, or to the nearest unit.

Limits of velocity in the original experiments are indicated in the table. If experiments were made at velocities of one foot or less, the table begins with one foot; otherwise it begins at the lowest velocity; and it ends with the highest velocity of the original experiments. A few exceptions are noted.

The heading of each set of experiments indicates briefly: the nominal or actual diameter in inches, and if determined, the actual diameter in feet; values of m and n in the formula $H_f = mV^n$; the age in service; the kind of coating; condition of the surface at the time of the experiments; length between piezometers, which was straight unless noted; the kind of joints ("B. and S." meaning modern bell and spigot); the method of determining the discharge; and the source from which the information here given was obtained.

Velocities are, or are assumed to be, higher than the critical velocity.

Abbreviations

A. S. C. E. means "Transactions of the American Society of Civil Engineers."

A. E. S. means "Journal of the Association of Engineering Societies."

N. E. W. W. means "Journal of the New England Water-works Association."

R. E. means Darcy's "Recherches Expérimentales Relatives au Mouvement de l'Eau dans les Tuyaux," 1857.

Asterisk (*) means values given in original experiments.

NOTE. — It is worth recording, as corroborating the experience of others who have made a similar study of pipe experiments, that the prevailing weak point in the majority of the experiments here given was not in the accuracy of measuring the volume of flow, but in the measurements of the heads, due chiefly to faulty piezometers or piezometric couplings.

SMALL BRASS PIPES

New, seamless drawn, straight, 6 to 57 feet, preceded and followed straight piece; special screw piezometric couplings. Measurement by we Temperature, 40° F.

Saph & Schoder; A. S. C. E. 1904, Vol. 51, p. 253 *et seq.*

The range of velocities here given indicates the highest velocity for e experiment, and approximately the lowest velocity to which the given va of m and n, which are *Saph & Schoder's* values, apply.

Lower velocities were included in the experiments, but are for the most p in the region of, or lower than, the critical velocity.

D = Actual Diameter in Feet		Velocity in Feet per Second									
		1	2	3	4	5	6	8	10	Highe Velocit	
D=.00893 m=102.8 n=1.795	C			74	76	78	79	82		83	
	H_f			739	1238	1850	2563	4296		5739	
D=.01261 m=70.7 n=1.771	C			76	78	80	82	85	87	89	1
	H_f			495	824	1223	1688	2811	4173	5679	
D=.01498 m=62.8 n=1.742	C			75	78	80	82	85	88	92	1
	H_f			426	703	1037	1424	2351	3467	6230	
D=.01846 m=47 n=1.743	C			78	81	83	85	89	91	95	1
	H_f			319	527	777	1068	1759	2601	4445	
D=.02176 m=89.0 n=1.731	C		75	80	83	85	87	91	94	96	1
	H_f		129	261	430	632	867	1427	2099	3053	
D=.02347 m=34.5 n=1.743	C	70	77	81	84	87	89	92	95	97	1
	H_f	34.5	116	234	387	570	784	1294	1909	2902	
D=.02679 m=29.4 n=1.741	C	71	78	82	85	88	90	93	96	98	1
	H_f	29.4	98.3	199	329	484	666	1073	1619	2027	
D=.03187 m=24.2 n=1.74	C	73	80	84	87	90	92	95	98	101	1
	H_f	24.2	80.8	164	270	398	547	902	1330	1906	
D=.04149 m=17.4 n=1.757	C	74	81	85	88	91	93	96	99	103	1
	H_f	17.4	61.3	120	199	294	404	672	994	2030	
D=.05251 m=13.4 n=1.716	C	75	83	88	92	95	97	101	105	107	1
	H_f	13.4	44.0	88.7	145	212	290	475	697	874	
D=.06881 m=8.8 n=1.738	C	82	89	94	98	101	103	107	110	112	1
	H_f	8.8	29.4	59.4	97.9	144	198	327	481	614	
D=.08785 m=6.6 n=1.755	C	83	90	95	99	101	103	107		108	
	H_f	6.6	22.3	45.4	75.2	111	153	254		288	
D=.10311 m=5.45 n=1.744	C	84	92	97	101	104	106			109	
	H_f	5.45	18.3	37.0	61.2	90.3	124			170	
D=.12484 m=4.20 n=1.755	C	87	95	100	104	106	109	113		114	
	H_f	4.20	14.2	28.9	47.7	70.7	97.5	162		199	
D=.1742 m=2.82 n=1.75	C	90	98	104	107	110				112	
	H_f	2.82	9.49	19.3	31.9	47.1				61.1	

LAP-WELDED STEEL AND WROUGHT-IRON PIPES

¼ Inch
0.02917 foot

$m = 35.2$ *
$n = 1.961$ *

New; galvanized; clean; 10.064 feet; screw; by weight; *Saph & Schoder*, 1903; *A. S. C. E.* 51, p. 253 *et seq.*

V	1.3	2	3	4	5	6	8	10	11.4
C	63	63	64	64	64	65	65	65	65
H_f	58.9	137	304	534	827	1182	2077	3218	4160

⅜ Inch
0.0405 foot

$m = 18.1$ *
$n = 1.91$ *

New; galvanized; clean; 17.032 feet; screw; by weight; *Saph & Schoder*, 1903; *A. S. C. E.* 51, p. 253 *et seq.*

V	1.7	2	3	4	5	6	8	10	10.8
C	76	76	78	79	79	80	81	82	82
H_f	49.9	68.0	148	256	392	555	961	1471	1705

½ Inch
0.05217 foot

$m = 13.2$ *
$n = 1.863$ *

New; galvanized; clean; 18.119 feet; screw; by weight; *Saph & Schoder*, 1903; *A. S. C. E.* 51, p. 253 *et seq.*

V	2.5	3	4	5	6	8	10	12.8
C	81	82	84	85	86	88	89	91
H_f	72.8	102	175	265	372	635	963	1525

¾ Inch
0.07083 foot

$m = 9.05$ *
$n = 1.802$ *

New; galvanized; clean; 17.496 feet; screw; by weight; *Saph & Schoder*, 1903; *A. S. C. E.* 51, p. 253 *et seq.*

V	2.6	3	4	5	6	8	10	11.4
C	87	88	91	93	94	97	99	101
H_f	50.6	65.5	110	165	229	384	574	726

1 Inch
0.08683 foot

$m = 8.2$ *
$n = 1.931$ *

New; galvanized; clean; 18.751 feet; screw; by weight; *Saph & Schoder*, 1903; *A. S. C. E.* 51, p. 253 *et seq.*

V	2.8	3	4	5	6	8	10	10.1
C	78	78	79	79	80	81	81	81
H_f	59.9	68.4	119	184	261	455	700	713

2 Inches
0.1719 foot

$m = 2.4$ *
$n = 1.85$ *

New; uncoated; clean; 62 feet; screw; by volume; *G. J. Davis, Jr.*, 1908; *A. S. C. E.* 62, p. 97 *et seq.*

V	1.5	2	3	4	5	6	8	10	14.7
C	102	104	107	109	111	113	115	117	121
H_f	5.08	8.65	18.3	31.2	47.1	66.0	113	170	347

2 Inches
0.17225 foot

$m = 3.15$ *
$n = 1.98$ *

Old; wrought iron; uncoated; rusted; 12.26 feet; screw; by volume; *Schoder & Gehring*, 1904–1908; *Eng. Record*, 58, p. 241.

V	1	2	3	4	5	6	8	10	Limits
C	86	86	87	87	87	87	88	88	of *V* not
H_f	3.15	12.4	27.7	49.0	76.3	109	193	301	stated

LAP-WELDED STEEL AND WROUGHT-IRON PIPES — Co

3 Inches
0.2600 foot

$m = 1.62$ *
$n = 1.91$ *

Old; wrought iron; uncoated; rusted; 19.97 feet volume; *Schoder & Gehring*, 1904–1908; *Eng. Recor*

V	1	2	3	4	5	6	8	
C	97	101	102	104	105	106	107	1
H_f	1.62	6.09	13.2	22.9	35.0	49.6	86.0	1

4 Inches
0.3398 foot

$m = 1.18$ *
$n = 1.95$ *

Old; wrought iron; uncoated; rusted; 22.15 fee volume; *Schoder & Gehring*, 1904–1908; *Eng. Recor*

V	1	2	3	4	5	6	8	
C	100	102	103	103	104	104	105	1
H_f	1.18	4.56	10.1	17.6	27.2	38.8	68.1	1

5 Inches
0.4222 foot

$m = 0.99$ *
$n = 1.91$ *

Old; wrought iron; uncoated; rusted; 48.04 feet volume; *Schoder & Gehring*, 1904–1908; *Eng. Recor*

V	1	2	3	4	5	6	8	
C	98	101	103	104	105	106	107	1
H_f	0.99	3.72	8.07	14.0	21.4	30.3	52.5	8

6 Inches
0.50625 foot

$m = 0.6$ *
$n = 1.94$ *

Old; wrought iron; uncoated, but glazed by u pipe; 46.1 feet; flange; by volume; *Schoder & Ge* 1908; *Eng. Record*, 58, p. 241.

V	1	2	3	4	5	6	8	
C	115	117	119	120	120	121	122	1
H_f	0.6	2.30	5.06	8.83	13.6	19.4	33.9	5

CAST-IRON PIPES

$3\frac{1}{8}$ Inches
0.2608 foot

$m = 3.56$
$n = 1.92$

Old; uncoated; with deposits; 328.1 feet; socl ume; *Darcy*, 1850; *R. E.*, Series 14, pp. 12 and 56.

V	1	2	3	3.7
C	66	68	69	69
H_f	3.56	13.5	29.3	43.9

$3\frac{5}{32}$ Inches
0.2628 foot

$m = 1.87$
$n = 1.98$

Old; uncoated; cleaned; 328.1 feet; socket; by preceding pipe after cleaning; *Darcy*, 1850; *R. E* pp. 12 and 56.

V	1	2	3	4	5
C	90	91	91	91	92
H_f	1.87	7.38	16.5	29.1	45.3

$3\frac{7}{32}$ Inches
0.2687 foot

$m = 1.83$
$n = 1.90$

New; uncoated: clean; 328.1 feet; socket; *Darcy*, 1850; *R. E.*, Series 16, pp. 12 and 56.

V	1	2	3	4	5	6	8
C	90	93	95	97	98	99	100
H_f	1.83	6.83	14.8	25.5	39.0	55.1	95.1

CAST-IRON PIPES — *Continued*

$5\frac{25}{64}$ Inches
0.4495 foot

$m = 0.849$
$n = 1.91$

New; uncoated; clean; 328.1 feet; socket; by volume; *Darcy*, 1850; *R. E.*, Series 17, pp. 12 and 58.

V	1	2	3	4	5	6	8	10	15.4
C	102	106	108	109	110	111	112	114	116
H_f	0.849	3.19	6.92	12.0	18.4	26.0	45.1	69.0	157

6 Inches

$m = 1.07$
$n = 1.83$

4 years; coal tar; unknown; 1170 feet; *B. & S.*; by nozzle; *E. B. Weston*, 1876–1877; *A. S. C. E.* 22, p. 18.

V	4.7	5	6	8	9.3
C	99	99	101	103	105
H_f	18.2	20.3	28.4	48.1	63.4

6 Inches

38 years; uncoated; with tubercles ½ inch to 1½ inches thick; service pipe in street; *B. & S.*; by Deacon meter; *Brackett*, 1887; *N. E. W. W.* 13, p. 343.

V	0.38	0.57	0.95	1.13	1.32	1.51	1.70
C	30	32	32	27	27	27	26
H_f	1.30	2.50	6.90	14.4	19.2	25.4	33.8

6 Inches

Same pipe as preceding, after it was cleaned by scraping, *Brackett*; *N. E. W. W.* 13, p. 344.

V	0.76	0.95	1.13	1.51	1.70	2.08	2.46
C	42	69	83	69	74	73	72
H_f	2.70	1.50	1.50	3.80	4.20	6.50	9.40

$7\frac{13}{32}$ Inches
0.6168 foot

$m = 0.623$
$n = 1.95$

New; uncoated; clean; 328.1 feet; socket; by volume; *Darcy*, 1850; *R. E.*, Series 18, pp. 12 and 58.

V	1	2	3	4	5	6	8	10	16
C	102	104	105	106	106	107	107	108	109
H_f	0.623	2.41	5.31	9.30	14.4	20.5	35.9	55.5	139

8 Inches
0.6672 foot

$m = 0.59$ *
$n = 1.91$ *

3 years; asphalt; not stated; 105 feet; *B. & S.*; by Venturi meter; *E. W. Schoder*, 1907; *A. S. C. E.* 62, pp. 67–96; and *Eng. Record*, 58, p. 241.

V	1	2	3	4
C	101	104	106	107
H_f	0.59	2.22	4.81	8.33

$9\frac{37}{64}$ Inches
0.7979 foot

$m = 0.904$
$n = 1.99$

Old; uncoated; with deposits; 328.1 feet; flange; by volume; *Darcy*, 1850; *R. E.*, Series 19, pp. 12 and 58.

V	1	2	3	4	5	6	8	10	12.6
C	75	75	75	75	75	75	75	75	75
H_f	0.904	3.59	8.05	14.3	22.2	32.0	56.7	88.4	140

Y

CAST-IRON PIPES — *Continued*

$9\frac{5}{8}$ Inches
0.8028 foot

Old; uncoated; cleaned; 328.1 feet; flange; by volume; the preceding pipe after cleaning; *Darcy*, 1850; *R. E.*, Series 20, pp. 12 and 58.

$m = 0.566$
$n = 1.96$

V	1	2	3	4	5	6	8	10	14.7
C	94	95	96	96	97	97	98	98	99
H_f	0.566	2.20	4.87	8.57	13.3	19.0	33.3	51.6	110

$11\frac{11}{16}$ Inches
0.9744 foot

Old; uncoated; very well cleaned; 328.1 feet; socket; by volume; *Darcy*, 1850; *R. E.*, Series 21, pp. 12 and 58.

$m = 0.395$
$n = 1.98$

V	1	2	3	4	5	6	8	10	10.4
C	102	103	103	[illegible]03	104	104	104	104	104
H_f	0.395	1.56	3.48	[illegible].15	9.56	13.7	24.3	37.7	40.8

12 Inches
1 foot

New; coal tar; clean; various lengths, straight; *B. & S.*; Pitot tube; *Williams, Hubbell, & Fenkell*, 1898–1901; *A. S. C. E.* 47, p. 187.

$m = .388$ *
$n = 1.78$ *

V	1	2	3	4	5
C	102	110	115	[illegible]18	121
H_f	0.388	1.33	2.74	[illegible].58	6.81

$12\frac{1}{16}$ Inches
1.004 feet

New; asphalt; clean; about 3 miles, with easy bends; in reservoir; *Iben*, 1874–1879; *H. S., Jr., Hydraulics*, p. 231.

$m = 0.58$
$n = 1.61$

V	1.6	2	3	3.1
C	91	95	103	103
H_f	1.24	1.77	3.40	3.59

12 Inches

Old; unknown; unknown; 8160 feet, six bends; *B. & S.*; unknown; *Thomas Duncan*; *Proc. Inst. C. E.*, 1852, p. 499.

$m = 0.42$
$n = 2.33$

V	1	2	2.5
C	98	87	84
H_f	0.42	2.11	3.55

16 Inches
1.335 feet

New; coal tar; clean; various lengths, straight; *B. & S.*; Pitot tube; *Williams, Hubbell, & Fenkell*, 1898–1901; *A. S. C. E.* 47, p. 187.

$m = 0.2114$ *
$n = 1.86$ *

V	1	2	3	4	5
C	119	126	129	132	134
H_f	0.211	0.767	1.63	2.78	4.22

$16\frac{15}{32}$ Inches
1.373 feet

New; varnish; clean; $4\frac{1}{2}$ to 6 miles, with easy bends; *B. & S.*; by volume; *Lampe*, 1869–1871; *H. S., Jr., Hydraulics*, pp. 233–234.

$m = 0.259$
$n = 1.82$

V	1.6	2	3	3.1
C	109	112	116	116
H_f	0.609	0.914	1.91	2.03

CAST-IRON PIPES — *Continued*

16 Inches

$m = 0.342$
$n = 2.10$

18 years; coal tar; tuberculated; 13,350 feet, four curves, three gates; *B. & S.*; by pumps; *Forbes*, 1891; *N. E. W. W.* 6, p. 164.

V	1	2
C	94	90
H_f	0.342	1.47

19$\frac{11}{16}$ Inches
1.6404 feet

$m = 0.229$
$n = 1.82$

New; uncoated; clean; 328.1 feet; socket; by volume; *Darcy*, 1850; *R. E.*, Series 22, pp. 12 and 58.

V	1.4	2	3	3.7
C	106	110	114	116
H_f	0.422	0.808	1.69	2.48

20 Inches

1 to 5 years; coal tar; not tuberculated; 75,000 feet; alignment very irregular; *B. & S.*; by pumps; *Brush*, 1882–1887; *A. S. C. E.* 19, p. 92.

V	2	3
C	115	110
H_f	0.729	1.80

28 Inches

New; coated; clean; 2006 feet, 2 curves; *B. & S.*; by weir; *I. W. Smith*, 1894; *A. S. C. E.* 36, p. 203.

$V = 7.51$; $C = 123$; $H_f = 6.34$.

30 Inches
2.497 feet

$m = 0.1125$ *
$n = 1.93$ *

New; coal tar; clean; various lengths, straight; *B. & S.*; Pitot tube; *Williams, Hubbell, & Fenkell*, 1898–1901; *A. S. C. E.* 47, pp. 187–188.

V	1	2	3
C	119	122	124
H_f	0.113	0.429	0.937

30 Inches

Coal tar; 5740 feet, several curves; *B. & S.*; by weir; *C. W. Sherman, Report Water Commissioner*, Boston, 1898, p. 123.
1 year old, $C = 111$. **10 years old**, $V =$ from 0.75 to 1.5, $C = 103$.

32 Inches

New; coated; clean; 21,460 and 9613 feet, with 17 curves; *B. & S.*; by weir; *I. W. Smith*, 1894; *A. S. C. E.* 36, p. 203.

$V = 5.75$; $C = 126$; $H_f = 3.10$

36 Inches
3.041 feet

Asphalt; about 1000 feet, with curves; *B. & S.*; by use of water in a reservoir; *Kuichling*, 1895–1899; *A. S. C. E.* 44, p. 56.

Age, years	1.28	1.30	3.06	3.37	3.91	4.47	5.04	5.34
V	4.20	4.24	4.13	4.02	4.03	4.03	4.08	4.08
C	129	125	99	66	70	75	80	82
H_f	1.39	1.51	2.28	4.85	4.34	3.76	3.45	3.26

In September, 1898, when the pipe was about 4.2 years old, it was entered and examined; extensive organic growths were found on the top and sides, the lower part was considerably tuberculated.

CAST-IRON PIPES — Continued

36 Inches

Laid in 1896; coal tar; 5500 feet, several curves; *B. & S.*; by weir; *C. W. Sherman*; *Reports Water Commissioner*, B 1897, p. 56; 1898, p. 128.

	1896	1896	1897
V	4.7	0.5 to 4.3	3.2
C	136	133	114

In 1898 the pipe was exam only a small amount of tu lation was found, but the interior surface was covere a slimy substance (*Polyzoo ericella*).

48 Inches

$m = 0.0656$
$n = 1.96$

½ year; "pitch"; clean; 3557 feet, with several c *B. & S.*; by volume; *A. F. Bruce*; *Proc. Inst. C. E* p. 414.

V	1.3	2	3	3.6
C	136	136	137	138
H_f	0.176	0.216	0.479	0.685

48 Inches
4.00 feet

$m = 0.0590$
$n = 1.89$

2 years; Angus Smith coating; clean; 1747 feet, 2 curves; *B. & S.*; by weir; *Stearns*, 1880; *A. S. C. E.* 14, See below.

V	1 †	3.7	4	5	6	(
C	130	140	141	142	144	1
H_f	0.059	0.699	0.811	1.24	1.74	1

† Below the limits of experiments.

48 Inches
4.00 feet

$m = 0.0880$ *
$n = 2.03$ *

17 years; Angus Smith coating; tuberculated; 1747 fee weir; same pipe as the one preceding 15 years later; *Fitzg* 1894–1895; *A. S. C. E.* 35, p. 241 *et seq.* See below.

V	1	2	3	3.5
C	107	106	105	105
H_f	0.088	0.359	0.818	1.12

48 Inches
4.00 feet

$m = 0.0820$ *
$n = 2.02$ *

17 years; Angus Smith coating; tuberculated; 1747 feet weir; a pipe laid parallel to the last, similar in every *Fitzgerald*, 1894–1895; *A. S. C. E.* 35, p. 241 *et seq.*

V	1	2	3	3.5	4	5	5.8
C	110	110	109	109	109	109	109
H_f	0.082	0.333	0.754	1.03	1.35	2.12	2.86

48 Inches
4.00 feet

$m = 0.0575$ *
$n = 1.91$ *

17 years; Angus Smith coating; cleaned; 1747 feet; by same pipe as Stearns's 48-inch after cleaning; see above. *gerald*, 1895; *A. S. C. E.* 35, p. 241 *et seq.*

V	1	2	3	3.5	4	5	6
C	132	136	139	140	140	142	143
H_f	0.0575	0.216	0.469	0.629	0.812	1.24	1.76

CAST-IRON PIPES — *Continued*

48 Inches

8 years; Angus Smith coating; partly tuberculated; 14,124 feet, with curves; *B. & S.;* by weir; *J. W. Gale, Inst. Engineers & Shipbuilders in Scotland,* 1869, Vol. 12, p. 136.

$V = 3.46$; $C = 112$; $H_f = 0.947$

61¼ Inches

$m = 0.04655$
$n = 1.816$

New; very smooth coating; clean; 1800 feet, with curves; *B. & S.;* not stated; *Fitzgerald,* 1900; *A. S. C. E.* 44, p. 88.

V	4.7	5	6	6.5
C	150	151	153	154
H_f	0.773	0.865	1.21	1.39

60 Inches

$m = 0.082$ *
$n = 2.1$ *

8 years; coated; tuberculated; 8215 feet, with curves; *B. & S.;* by pumps; *Fenkell,* 1903; *A. S. C. E.* 51, p. 325.

V	0.2	0.6	1.0	1.1
C	107	101	99	99
H_f	0.00279	0.028	0.082	0.100

RIVETED STEEL OR SHEET-IRON PIPES

3 Inches
0.2502 foot

$m = 2.49$
$n = 1.89$

New; galvanized; clean; 24.9 feet; taper; by weight; *Giltner & Ketchum,* 1899–00; reported by *G. S. Williams, A. E. S.* 26, p. 181.

V	1	2	2.3
C	80	83	84
H_f	2.49	9.23	12.0

3¼ Inches
0.271 foot

$m = 1.555$
$n = 1.80$

New; asphalt; clean; 328.1 feet; screw coupling; by volume; *Darcy,* 1850; *R. E.*, Series 8, pp. 12 and 52.

V	1	2	3	4	5	6	8	10	12.8
C	97	104	109	112	114	117	120	123	126
H_f	1.56	5.42	11.2	18.9	28.2	39.1	65.7	98.1	153

4 Inches
0.3403 foot

$m = 1.34$ *
$n = 1.85$ *

New; asphalt; clean; standard spiral riveted; 80.1 feet; flange; by volume; *Schoder & Gehring; Eng. Record,* 1908, 58, p. 241.

V	1	2	3	4	5	6	8	10	Limits
C	94	99	102	104	106	107	109	111	of V not
H_f	1.34	4.83	10.2	17.4	26.3	36.9	62.8	94.9	stated

6 Inches
0.4953 foot

$m = 0.80$ *
$n = 1.85$ *

New; asphalt; clean; standard spiral riveted; 60 feet; flange; by volume; *Schoder & Gehring; Eng. Record,* 1908, 58, p. 241.

V	1	2	3	4	5	6	8	10	Limits
C	100	106	109	111	113	115	117	119	of V not
H_f	0.80	2.88	6.11	10.4	15.7	22.0	37.5	56.6	stated

RIVETED STEEL OR SHEET-IRON PIPES — *Continued*

6 Inches
0.4893 foot

$m = 0.97$ *
$n = 1.92$ *

New; asphalt; clean; double extra heavy spiral riveted; 73.55 feet; flange; by volume; *Schoder & Gehring*; *Eng. Record*, 1908, 58, p. 241.

V	1	2	3	4	5	6	8	10	Limits
C	92	94	96	97	98	99	100	101	of V not
H_f	0.97	3.67	7.99	13.9	21.3	30.3	52.6	80.7	stated

7$\frac{23}{32}$ Inches
0.643 foot

$m = 0.605$
$n = 1.77$

New; asphalt; clean; 328.1 feet; screw coupling; by volume; *Darcy*, 1850; *R. E.*, Series 9; pp. 12 and 52.

V	1	2	3	4	5	6	8	10	19.7
C	101	110	115	119	122	125	129	132	143
H	0.605	2.06	4.2[illegible]	7.04	10.5	14.4	24.0	35.6	118

10$\frac{59}{64}$ Inches
0.9105 foot

$m = 0.501$
$n = 1.82$

5 years; uncoated; cl[illegible] about 700 feet, with curves; taper; by weir; *H. Smith, Jr.*, [illegible]; *Hydraulics*, pp. 241 and 302.

V	[illegible].7	5	6	8	10
C	[illegible]08	108	110	113	115
H_f	[illegible]38	9.38	13.1	22.1	33.1

11$\frac{7}{32}$ Inches
0.935 foot

$m = 0.414$
$n = 1.78$

New; asphalt; clean; 32[illegible]1 feet; screw coupling; by volume; *Darcy*, 1850; *R. E.*, Seri[illegible]10; pp. 12 and 54.

V	1.3	2	[illegible]	4	5	6	8	10	10.5
C	105	110	1[illegible]	[illegible]18	121	124	128	131	132
H_f	0.660	1.42	2.[illegible]	[illegible].88	7.26	10.1	16.8	24.9	27.2

12$\frac{43}{64}$ Inches
1.056 feet

$m = 0.351$
$n = 1.92$

5 years; uncoated; clean; about 700 feet, with curves; taper; by weir; *H. Smith, Jr.*, 1876; *Hydraulics*, pp. 241 and 302.

V	4.6	5	6	8	10	10.7
C	110	111	112	113	114	114
H_f	6.58	7.72	10.9	19.0	29.2	33.2

14$\frac{12}{64}$ Inches
1.023 feet

$m = 0.310$
$n = 1.86$

5 years; uncoated; clean; about 700 feet, with curves; taper; by weir; *H. Smith, Jr.*, 1876; *Hydraulics*, pp. 241 and 302.

V	4.4	5	6	8	10	12
C	114	115	116	119	120	122
H	4.88	6.19	8.69	14.8	22.5	31.5

16 Inches

New; asphalt; clean; 16,416 feet, with curves; cylinder; in reservoir; *Adams*, 1895; *A. S. C. E.* 36, p. 25.

$V = 4.58$; $C = 112$; $H_f = 5.00$.

30 Inches

New; asphalt and coal tar; clean; 308 miles, with curves; cylinder; by pumps; *Unwin's Hydraulics*, p. 193.

V	1.89	2.12
C	116	116
H_f	0.426	0.530

RIVETED STEEL OR SHEET-IRON PIPES—*Continued*

33 Inches

New; asphalt; clean; 34,176 feet, with curves; taper; by weir and reservoir; *I. W. Smith*, 1894; *A. S. C. E.* 36, p. 203.

$V = 6.12$; $C = 123$; $H_f = 3.57$.

35 Inches

New; asphalt; clean; 39,829 feet, with curves; same as 33-inch.

$V = 5.44$; $C = 127$; $H = 2.53$.

36 Inches

$m = 0.149$
$n = 1.76$

New; asphalt; clean; 25,000 feet; cylinder joints; by weir; *Herschel*, 1892; *115 Experiments*, p. 28.

							Same, 4 years old	
V	1	2	3	4	5	5.7	**1896**	4.93
C	95	103	107	112	115	117		106
H_f	0.149	0.505	1.03	1.71	2.53	3.19		2.87

38 Inches
3.167 feet

Built in 1893–1894; asphalt and Sabin's varnish; 46,339 feet, with curves; cylinder; in reservoir; *Kuichling, Report Executive Board*, Rochester, N. Y., 1897, pp. 40–42.

	June 9 to 15, 1896								
V	0.490	0.612	0.758	0.888	0.965	1.06	1.13	1.19	1.27
C	95	95	110	111	97	106	111	112	110
H_f	0.0337	0.0508	0.0605	0.0810	0.125	0.126	0.131	0.142	0.166

Oct. 1896	July 6-17, 1897								
1.22	0.505	0.637	0.814	0.897	1.02	1.11	1.15	1.22	1.24
102	113	114	117	112	112	115	117	115	109
0.181	0.0254	0.0397	0.0615	0.0819	0.105	0.117	0.123	0.142	0.163

Another part of the same line, 91,641 feet.

	Oct. '95	Dec. '95	July '97	Nov. '97
V	3.27	3.23	3.25	3.15
C	117	114	116	112
H_f	0.994	1.01	0.999	0.997

Another part of the same line, 45,394 feet. This piece is connected directly with the Rochester 36 cast-iron pipe, gauging was simultaneous; compare them.

Age in years	1.28	1.30	3.06	3.37	3.91	4.47	5.04	5.34
V	3.88	3.91	3.81	3.71	3.72	3.71	3.77	3.76
C	109	109	106	107	106	106	108	108
H_f	1.59	1.61	1.63	1.53	1.57	1.55	1.54	1.54

The pipe was examined when it was about 4.2 years old; only slight tuberculation was found.

42 Inches

New; asphalt; clean; 50,965 feet, with curves; taper; by weir and reservoir; *I. W. Smith*, 1894; *A. S. C. E.* 36, p. 203.

$V = 3.78$; $C = 116$; $H_f = 1.21$.

RIVETED STEEL OR SHEET-IRON PIPES — *Continued*

42 Inches
3.5 feet

$m = 0.111$
$n = 1.91$

New; asphalt; clean; 49,833 to 81,139 feet, with (taper; by Venturi meter; *Herschel*, 1896; *115 Experime* 29 and 53.

V	1	2	3	4	5
C	101	105	107	108	109
H_f	0.111	0.417	0.905	1.57	2.40

48 Inches
3.96 feet

$m = 0.12$
$n = 1.82$

New; asphalt; clean; 24,648 feet, with curves; ta Venturi meter; *Herschel*, 1896; *115 Experiments*, pp. 53.

V	3	4	4.7
C	101	104	105
H_f	0.886	1.50	2.01

48 Inches
3.94 to 3.96 feet

$m = 0.0896$
$n = 1.92$

New; asphalt; clean; 24,630 to 74,396 feet, with cylinder; by Venturi meter; *Herschel*, 1892; *115 Expe* pp. 27 and 52.

V	2	3	4	5	5.5
C	109	111	112	113	114
H_f	0.339	0.738	1.28	1.97	2.37

48 Inches

$m = 0.126$
$n = 1.80$

4 years; part of same line; *Herschel*, 1896; below P Notch; by Venturi meter.

V	3	4	5	6
C	100	103	105	107
H_f	0.910	1.53	2.28	3.17

48 Inches

$m = 0.146$
$n = 1.80$

4 years; another part of same; *Herschel*, 1896; above ton Notch; by Venturi meter.

V	3	4	4.6
C	93	96	97
H_f	1.05	1.77	2.28

72½ Inches
6.042 feet

$m = 0.0563$
$n = 2.01$

New; asphalt; unknown; about 4400 feet, with butt joint; by Venturi meter; *Marx, Wing, & Hoskin* *A. S. C. E.* 40, p. 493.

V	1	2	3	3.8
C	108	108	108	108
H_f	0.0563	0.227	0.512	0.824

72½ Inches
6.042 feet

$m = 0.0941$
$n = 1.72$

2 years; asphalt; unknown; 4427 feet, with curves same pipe as last; by Venturi meter; *Marx, Wing, &* 1899; *A. S. C. E.* 44, p. 46.

V	1	2	3	4	5	5.3
C	84	92	98	102	105	106
H_f	0.0941	0.310	0.623	1.02	1.50	1.66

RIVETED STEEL OR SHEET-IRON PIPES — *Continued*

103⅜ Inches
8.615 feet

5 years; uncoated; rusty, not tuberculated; 153 feet; cylinder; by weir; *Herschel*, 1887; *115 Experiments.*

$m = 0.0342$
$n = 2.14$

V	1	2	3	4	4 5
C	117	111	108	106	105
H_f	0.0342	0.151	0.359	0.664	0.855

WOODEN STAVE PIPES

14 Inches
1.171 feet

New; redwood; clean; 3436 to 9912 feet, with curves; by weir and volume; *Adams; A. S. C. E.*, 1898, 40, p. 545.

$m = 0.30$
$n = 1.74$

V	1	1.5
C	107	113
H_f	0.300	0.607

18 Inches

New; yellow fir; clean; 23,310 feet, with curves; in reservoir; *Adams*, 1895; *A. S. C. E.* 36, p. 25.

$$V = 3.61; \quad C = 133; \quad H = 1.96.$$

44½ Inches

Less than 1 year; clean; 4041 feet, with curves; by 7½ inch Haskell current meter; *T. A. Noble; A. S. C. E.*, 1902, 40, p. 143.

$m = 0.125$
$n = 1.72$

V	3.5	4	4.8
C	111	113	116
H_f	1.07	1.37	1.87

56$\frac{1}{16}$ Inches

Less than 1 year; some growths (spongilla); 2679 feet, with curves; 7½ Haskell meter; *T. A. Noble; A. S. C. E.*, 1902, 40, p. 143.

$m = 0.0818$
$n = 1.72$

V	2.3	3	4	4.7
C	117	121	126	129
H_f	0.343	0.541	0.888	1.17

72½ Inches

New; Douglas fir; 2,710 feet, with bends; by Venturi meter; *Marx, Wing, & Hoskins*, 1897; *A. S. C. E.* 40, p. 512.

$m = 0.0623$
$n = 1.71$
Lower V's omitted

V	1.2	2	3	3.6
C	106	114	121	124
H_f	0.0851	0.204	0.408	0.557

72½ Inches

2 years; same pipe as last; 22,672 feet, with bends; by Venturi meter; *Marx, Wing, & Hoskins*, 1899; *A. S. C. E.* 44, p. 48.

$m = 0.0473$
$n = 1.96$

V	1.2	2	3	4	5	5.3
C	119	120	121	122	122	122
H_f	0.0676	0.184	0.407	0.716	1.11	1.24

TABLE L

VALUES OF f AND C FOR NEW CAST-IRON PIPES; FROM DARCY'S EXPERI
BY WESTON'S FORMULA. $f = .01989 + \frac{.001666}{D}$; $C = \frac{16.04}{(f)^{\frac{1}{2}}}$

Diameters		f	C	Diameters		f
Inches	Feet (D)			Inches	Feet	
¾	.0625	.0466	74	6	.500	.0232
1	.0833	.0399	80	8	.667	.0224
1¼	.104	.0359	85	10	.833	.0219
1½	.125	.0332	88	12	1.00	.0216
2	.167	.0299	93	14	1.17	.0213
2½	.208	.0279	96	16	1.33	.0211
3	.250	.0266	98	18	1.50	.0210
4	.333	.0249	102	20	1.67	.0209

TABLE LI

VALUES OF COEFFICIENTS C OF H. SMITH, JR., SUITABLE ONLY FOR NEW SMOOTH PIPES, HAVING NO SHARP BENDS. ALSO EQUIVALENT VALUES OF $f = \frac{8g}{C^2}$; AND THE FRICTION HEAD PER 1000 FEET (H_f)

Diameters			Mean Velocity (V) in Feet per Second							
Inches	Feet D		1	2	3	4	5	7	10	15
⅝ (about)	.05	C		78	82	86	88	90	91	91
		f		.0423	.0383	.0348	.0332	.0318	.0311	.0311
		H_f		52.6	107	173	258	484	966	2170
1¼ (about)	.10	C	80	89	94	97	99	102	105	105
		f	.0402	.0325	.0291	.0273	.0263	.0247	.0233	.0233
		H_f	6.25	20.2	40.7	68.0	102	188	363	816
12	1.0	C	96	104	109	112	114	118	121	124
		f	.0279	.0238	.0217	.0205	.0198	.0185	.0176	.0167
		H_f	.434	1.48	3.03	5.10	7.69	14.1	27.3	58.5
18	1.5	C	103	111	116	119	121	125	129	132
		f	.0243	.0209	.0191	.0182	.0176	.0165	.0155	.0148
		H_f	.251	.866	1.78	3.01	4.55	8.36	16.0	34.4
24	2.0	C	109	116	121	124	127	130	135	138
		f	.0217	.0191	.0176	.0167	.0160	.0152	.0141	.0135
		H_f	.169	.594	1.23	2.08	3.10	5 80	11.0	23.6
30	2.5	C	113	120	125	128	131	135	139	143
		f	.0201	.0179	.0165	.0157	.0150	.0141	.0133	.0126
		H_f	.125	.444	.922	1.56	2.33	4.30	8.28	17.6
36	3.0	C	117	124	128	132	134	138	143	147
		f	.0188	.0167	.0157	.0148	.0143	.0135	.0126	.0119
		H_f	.0974	.347	.732	1.22	1.85	3.43	6.52	13.9
42	3.5	C	120	127	131	135	137	142	146	151
		f	.0179	.0160	.0150	.0141	.0137	.0128	.0121	.0113
		H_f	.0794	.285	.599	1.00	1.52	2.78	5.38	11.3
48	4.0	C	123	130	134	137	140	145	150	155
		f	.0170	.0152	.0143	.0137	.0131	.0122	.0114	.0107
		H_f	.0661	.237	.501	.852	1.28	2.33	4.44	9.36
60	5.0	C	128	134	139	142	145	149	155	
		f	.0157	.0143	.0133	.0128	.0122	.0116	.0107	
		H_f	.0488	.178	.373	.635	.950	1.77	3.33	
72	6.0	C	132	138	142	146	148	153		
		f	.0148	.0135	.0128	.0121	.0117	.0110		
		H_f	.0383	.140	.298	.501	.759	1.40		
84	7	C	135	141	145	149	151			
		f	.0141	.0129	.0122	.0116	.0113			
		H_f	.0314	.115	.245	.412	.627			
96	8	C	138	143	148	151	154			
		f	.0135	.0126	.0117	.0113	.0108			
		H_f	.0263	.0978	.205	.351	.527			

TABLE LII

VALUES OF C AND f CORRESPONDING TO FANNING'S COEFFICIENT, $\left(\text{WHICH WAS EQUAL TO } \frac{f}{4}\right)$, APPLICABLE ONLY TO NEW SMOOTH STRAIGHT PIPES OR CHANNELS

Diameters			Mean Velocity (V) in Feet per Second								
Inches	Feet		0.5	1	2	3	4	5	10	15	20
½	.0417	C	78	82	87	90	93	95	101	104	106
		f	.0418	.0381	.0340	.0317	.0300	.0287	.0250	.0237	.0231
¾	.0625	C	80	84	88	91	94	96	102	105	106
		f	.0405	.0366	.0329	.0308	.0292	.0280	.0247	.0235	.0229
1	.0833	C	80	85	90	93	95	97	103	105	106
		f	.0398	.0353	.0317	.0300	.0285	.0274	.0245	.0234	.0228
1½	.125	C	82	87	91	94	96	98	103	106	107
		f	.0384	.0343	.0310	.0292	.0278	.0268	.0241	.0231	.0226
2	.167	C	84	88	93	95	97	99	104	106	107
		f	.0364	.0330	.0301	.0284	.0272	.0263	.0237	.0228	.0223
3	.250	C	85	90	95	97	99	101	105	107	108
		f	.0354	.0317	.0288	.0273	.0263	.0254	.0232	.0224	.0219
4	.333	C	87	92	96	99	100	102	107	108	110
		f	.0340	.0306	.0279	.0265	.0255	.0247	.0226	.0219	.0214
6	.500	C	90	94	99	101	103	104	108	110	111
		f	.0317	.0289	.0264	.0252	.0243	.0236	.0219	.0212	.0208
8	.667	C	93	97	101	103	105	106	110	111	113
		f	.0296	.0275	.0253	.0242	.0234	.0227	.0212	.0207	.0202
10	.833	C	95	99	103	105	107	108	112	113	114

14	1.17	C	100	103	107	109	110	111	115	116	117
		f	.0256	.0241	.0225	.0217	.0211	.0207	.0196	.0192	.0188
16	1.33	C	103	105	109	111	112	113	116	117	118
		f	.0244	.0232	.0218	.0210	.0205	.0201	.0192	.0188	.0184
18	1.50	C	104	107	110	112	114	115	117	119	119
		f	.0236	.0224	.0211	.0204	.0199	.0196	.0188	.0183	.0181
20	1.67	C	106	109	112	114	115	116	118	119	120
		f	.0229	.0216	.0204	.0198	.0194	.0191	.0184	.0180	.0177
24	2.00	C	110	113	116	117	118	119	121	122	123
		f	.0212	.0202	.0193	.0187	.0184	.0182	.0176	.0173	.0170
30	2.50	C	115	118	120	121	122	123	125	126	126
		f	.0194	.0186	.0179	.0175	.0173	.0171	.0166	.0163	.0161
36	3.00	C	120	123	125	125	126	126	128	129	130
		f	.0177	.0171	.0166	.0164	.0162	.0161	.0156	.0154	.0152
42	3.50	C	125	127	128	129	130	130	132	133	133
		f	.0164	.0160	.0156	.0154	.0153	.0152	.0148	.0146	.0145
48	4.00	C	130	131	132	133	133	134	135	136	137
		f	.0153	.0150	.0147	.0146	.0145	.0144	.0141	.0139	.0138
54	4.50	C	134	135	136	136	137	137	139	139	140
		f	.0144	.0142	.0140	.0138	.0137	.0137	.0134	.0133	.0132
60	5.00	C	137	138	139	140	140	140	142	143	143
		f	.0137	.0135	.0133	.0132	.0131	.0131	.0128	.0127	.0125
72	6.00	C	143	144	145	146	147	147	148	148	148
		f	.0126	.0124	.0122	.0120	.0120	.0119	.0117	.0117	.0117
84	7.00	C	148	150	151	151	152	152	153	154	154
		f	.0117	.0115	.0113	.0112	.0112	.0111	.0109	.0109	.0108
96	8.00	C	154	155	156	156	156	157	157	157	158
		f	.0109	.0107	.0106	.0106	.0105	.0105	.0104	.0104	.0103

TABLE LIII

VALUES OF f AND C FOR VERY SMOOTH PIPES SUCH AS BRASS OR LEAD

$$f = .0126 + \frac{.0315 - .06\,D}{V^{\frac{1}{2}}} \text{ (WESTON'S FORMULA)}. \quad C = \frac{16.04}{(f)^{\frac{1}{2}}}$$

H_f = friction head for 1000 feet of pipe. G = discharge in gallons per minute

DIAMETERS	MEAN VELOCITY (V) IN FEET PER SECOND										
Inches	1	1.5	2	3	4	6	8	10	15	20	
$\frac{1}{2}$	.0416	.0363	.0331	.0293	.0271	.0244	.0229	.0218	.0201	.0191	f
	79	84	88	94	97	103	106	109	113	116	C
	15.5	30.5	49.4	98.5	161.8	328.3	545.7	812.3	1687	2848	H_f
	0.61	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4.90	6.12	9.06	12.24	G
$\frac{5}{8}$	.0410	.0358	.0327	[illegible]	[illegible]	[illegible]	.0226	.0216	.0200	.0189	f
	79	85	89	[illegible]	[illegible]	[illegible]	107	109	113	117	C
	12.2	24.0	39.0	[illegible]	[illegible]	[illegible]	432.4	643.9	1343	2262	H_f
	.96	1.44	1.91	[illegible]	[illegible]	[illegible]	7.65	9.56	14.34	19.13	G
$\frac{3}{4}$	.0404	.0353	.0322	[illegible]	[illegible]	[illegible]	.0224	.0214	.0198	.0188	f
	80	85	89	[illegible]	[illegible]	[illegible]	107	110	114	117	C
	10.0	19.7	32.1	64.1	[illegible]	[illegible]	356.8	531.7	1108	1871	H_f
	1.38	2.07	2.75	4.13	[illegible]	[illegible]	11.02	13.77	20.66	27.54	G
1	.0391	.0343	.0313	.0279	[illegible]	[illegible]	.0220	.0210	.0195	.0185	f
	81	87	91	96	[illegible]	[illegible]	108	111	115	118	C
	7.3	14.4	23.4	46.8	77.2	[illegible]	262.3	391.4	819	1382	H_f
	2.45	3.67	4.90	7.34	9.79	14.69	19.58	24.48	36.72	48.96	G
$1\frac{1}{4}$	.0379	.0332	.0305	.0272	.0252	.0229	.0215	.0206	.0191	.0182	f
	82	88	92	97	101	106	109	112	116	119	C
	5.6	11.1	18.2	36.5	60.2	123.1	205.6	307.2	641.4	1089	H_f
	3.83	5.78	7.65	11.48	15.30	22.95	30.60	38.25	57.38	76.5	G
$1\frac{1}{2}$	.0366	.0322	.0296	.0265	.0246	.0224	.0211	.0202	.0188	.0180	f
	84	89	93	99	102	107	110	113	117	120	C
	4.6	9.0	14.7	29.6	48.9	100.3	167.8	251.1	526.1	893.8	H_f
	5.51	8.26	11.02	16.52	22.03	33.05	44.06	55.08	82.62	110.2	G
2	.0341	.0302	.0278	.0250	.0234	.0214	.0202	.0194	.0182	.0174	f
	87	92	96	101	105	110	113	115	119	122	C
	3.2	6.3	10.4	21.0	34.8	71.8	120.6	180.9	381.9	649.5	H_f
	9.79	14.69	19.58	29.38	39.17	58.75	78.34	97.92	146.9	195.8	G
$2\frac{1}{2}$	.0316	.0281	.0260	.0236	.0221	.0204	.0193	.0186	.0175	.0165	f
	90	96	99	104	108	112	115	118	121	124	C
	2.4	4.7	7.8	15.8	26.4	54.7	92.3	138.9	293.9	502.9	H_f
	15.30	22.95	30.60	45.90	61.20	91.80	122.4	153.0	229.5	306.0	G
3	.0291	.0261	.0243	.0221	.0209	.0193	.0184	.0178	.0168	.0163	f
	94	99	103	108	111	115	118	120	124	126	C
	1.8	3.6	6.0	12.4	20.7	43.3	73.4	110.8	249.1	405.2	H_f
	22.03	33.05	44.06	66.1	88.13	132.2	176.3	220.3	330.5	440.6	G
$3\frac{1}{2}$	.0266	.0240	.0225	.0207	.0196	.0183	.0175	.0170	.0162	.0157	f
	98	104	107	111	115	119	121	123	126	128	C
	1.4	2.9	4.8	9.9	16.7	35.1	59.9	90.8	194.3	335.4	H_f
	30.0	45.0	60.0	90.0	120	180	240.0	300.0	450.0	600.0	G

Values of f, H_f, and G taken from tables by Weston, *Trans. Am. Soc. C. E.*, Vol. 22, pp. 55 and 65. Values of C computed from Weston's f.

Problems

1. The mean velocity of flow in a new 18-inch cast-iron pipe is 6 feet per ond. Determine slope of the hydraulic grade line.

2. The discharge through a 12-inch cast-iron pipe is 1500 gallons per nute. Compute the head lost in friction in 2000 feet.

3. What discharge in gallons per day would be given by a system of four oot cast-iron pipes, 120 feet long, laid level, with a head of 100 feet on the ater?

4. What head is necessary to deliver 1,330,000 gallons per day through a oot cast-iron pipe, 17,684 feet long? (*a*) When the pipe is new. (*b*) When e pipe is 20 years old.

5. Water is raised by a pump through an actual vertical distance of 100 et, and delivered at the rate of 8.124 million gallons per day through a 42- ch cast-iron pipe 5 miles long. To raising the water what height is the ork of the pump equivalent?

6. The elevation of the water surface in a storage reservoir, *A*, is 78 feet. a distribution reservoir, *B*, it is 60 feet. Distance between them is 3 miles. hey are connected by a 2.5-foot new cast-iron pipe. What will be the dis- arge in cubic feet per second? Compute the elevation of water in a piezom- er tube 10,000 feet distant from *A*.

7. The water surface in a storage reservoir, *A*, is at elevation 310 feet. In istribution reservoir, *B*, it is 298.5 feet. Distance between them is 3 miles. ey are connected by a 3-foot cast-iron pipe. Compute the discharge in cubic t per second. (*a*) When pipe is new. (*b*) When 20 years old.

8. Two reservoirs are connected by a 15-inch pipe 4 miles long. In each rvoir the center of the pipe is 20 feet below the water surface. For the first distance, the pipe is laid on a grade of 1 in 660; the next $\frac{1}{4}$, 1 in 330; for last $\frac{1}{4}$, level. Determine the discharge in gallons per minute. Compute ght of piezometer reading at each change of grade and reduce to pounds per are inch.

9. A tank of 15,000 gallons capacity is 30 feet above the street. The pres- e on the street main is 30 pounds per square inch. Between the main and tank is a pipe 1000 feet long. What diameter must it have to fill the tank 30 minutes? With this diameter what head would be needed to fill tank in minutes?

10. A tank of 5000 gallons capacity is 28.6 feet above the street. Head on street main is 30 feet. Between the main and the tank is a pipe 200 feet g. What diameter must this pipe have to fill the tank in 30 minutes?

11. A riveted steel pipe 60 inches in diameter is 25,000 feet long. At 5000 t from the inlet 1,000,000 gallons per day are taken off; at 10,000 feet, 00,000 gallons; at 15,000 feet, 4,000,000 gallons; at 20,000 feet, 3,000,000 llons; and from the end 6,000,000 gallons per day. (*a*) Compute and plot e hydraulic grade line. (*b*) Compute also for a wooden stave pipe.

12. A double line of 30-inch cast-iron pipe, 20,000 feet long, with an available head of 120 feet, is to be replaced by a single line of larger size, to carry the same amount of water. What will be the required diameter, using the nearest commercial size above that given by the computation?

13. Water is raised by a pump through an actual vertical distance of 100 feet, and is to be delivered at the rate of 8,124,000 gallons per day through: (*a*) a 4-foot cast-iron pipe; or (*b*) a 2-foot cast-iron pipe; in each case 10 miles long. The work of the pump is equivalent to lifting the water to what height in each case?

14. What diameter of cast-iron pipe will be required to discharge 7,000,000 gallons per day under a head of 100 feet, through a length of 52,800 feet, making a reasonable allowance for deterioration in use?

15. What will be the velocity through a riveted steel pipe 40,000 feet long, 8 feet in diameter, under a head of 50 feet? (*a*) New. (*b*) 15 years old.

16. A cast-iron pipe 42 inches in diameter is used as an inverted siphon to connect two ends of an aqueduct, 3000 feet apart, and is to deliver 80 cubic feet per second. What should be the difference in elevation of the two ends of the aqueduct?

17. What pressure in pounds per square inch would be required at the street end of a house-service pipe of galvanized welded steel pipe $1\frac{1}{4}$ inches in diameter, to deliver 8 gallons per minute at a faucet, 400 feet distant? Assume five elbows.

18. What is the loss of head at entrance for an 18-inch cast-iron pipe having the inlet flush with the wall of a dam, if discharging 5000 gallons per minute?

19. Determine the loss of head in a bend of 45 degrees with a radius of 50 feet, in a 42-inch pipe discharging 10,000 gallons per minute.

20. The slope of a hydraulic grade line is 0.001. Compute the diameter of a galvanized steel pipe to deliver 470 cubic feet per minute.

21. 7480 gallons per minute are pumped through a pipe line composed of 3000 feet of 36-inch, 1000 feet of 30-inch, and 2000 feet of 24-inch cast-iron pipes. Compute the friction head in each length.

22. Compute the discharge in cubic feet per second in each of the following independent pipe lines (head of 150 feet in all cases): (*a*) 6000 feet of 24-inch pipe; (*b*) 2000 feet of 24-inch pipe, and 4000 feet of 48-inch pipe; (*c*) 2000 feet of 24-inch, 1000 feet of 12-inch, 2000 feet of 18-inch, and 1000 feet of 6-inch.

23. Two reservoirs, having a difference in elevation of 250 feet, are connected by a 4-foot pipe 10,000 feet long. At 1000 feet from the upper reservoir, a pipe 2 feet in diameter and 4900 feet long leads to another reservoir, at an elevation 130 feet lower than the upper reservoir. What is the discharge into each reservoir and what is the piezometer reading at the point of divergence?

24. Three mains separate at a point A, to which the water is brought in a single 36-inch cast-iron main. The first is 6 inches; the second, 12 inches; and the third, 24 inches; each 1 mile long. (*a*) What head measured from the

center of the pipe at A is necessary, at a reservoir 2 miles away, to deliver 8400 gallons per day at A, the pressure at A to be 100 pounds per square inch? (*b*) What will be the discharge at the outlet of each distribution pipe if they all discharge into atmospheric pressure, and have their outlets at the same elevations as the point A?

25. A distribution system of cast-iron pipes is laid out in the form of a gridiron, measuring 300 feet each way to points of intersection.

The pipes are of two sizes, being 8-inch in one direction and 6-inch in the other, which is at right angles to that first mentioned. These pipes are connected throughout and there are no dead ends.

Assuming, as in the case of a fire, that it is required to deliver 1500 gallons per minute under a hydrant pressure of 70 pounds per square inch at point B, whose coördinates are 1800 feet by 1800 feet from the point of supply A, determine the necessary pressure at the point A, if A and B are at the same elevation.

CHAPTER XVI

THE FLOW OF WATER IN OPEN CHANNELS

382. Open channels are natural or artificial waterways of any form in which the flowing water has its upper surface exposed to the air. Brooks, rivers, canals, troughs, and also, if only partly filled, covered conduits or pipes are open channels.

UNIFORM FLOW

383. **Uniform, steady flow** in open channels will exist when the following conditions occur simultaneously: if the volume of flow is constant for successive equal intervals of time, if the dimensions and outlines of the channel are constant from place to place, if the lining is of uniform material and smoothness, and the slope of the bottom is constant. Then in successive sections the cross-sectional areas of the stream and the mean velocity will be constant, and the slope of the water surface will be parallel to the slope of the bottom; and the slope,

$$S=\frac{h_f}{L}.$$

h_f = the loss of head in friction, which in this case is the difference in elevation of the surface of the water at two cross sections of a stream separated by a distance L.

The simultaneous occurrence of all conditions requisite for uniform steady flow is in reality rare. A change from a steady flow of a given volume to that of a different volume in the same channel will alter (perhaps materially) the slope of the water surface, which will no longer be parallel to the slope of the bottom, hence the areas of, and the mean velocity in successive areas of, the stream will no longer be constant. Apparently slight variations in the character of the lining, the shape of the channel, or the slope of the bottom may produce marked changes in the resistance to flow, and the conditions which depend upon it.

As the surface of the water in an open channel is free to rise or fall, the stream, in adapting itself to variations or changes, produces a great variety of combinations of the factors causing flow. To correlate experimental results is often intricate and difficult; and except for channels of regular form, and very uniform linings, the formulation and reapplication of experimental results are not always trustworthy.

Empirical formulas must, however, be used, together with direct recourse in important cases to experimental data; and the accuracy of the result will depend largely upon the closeness with which a case under consideration fits the available experimental data.

384. The Chezy formula. If it be assumed that the loss of head varies as the square of the mean velocity, Chezy's formula, or some equivalent expression, is most commonly used in calculating discharge, namely:

$$V = C(RS)^{\frac{1}{2}}. \tag{1}$$

$$Q = AV = AC(RS)^{\frac{1}{2}}. \tag{2}$$

$$S = \frac{V^2}{C^2R}. \tag{3}$$

$$h_f = \frac{V^2L}{C^2R}. \tag{4}$$

V = mean velocity of flow in a stream cross section, feet per second.

A = area of cross section, square feet.

R = mean hydraulic radius, feet.

S = sine of slope of water surface; or, as more commonly used, the slope of the bottom of the channel.

C = an empirical coefficient depending upon V, R, the degree of roughness of the lining of the channel; and probably to some extent upon the slope.

h_f = head lost in friction in a length L (both in feet).

385. Exponential formulas. The resistance to flow has repeatedly been proved to be not always proportional to the square of the mean velocity, but to some other power of V, varying from

1.75 to 2.10 or higher; nor can the relation between velocity, slope, and the mean hydraulic radius be represented as a definite general law.

By classifying channels according to the roughness of their linings, it may be shown that if accurate experimental data covering the required conditions are available, the following expression, or some equivalent form, will give the true relation between S, V, and R for a particular channel.

$$S = \kappa \frac{V^n}{R^m}. \tag{5}$$

κ = a coefficient of roughness.
n = exponent of V.
m = exponent of R.

κ, m, and n will depend upon the roughness of the lining and to a limited extent upon the form of the channel.

Such a formula requires a separate determination of κ, m, and n for many classes of channels, and will be most accurate if based upon a large number of observations of nearly equal precision for each class; and means, in effect, a separate formula for each class. Such a procedure is logical and may ultimately be the prevailing practice; requiring, however, a great increase in existing data. At the present time the most generally used formula of this type is the Williams and Hazen formula given in § 343.

386. Coefficient C for the Chezy formula. The coefficient C is a variable, usually assumed to depend on the degree of roughness or smoothness of the channel lining, the mean hydraulic radius, the mean velocity of flow, and the slope. Whether C depends upon, or is independent of, the slope, is an open question; C may be determined directly from experiments, or by the use of an empirical formula.

An experimental determination of C comprises a measurement of the slope, the discharge, the dimensions of a stream and the channel in which it is contained, from which the following computation will give C for a given flow:

$$\frac{Q}{A} = V; \quad \frac{\text{Area}}{\text{Wetted Perimeter}} = R; \quad \frac{h_f}{L} = S.$$

Then since $V = C\left(R\frac{h_f}{L}\right)^{\frac{1}{2}}$; $C = V\left(\frac{L}{Rh_f}\right)^{\frac{1}{2}}$. (6)

The results of observations on a series of steady flows, if made with accuracy, will, if plotted on coördinate paper, allow the drawing of a curve or curves which will be a graphic representation of the law of flow in the experimental channel. From such a curve values of C may be taken within the range of the experiments, with as high a degree of precision as the experiments themselves warrant.

Such experimental results are the most trustworthy guides for obtaining accurate coefficients for other similar channels; but very close similarity is infrequent. To be of use the results of experiments must, in general, be formulated. The formula most widely used is Kutter's; but for smooth channels of small hydraulic radius Bazin's is especially good; and in general is as good as if not better than the Kutter formula.

287. Kutter's formula for calculating C. E. Ganguillet and W. R. Kutter, two Swiss engineers, published,* as the result of their studies of all the then (about 1870) available gaugings of the flow of water in channels, a general formula for computing the values of C, intended to be applicable to any kind of channel closed or open. The formula proposed by them is:

$$C = \frac{41.66 + \frac{1.811}{n} + \frac{.00281}{S}}{1 + \left(41.66 + \frac{.00281}{S}\right)\frac{n}{(R)^{\frac{1}{2}}}}. \tag{7}$$

S = sine of the slope.

R = the mean hydraulic radius.

n = a coefficient of roughness determined by experiment, or estimated.

The more common values given to the coefficients of roughness n are as follows:

n	*Character of the lining of a channel.*
.009	very smooth boards carefully joined; very smooth glass or enameled tubes.
.010	planed boards; neat cement plaster.
.011	cement mortar $\frac{1}{3}$ sand.

* *Flow of Water in Rivers and Other Channels*, translated by Hering and Trautwine. Wiley & Sons.

.012	common boards unplaned, well joined; new best qu ity brick masonry.
.013	new ashlar; ordinary, good brick masonry; well-la vitrified sewer pipes.
.015	dirty cement, brick and vitrified pipe sewers; poorly laid vitrified pipes; canvas lining on frame
.017	good rubble masonry and rough ashlar with cemer joints; rough brick masonry.
.02	clean canals in firm gravel; rough rubble (dry c mortar).
.025	natural or artificial channels in earth of regular forr and free from weeds or detritus.
.03–.04	channels in earth, with detritus or aquatic plant varying from good to bad condition.
.04–.05	natural streams of irregular form with flood flow.

Table LVIII (at the end of this chapter) contains values of computed by Kutter's formula for slopes of .001, .0002, .0001.

388. Kutter's diagram. The computation of values of C tedious, but may be facilitated by the use of a graphical cha as proposed by Ganguillet and Kutter.* From this chart al the mean velocity may be graphically determined by a methc proposed by Hering and Trautwine.*

389. Church's diagrams. I. P. Church has published a set diagrams on logarithmic paper covering the ordinary range practice from which V may be taken very simply.†

Example. Given an open channel of rectangular cross se tion, 10 feet wide, the water 20 feet deep, the slope .0001, a $n = .015$. Find R, C, V, and Q.

$$R = \frac{20 \times 10}{10 + 20 + 20} = 4 \text{ feet.}$$

By computation $$C = \frac{41.66 + \dfrac{1.811}{.015} + \dfrac{.00281}{.0001}}{1 + \left(41.66 + \dfrac{.00281}{.0001}\right)\dfrac{.015}{2}} = \frac{190.5}{1.523} = 125.$$

* See *Flow of Water*, etc., by Ganguillet and Kutter. Translated by He and Trautwine.

† *Diagrams of Mean Velocity of Uniform Motion of Water in Open Chan* Wiley & Sons.

Then, $$V = 125.1\left(4 \times \frac{1}{10000}\right)^{\frac{1}{2}} = 2.50 \text{ feet per second.}$$

By diagram, $C = 125$, $V = 2.5$, $Q = AV = 200 \times 2.5 = 500$ cubic feet per second.

390. Limitations of Kutter's formula. There is a wide range in the magnitude of the streams on which this formula is based (from hydraulic radii of 0.28 to 74.4 feet) ; but a study of the data on which the formula is based, as given in the author's book, has led to the following conclusions:

That, for hydraulic radii greater than 10 feet, or velocities higher than 10 feet per second, or slopes flatter than 1 in 10,000, the formula should be used with great caution. For hydraulic radii greater than 20 feet, or velocities higher than 20 feet per second, but little confidence can be placed in results.

That, considering the variable accuracy of the data on which the formula is based, results should not be expected to be consistently accurate within less than about 5 per cent.

That, for any slope steeper than .001 the values of C computed for $S = .001$ may be used with errors less than the probable error in the ordinary use of the Kutter formula.

That between slopes of .001 and .004 the maximum variation (at the extreme values of n and R) in C is about 4 per cent; for such values as fall within the range of ordinary practice the maximum variation is but 2 per cent.

That between slopes of .0004 and .0002 the maximum variation is about 5 per cent, but for such values as fall within the range of ordinary practice the maximum is less than 3 per cent.

That for higher values of S the divergence in the values of C increases; but the occasions when slopes flatter than .0004 are to be considered in design are not common, and when they do occur they are usually for structures of such high character that they warrant special study and some basis in addition to a general empirical coefficient. And considering that a degree of precision of .001 is rarely exceeded in leveling for ordinary construction work, and that in picking out the value of n a variation of .001 for small values of n and R may change the value of C as much as 17 per cent, and for moderate values as much as 5 to 8 per cent, it should be obvious that hair-splitting calculations with the

Kutter formula are a needless waste of time, producing merely mechanical accuracy instead of a high degree of precision.

391. Logarithmic diagram for the Kutter formula. In view of these many limitations to securing a high degree of precision by the use of the Kutter formula a logarithmic diagram * for solving problems is proposed, which is based on three curves for every desired value of n; one for $S = .001$, to be used for slopes from 1.0 to .0005; one for $S = .0002$, and one for $S = .0001$. Intermediate values may be closely estimated; and in any case but very slight error is involved in overlooking variations in C due to slight changes in S.

The Chezy formula is

$$V = C(RS)^{\frac{1}{2}} = CR^{\frac{1}{2}}S^{\frac{1}{2}}.$$

In the Kutter formula (7): let

$$y = 41.66 + \frac{1.811}{n} + \frac{.00281}{S}; \tag{8}$$

and

$$x = \left(41.66 + \frac{.00281}{S}\right)n. \tag{9}$$

Then

$$C = \frac{y}{1 + \dfrac{x}{R^{\frac{1}{2}}}} = \frac{yR^{\frac{1}{2}}}{R^{\frac{1}{2}} + x} = \frac{yR}{(R^{\frac{1}{2}} + x)R^{\frac{1}{2}}} \tag{10}$$

And

$$V = \frac{yR^{\frac{1}{2}}}{R^{\frac{1}{2}} + x} \times R^{\frac{1}{2}} \times \frac{1}{\left(\dfrac{1}{S}\right)^{\frac{1}{2}}} = \frac{yR}{(R^{\frac{1}{2}} + x)\left(\dfrac{1}{S}\right)^{\frac{1}{2}}}. \tag{11}$$

For logarithmic computations:

Formula (10) becomes:

$$\text{Log } C = \log\left(\frac{y}{R^{\frac{1}{2}} + x}\right) + \log R^{\frac{1}{2}} = \log\left(\frac{yR}{R^{\frac{1}{2}} + x}\right) - \log R^{\frac{1}{2}}. \tag{12}$$

Formula (11) becomes:

$$\text{Log } V = \log\left(\frac{y}{R^{\frac{1}{2}} + x}\right) + \log R - \log\left(\frac{1}{S}\right)^{\frac{1}{2}}. \tag{13}$$

And the discharge

$$\text{Log } Q = \log A + \log V. \tag{14}$$

* This diagram is new so far as the authors know.

Values of R sufficient in number to locate the curve are assumed and corresponding values of $\left(\frac{y}{R^{\frac{1}{2}}+x}\right)$ are computed.* On logarithmic paper, using the left-hand edge as the Y–Y axis, and the bottom edge as the X–X axis, points are plotted having for abscissas values of R; and for ordinates values of $\left(\frac{y}{R^{\frac{1}{2}}+x}\right)$; through these points a smooth curve is drawn and labeled with the proper n and S. The scale for R begins at the origin with .10, and the scale for $\left(\frac{y}{R^{\frac{1}{2}}+x}\right)$ with 10.

If on logarithmic paper a straight line be drawn from any number on the Y–Y axis to the same number on the X–X axis, this number will be the product of the two rectilinear coördinates of any point on this line. All such lines will be parallel and will make an angle of 315° with the X–X axis. Such lines will be called ***product lines***. For this diagram the scale of numbers for these lines should, in order to give directly the products of R by $\left(\frac{y}{R^{\frac{1}{2}}+x}\right)$, begin at $1=\left(\frac{1}{10}\times 10\right)$ at the origin. The scales of the ***product lines*** are marked on the edges of the diagram by short diagonals which also indicate their inclination.

A straight line (AB) drawn through the origin at 45° with the X–X axis will have every point equidistant from both axes; and may thus be used to determine the square roots of numbers. The line AB is graduated for this purpose by its intersections with the product lines, and has, therefore, the same scale; but the subdivisions are labeled, not with numbers themselves, but with their reciprocals. Thus at 1000 the scale on AB reads .001, at 50 it reads .02; the two coördinates of .001 are seen to be 31.6, and of .02, 7.07, obviously the square roots of 1000 and 50, the reciprocals of .001 and .02. The line AB is thus graduated and labeled "Scale of Slopes."

Figure 131 is such a logarithmic diagram for computing dis-

* Since $\frac{y}{R^{\frac{1}{2}}+x}=\frac{C}{R^{\frac{1}{2}}}$ the ordinates may be readily gotten by dividing values of C in Table LVIII by the corresponding values of $R^{\frac{1}{2}}$. The values of C in this table were computed by the reverse process.

charge in open channels by the use of the combined Chezy and Kutter formulas. This figure will be found in the Appendix.

To find V. The value of the *product line* drawn through that point on the given n and S curve which has for its abscissa the given R is the product of R by $\left(\frac{y}{R^{\frac{1}{2}}+x}\right)$. To this *product line* drop a perpendicular either horizontally or vertically from the given point on AB corresponding to the given slope; the other coördinate $\left(\text{the first being } \left(\frac{1}{S}\right)^{\frac{1}{2}}\right)$ is the value of the mean velocity, V. The decimal point for V is determined by inspection, as both the approximate values of the *product line* and of $\left(\frac{1}{S}\right)^{\frac{1}{2}}$ are easily noted.

To find Q. Follow the perpendicular through V thus found, to a perpendicular through a number on the other axis corresponding to the area (A); at their intersection draw a *product line* and at the margin read its value, which is the discharge (Q).

To find C. Trace (by eye) a horizontal line through that point on the given n and S curve which has for its abscissa the given value of R, to an intersection with a vertical through $(R)^{\frac{1}{2}}$ read on the R scale. Through the point of intersection draw product line to the nearest margin and read the value of

This is merely multiplying $\left(\frac{y}{R^{\frac{1}{2}}+x}\right)$ by $R^{\frac{1}{2}}$, which gives C.

In using the diagram a large 45° triangle is convenient; but any straightedge will answer, and the direction of product lines can be fixed as well by eye by noting equal intersections of the margins or intermediate lines. The horizontal and vertical lines can be traced by eye. One setting only is required to find V and C, and two settings to find Q.

Other computations may be made on the diagram, as, for example, if C, R, and S are given, to find V or Q; but in this the diagram has but little if any advantage over a good table of square roots and reciprocals and a slide rule.

Example. Given the area of the cross section of a stream $(A) = 42.5$ square feet; the mean hydraulic radius $(R) = 4.4$ feet

the slope (S) $= .0015$; and $n = .013$. Find the velocity (V), the discharge (Q), and the value of C.

To find V. The vertical through $R = 4.4$ cuts the curve for $S = .001$ and $n = .013$ at D; the product line through D is cut at E by a horizontal from $S = .0015$. The abscissa of E is 11.7, the velocity V.

The other coördinate of E is 25.8 $\left(\text{the square root of } \frac{1}{S}\right)$. The same result but on the other scales would result by dropping a perpendicular to the product line.

To find Q. Continue the perpendicular through E to its intersection at F with a horizontal through 42.5 ($= A$). The product line through this point reads 497, the discharge (Q) in cubic feet per second.

To find C. Trace a horizontal through D to I, its intersection with a vertical through 2.097 ($R^{\frac{1}{2}} = 2.097$); a product line I–J through this point reads $C = 145$.

392. Bazin's formula for calculating C. Bazin proposed * a formula for computing the flow in open channels, which in distinction to an earlier one is called Bazin's "new formula." The formula itself is for computing C for use in the Chezy formula.

$$V = \frac{157.6}{1 + \frac{\Gamma}{R^{\frac{1}{2}}}}(RS)^{\frac{1}{2}}. \tag{15}$$

$$C = \frac{157.6}{1 + \frac{\Gamma}{R^{\frac{1}{2}}}} = \frac{157.6\, R^{\frac{1}{2}}}{(R)^{\frac{1}{2}} + \Gamma}. \tag{16}$$

Γ † = coefficient of roughness.

R = mean hydraulic radius.

S = sine of slope.

This formula eliminates the variation in C with changes in S (in which it differs from the Kutter formula), and makes the variations in C depend only upon variations in the mean hydraulic radius, and in the coefficient of roughness.

* *Ann. des Ponts et Chaussées*, Mém. et Doc., 1897, 4th trimestre, p. 20 *et seq.*

† Bazin's symbol was "γ" (small gamma); to avoid confusion "Γ" (capital gamma) is here used.

VALUES OF Γ

Class I	$\Gamma = .109.$	Very smooth surfaces; neat cement; planed wood.
Class II	$\Gamma = .290.$	Smooth surfaces; planks, bricks, ashlar.
Class III *a*	$\Gamma = .833.$	Rough surfaces; rubble masonry.
Class III *b*	$\Gamma = 1.54.$	Canals with mixed linings; very regular earth or paved with stones.
Class IV	$\Gamma = 2.35.$	Earth canals in ordinary condition.
Class V	$\Gamma = 3.17.$	Earth canals in bad condition.

This formula represents the results of a very careful study of existing data and is extremely valuable in designing relatively small channels and those with very smooth linings; and meets the conditions of design for all open channels as well as, if not better than any general formula available. It has the additional merit of simplicity.

Table LIX (at the end of this chapter) gives values of C computed by Bazin's formula.

393. Logarithmic diagram for Bazin's formula. For logarithmic computation Bazin's formula (15) may be stated:

$$\operatorname{Log} V = \log\left(\frac{157.6}{R^{\frac{1}{2}} + \Gamma}\right) + \log R - \log\left(\frac{1}{S}\right)^{\frac{1}{2}}. \qquad (17)$$

The computations, plotting, and use of a logarithmic diagram for using Bazin's formula are similar to those described for use with Kutter's formula (§ 391). Values of R as abscissas are plotted on logarithmic paper against corresponding values of $\left(\frac{157.6}{R^{\frac{1}{2}} + \Gamma}\right)$, and through the points thus located a smooth curve is drawn, one for each value of Γ. By means of these curves a complete solution for V, Q, and C may be made as described in connection with the Kutter logarithmic diagram.

Figure 132 is a logarithmic diagram for Bazin's formula. This figure will be found in the Appendix.

Example: Given $A = 42.5$ square feet, $R = 4.4$, $S = .0015$, $\Gamma = .29$.

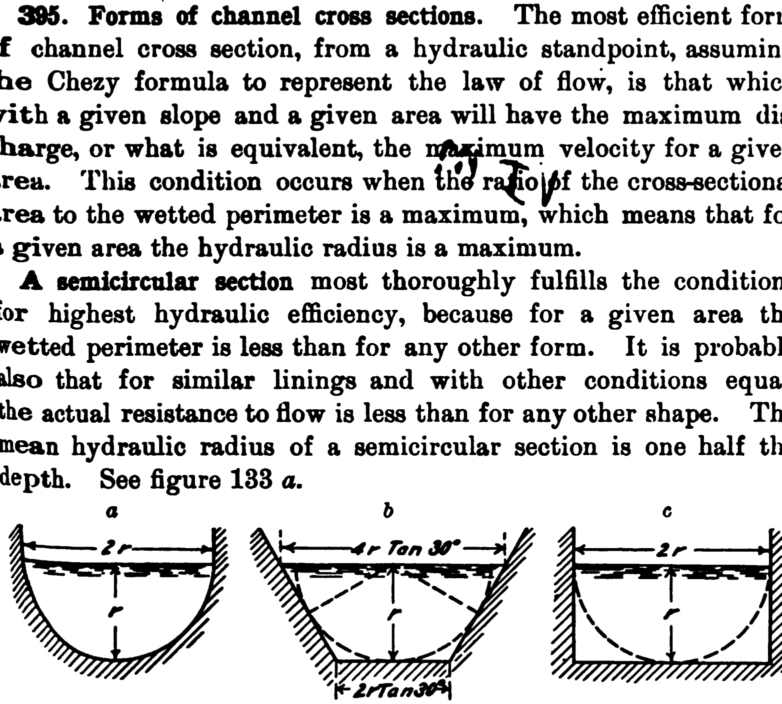
Find V, Q, and C.

From the diagram $V = 11.3$, $Q = 480$, $C = 140$.

394. Experimental results. Table LVII (at the end of this chapter) contains a few experimental results on open channels. Numbers (1) to (17) illustrate the comparative effect of various linings in channels of about the same capacity. Numbers (18) to (21) are examples of large masonry aqueducts. Owing to the large number and variety of experiments, no attempt has been made to show even a representative set of experiments. In designing important open channels a careful study should be made of original records of experiments.

395. Forms of channel cross sections. The most efficient form of channel cross section, from a hydraulic standpoint, assuming the Chezy formula to represent the law of flow, is that which with a given slope and a given area will have the maximum discharge, or what is equivalent, the maximum velocity for a given area. This condition occurs when the ratio of the cross-sectional area to the wetted perimeter is a maximum, which means that for a given area the hydraulic radius is a maximum.

A semicircular section most thoroughly fulfills the conditions for highest hydraulic efficiency, because for a given area the wetted perimeter is less than for any other form. It is probable also that for similar linings and with other conditions equal, the actual resistance to flow is less than for any other shape. The mean hydraulic radius of a semicircular section is one half the depth. See figure 133 *a*.



SEMICIRCULAR HALF HEXAGON HALF SQUARE

FIG. 133.

For (*a*) $A = \frac{\pi r^2}{2}$; $w.p. = \pi r$; $R = \frac{\pi r^2}{2\pi r} = \frac{r}{2}$;

(*b*) $A = 3r^2 \tan 30°$; $w.p. = 6r \tan 30°$; $R = \frac{3r^2 \tan 30°}{6r \tan 30°} = \frac{r}{2}$;

(*c*) $A = 2r^2$; $w.p. = 4r$; $R = \frac{2r^2}{4r} = \frac{r}{2}$.

(polygons, one half a regular hexagon or one half a squ… …e best of their particular shapes for hydraulic efficie… …ach having a mean hydraulic radius equal to one half the depth. See figures 133 *b* and 133 *c*.

For trapezoidal sections having a level bottom line, and two inclined side walls, but at some angle of inclination other than 60°, like figure 134 *d*, the best form will be that in which all the faces are tangent to an inscribed semicircle with its center in the water surface. R can be proved to be equal in all such sections to $\frac{r}{2}$.

Any section formed as the lowe… …alf of a regular polygon of an even number of sides, and each of… …s faces tangent to a semicircle

FIG. 134.

with its center in the water surface, will have its mean hydraulic radius equal to one half the radius of the inscribed circle. (See figures 134 *e* and 134 *f*.)

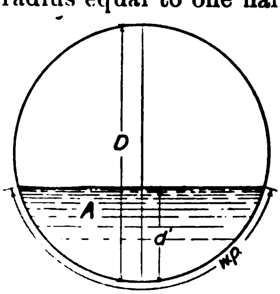


FIG. 135. — Circular Channel partly Filled.

For circular channels, a common form for sewers, the area and the mean hydraulic radius at different depths are given in Table LXV (in Appendix) for a diameter of one, by means of which desired values may be gotten for any other diameter by multiplying by the square of the given diameters to compute the area, and directly by the diameter to compute the mean hydraulic radius. Figure 135 explains the symbols in Table LXV.

The maximum value of R occurs when the depth $d' = .81\,D$.

The maximum discharge occurs not where R is a maximum, but where $R^{\frac{1}{2}} \times A$ is maximum, which is where the depth $= .95\,D$.

396. Factors determining the shapes of channels. The shapes of channels are not infrequently fixed by many important considerations other than to secure best hydraulic efficiency.

The question of cost of construction and maintenance, the choice of materials available for supporting or lining the channel, the limits to the slope of the bottom or its depth imposed by the character of the material or its surroundings, the frequently necessary provision for cleaning or inspecting, the securing of a good form of section in a channel both for large and small flows, the prevention of scouring or silting, and if in rock whether to use a large rough section or to line it; these and many other considerations give rise to a great choice of cross sections. The best form for many channels can be settled only by a thorough study of the hydraulic efficiency in connection with cost and permanency.

397. The effect of ice covering on the flow in an open channel. In northern climates, in the winter time, natural streams as well as power canals may be covered more or less continuously with ice which rests on the water surface. In such cases the ice adds to the wetted perimeter of the stream and increases the resistance to flow. The effect of ice covering may best be determined by gaugings with a current meter when the ice is in place; if gaugings are not feasible and computations of discharge by empirical formulas are necessary, the wetted perimeter should be increased by the total top width of the water surface. In addition to this, the area of the section is usually diminished by the thickness of the ice. In the design of channels for such conditions, the reduction should be provided for in fixing the dimensions.

398. The effect of minute growths and deposits upon the carrying capacity of channels. This question has been given considerable importance, due to the measurements in the Sudbury and Wachusett aqueducts, after being in commission for a number of years, when the carrying capacity of these aqueducts was found to be about 10 per cent less than when first used. This loss can be considerably reduced by cleaning, and this cleaning will probably be a part of the maintenance of every big aqueduct in the future.

399. The effect of anchor ice and frazil ice. Anchor ice is an ice formation on the stream bed. Frazil or needle ice forms in streams where the velocity is too great for surface ice to form, and will settle and build upon anchor ice or upon the bottom, or be carried along by the current just about in suspension until it is caught by some obstruction and there builds up. The effect of

anchor ice and frazil ice is to diminish the stream area, roughen the bed, and greatly retard if not stop all flow. To provide for this, ample allowance should be made by enlarging the channel to such an extent that the surface ice will form readily and protect the water; and some means, such as a large sluice, should be provided for drawing off the anchor and frazil ice, if it forms.

400. Distribution of velocities in the cross section of open channels. In straight channels without disturbing influences, it may be stated that in general the velocity is least at the perimeter, and increases toward the center from the sides, and from the bottom toward the top. The maximum velocity is usually found somewhere between mid depth and the surface, about in the middle of the channel.

In channels with curved alignment, or so near curves as to feel their effects, the distribution of velocities as compared with that of straight channels is more or less modified; for example, the velocity may be a maximum at or near the outside of the curve. The discharge from submerged wheels, orifices, gates, or entering streams, either above or below a stream section, may cause an entire redistribution of velocities at the point. A great many measurements based upon the determination of a few velocities have been vitiated by the effect of such disturbances.

401. Diagram of equal velocities. The relation of velocities in any stream cross section may be shown by plotting on cross section

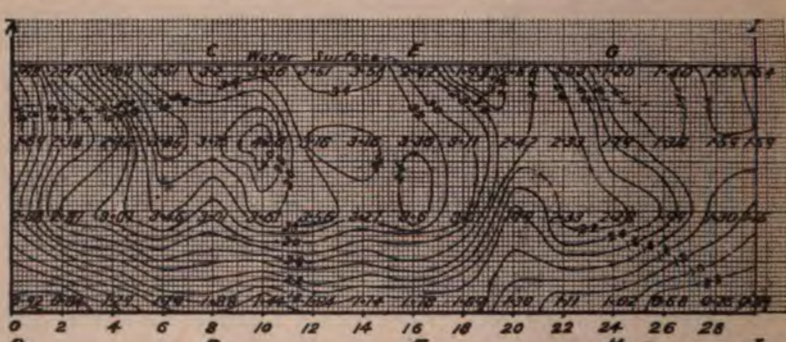


FIG. 136. — Diagram showing Curves of Equal Velocity in a Power Canal.

paper the outline of the section, and the velocities at the points where actual observations have been made. On this plot, by a

process like that of interpolating contours on topographic maps, a series of lines may be drawn, each one representing approximately the locus of all points having the same velocity. Such lines may be called contours of equal velocity, their area determined by planimeter, and from these areas the discharge by ordinary rules of mensuration. Figure 136 is such a diagram, in this case of a power canal; on account of the location of a closed feed gate on one side of the channel above this section, the maximum velocity is located away from the center of the channel.

402. A vertical velocity curve is a curve drawn through plotted points of actual observation in a vertical plane parallel to the axis

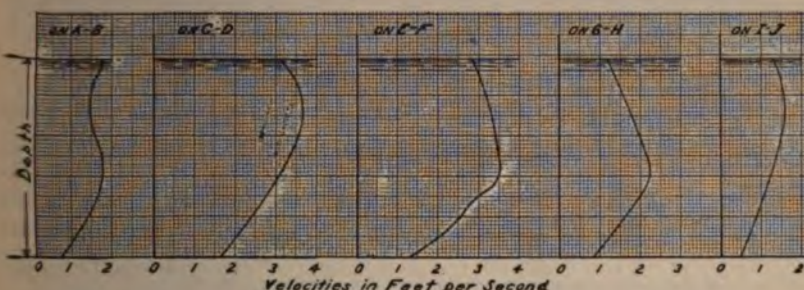


FIG. 137. — Vertical Velocity Curves in a Power Canal (taken from Fig. 136).

of flow, with proportional depths as ordinates, and velocities as abscissas. Figure 137 is a set of such curves taken at several sections from figure 136.

403. A horizontal velocity curve is a curve drawn through points of actual observations in a horizontal plane parallel to the axis of flow, with distances from one side as ordinates, and velocities as abscissas. Figure 138 shows such a curve in a straight channel; figure 139 shows such a curve at a bend in a channel. A rod float traverse or a current meter integration in sections is a curve of this kind.

404. Use of velocity curves. From a study of such curves, it is possible to find some fairly definite relations between mean velocity and the velocity at given points in any particular channel, and for a series of steady flows. Although a thorough study of experimental results has yielded formulated relations between

velocities at given points and mean velocities, presumably of general application, such formulas should be used with extreme

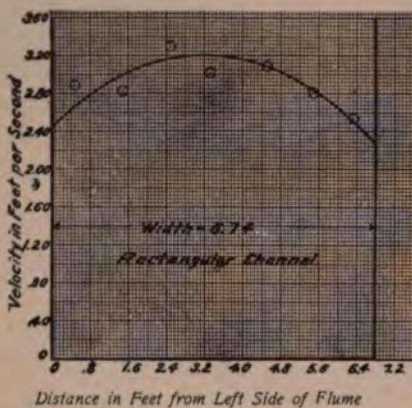


FIG. 138. — Horizontal Velocity Curve in a Straight Channel.

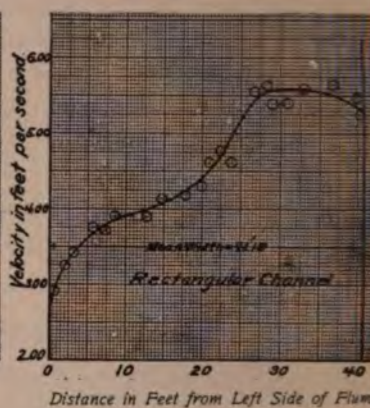


FIG. 139. — Horizontal Velocity Cur at a Bend in a Channel.

reluctance, and only where actual measurements can not be mad by current meter or other apparatus. One actual traverse ma establish thi ratio better thar it can be calcu lated in ad vance.

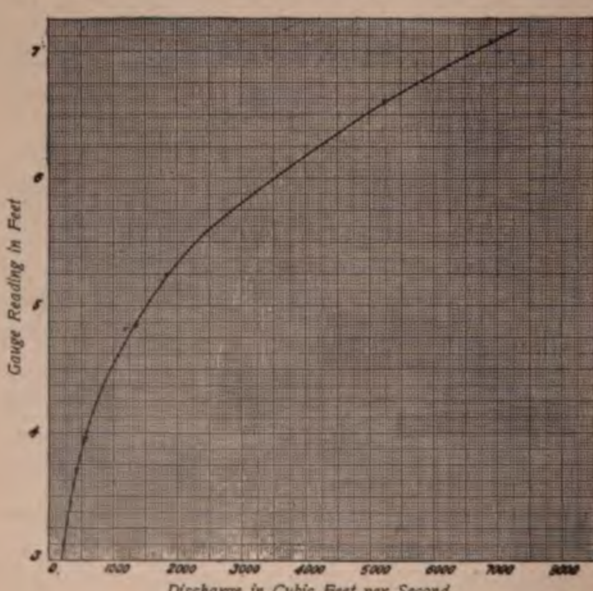


FIG. 140. — Rating Curve for a Stream.

405. A dis charge or ratin curve for an ope channel. For a open channe which maintain its cross sectior stable, that is does not erod or silt, and which has a fairly uni form section fo some consider able length,

moderately definite relation may be determined between the height of the water surface and the discharge.

The process of rating a stream or establishing a rating station consists in measuring the flow at various stages of the height of the water surface. The results of such measurements plotted will give a rating curve, from which, if the measurements have covered a suitable range, the discharge may be gotten at any time with such accuracy as the original measurements warrant by observing the gauge indicating the height of the water, and entering the curve with this reading. Figure 140 is such a curve.

Variable Flow in Open Channels

406. If the cross-sectional area of an open channel changes from place to place, instead of being uniform, the problem of computing the flow will be modified, and may become, according to given conditions, complicated, more or less uncertain, or very difficult of solution.

407. Steady non-uniform flow in artificial channels. Artificial channels, if not uniform, are, for the most part, either combinations of independent channels each of different area but within themselves uniform, and connected by an enlarging or reducing channel of relatively short length ; or two or more lengths of the same uniform channel connected by bends.

408. The loss of head in non-uniform artificial channels may be computed as the sum of the friction heads in each of the parts which are straight and uniform within themselves, and of the losses of head due to enlargements, contractions, bends, or any other irregularity between the uniform parts which produce eddies or other disturbances.

409. For enlargements or contractions the losses of head may be computed by methods similar to those indicated for pipes under pressure, or the losses may be provided for in assuming a sufficiently low value of C.

410. For bends in open channels no satisfactory solution for the losses of head can be offered ; the loss is frequently assumed to be $.5\frac{V^2}{2g}$.

411. *…mple.* Consider a channel composed of two parts, both recta…gular channels lined with planed boards (see figure 141); one …rom B to C is 1000 feet long, 6 feet wide, and $3\frac{1}{3}$ feet deep; the other from C to D is 2000 feet long, 5 feet wide, and

Fi…

2 feet deep. Assume … ion at C abrupt. If the discharge is 40 cubic … and the slope of water surface and bottom are the same … lost head between B and D, and the difference in … he water surface at B and D.

By Bernoulli'…

$$\ldots h_1 + \frac{V_1^2}{2g} + h_\lambda.$$

Then the lost head, $\quad h_\lambda = h_0 - h_1 + \frac{V_0^2}{2g} - \frac{V_1^2}{2g}.$

Assuming that the losses are due only to friction and to changes in velocity, the lost head may be expressed thus:

The lost head between B and D,

$$h_\lambda = \frac{L_0 V_0^2}{C_0^2 R_0} + \frac{L_1 V_1^2}{C_1^2 R_1} + \xi_\kappa \frac{V_1^2}{2g},$$

and the difference in elevation between B and D, which is

$$h_0 - h_1 = h_\lambda + \left(\frac{V_1^2}{2g} - \frac{V_0^2}{2g}\right).$$

Since $\quad V_0 = \frac{40}{3\frac{1}{3} \times 6} = 2$; and $V_1 = \frac{40}{2 \times 5} = 4$;

and $\quad R_0 = \frac{6 \times 3\frac{1}{3}}{3\frac{1}{3} + 3\frac{1}{3} + 6} = 1.58$; and $R_1 = \frac{10}{2 + 2 + 5} = 1.11$;

and by Kutter's formula, $n = .01$, $S = .001$ (assumed);

$$C_0 = 166 \text{ and } C_1 = 158;$$

so since the ratio of the two areas is 0.5, the factor for con-action loss, $\xi_\kappa = .298$ (§ 364, Table XLI):

$$\text{Then } h_\lambda = \frac{1000 \times 2^2}{166^2 \times 1.58} + \frac{2000 \times 4^2}{158^2 \times 1.11} + .298 \frac{4^2}{64.32}$$

$$= \quad .092 \quad + \quad 1.155 \quad + \quad .074 \quad = 1.32 \text{ feet.}$$

The values of the slope assumed (.001) gave excessive values C. Taking values of C for the slopes computed (.092 and .58 r 1000), a final solution gives

$$h_\lambda = \frac{1000 \times 2^2}{160^2 \times 1.58} + \frac{2000 \times 4^2}{150^2 \times 1.11} + .298 \frac{4^2}{64.32} = 1.45 \text{ feet.}$$

The difference in elevation of B and D,

$$h_0 - h_1 = 1.45 + \left(\frac{4^2}{2g} - \frac{2^2}{2g}\right) = 1.64.$$

NOTE. These trial computations can be done very quickly on the loga-hmic diagram for Kutter's formula (figure 131), by the inverse process of ding V. Here V is known and S is desired. As $V = \frac{CR^{\frac{1}{2}}}{\left(\frac{1}{S}\right)^{\frac{1}{2}}}$, therefore $\left(\frac{1}{S}\right)^{\frac{1}{2}}$ $\frac{CR^{\frac{1}{2}}}{V}$; the process being to divide by V the product line through the given and the given n and S curve.

The chief difficulty about problems of this kind is the deter-ination of the head lost at the place where changes occur, a gligible factor if the changes are gradual. For the most part, dealing with variable flow in open channels, no distinction can made between head lost in friction and the difference in eleva-n; the practice is to decrease the value of C enough to cover h losses.

112. Steady non-uniform flow in natural channels. Natural innels for the most part comprise a succession of varying cross-tions, never wholly uniform, but more or loss approximating iformity. Coefficients derived from observations on uniform w do not exactly fit variable flow; nor does a coefficient for variable flow fit another; and though estimates must be de, they may be far afield unless some actual measurements are ailable for the stream under consideration. Often the prob-ns concerning the carrying capacity of natural streams have to

do with extreme conditions, which may readily be estimated, but with a vague knowledge of just how accurate the results may be. The following example will illustrate a problem common to river hydraulics, and shows a method bv which this problem and similar problems may be solved.

413. *Example. A dam was proposed to create a waterfall* for a power station at a certain point on a given river. Before constructing the dam certain questions concerning the effect on the river of building the dam had to be answered; among them (*a*) *how high would the water rise at the dam;* and (*b*) *how high at a point about eight miles upstream* where there is an existing water-power plant with the running of which interference was not legally permissible.

The first question (*a*) could be answered by computing, by weir formulas, the head of water on the crest of the dam necessary to discharge the maximum known or predictable freshet.

The second question (*b*) could be answered only by measuring actual cross sections, and computing and plotting the hydraulic grade line of the river as it probably would be after the dam was built.

The steps in this process cover about the following:

The location and elevation of all high-water marks, especially those of the greatest flood flow.

Profiles of the water surface at the greatest and ordinary yearly flood flows and a profile of the stream bed on its axis.

Cross sections of the bed of the river, taken wherever any marked changes in section occurred and at enough intervals between such places as would approximately give the mean area. Such cross sections should include the banks to the highest elevation to which the river could possibly rise after the dam was built.

Determination of flood flows from records.

Computations (from the accumulated information) of values of C for successive sections of the stream, to be used in computing the hydraulic grade line for the assumed conditions.

The maximum flood flow in this case was known to be about 63,800 cubic feet per second; such a flood was likely to occu

only about two or three times in a century. The yearly maximum was known to be about 26,000 cubic feet per second.

From a study of the profiles and cross sections it was found that the river from the proposed dam to the water power eight miles upstream might be reasonably divided into 16 lengths. See columns (1) and (2) in Table LIV.

By means of the profiles and the cross sections, the mean area in each length was determined. See column (3).

From the profiles of the water surface the rise in water surface for each length was taken in this case for maximum flood flow. See column (4).

From the cross sections the mean wetted perimeter was scaled for each length. See column (5).

TABLE LIV

CALCULATED VALUES OF C, FOR $Q = 63{,}800$ CUBIC FEET PER SECOND

(1) No. of Sections	(2) L (ft.) Distance between Sections	(3) A (sq. ft.) Mean Area below Hyd. Gr. Line between Sections	(4) h_λ (ft.) The Rise between Sections or Head Lost	(5) $w.p.$ Wetted Perimeter	(6) V (ft. per sec.) $V = \frac{63800}{A}$	(7) R (ft.) $R = \frac{A}{w.p.}$	(8) C $C = V\left(\frac{L}{Rh_\lambda}\right)^{\frac{1}{2}}$
0							
1	2,600	13,510	0.45	440	4.73	30.7	64.9
2	2,400	18,700	0.50	585	3.41	32.0	41.8
3	1,800	13,610	0.20	480	4.68	28.4	83.7
4	2,000	19,700	0.40	640	3.24	30.8	41.3
5	2,500	18,100	0.50	625	3.53	29	46.3
6	2,200	19,390	0.40	570	3.39	34	41.7
7	900	11,390	0.10	410	5.64	27.6	102.4
8	1,600	17,070	0.40	618	3.73	27.7	44.8
9	3,900	15,580	0.70	635	4.10	24.5	61.9
10	1,500	16,560	0.40	680	3.85	24.4	47.7
11	2,900	19,070	0.50	885	3.33	21.6	54.7
12	2,600	10,190	0.60	450	6.25	22.6	86.0
13	2,400	13,700	1.20	545	4.66	25.1	41.6
14	3,000	15,120	1.40	655	4.22	23.1	40.6
15	2,100	12,430	1.00	495	5.15	25.1	47.1
16	1,200	13,180	0.40	525	4.83	25.1	52.8
	35,600		9.15				

The mean velocity in each length was next computed for the greatest flood by dividing the mean area of each length into 63,800. Thus for Section 1 the mean velocity $V = \frac{63800}{13510} = 4.73$. See column (6).

The mean hydraulic radius for each length was computed from the area and wetted perimeter $R = \frac{A}{w.p.}$. Thus for Section 1,

$$R = \frac{13510}{440} = 30.7. \quad \text{See column (7).}$$

The values of $C = V\left(\frac{L}{Rh_x}\right)^{\frac{1}{2}}$. For Section 1,

$$C = 4.73\left(\frac{2600}{30.7 \times .45}\right)^{\frac{1}{2}} = 64.9. \quad \text{See column (8).}$$

The results of these computations are shown in Table LIV.

Values of the coefficient C being thus determined, a computation of the hydraulic grade line as it probably would be after the dam was built showed that at the time of the greatest freshet the height of the water eight miles upstream would not affect the power plant there more than without the new dam on account of natural obstruction still farther down river. In other words the plant was flooded out in either case, and this would occur only at long intervals. As the same method of computation was used in estimating an abnormal freshet as for a yearly freshet, the computations for a discharge of 26,000 cubic feet per second only are given, because the annual freshet was in this case of more importance than an abnormal one.

As the extreme effect of this dam with a normal spring freshet was desired, instead of taking the individual values of C for each section as shown in Table LIV, the average value of C for the smaller ten computed values in Table LIV for a larger discharge were taken, in this case 45.*

Beginning at a height on the crest computed for this flood flow, the mean area between successive sections was recomputed by marking on each cross section the level of the water surface where a trial slope of .8 foot per mile cut each section, and again recomputed as successive trials indicated more nearly the probable grade line.

* This value is of interest as being about one half of the value of C commonly ascribed to such streams.

Thus step by step the grade was computed, starting from an elevation of 125.0 for high water at the dam. The results of these calculations are given in Table LV.

TABLE LV

CALCULATED SLOPE OF THE RIVER FOR $Q = 26{,}000$ CUBIC FEET PER SECOND, USING $C = 45$

(1) No. of Sections	(2) L (ft.) Distance between Sections	(3) A (sq. ft.) Mean Area below Hyd. Gr. Line between Sections	(4) $w.p.$ Wetted Perimeter	(5) V (ft. sec.) $V = \frac{26000}{A}$	(6) R (ft.) $R = \frac{A}{w.p.}$	(7) h_λ Rise between Sections $= \frac{LV^2}{C^2R}$	(8) Calculated Elevations of Water Surface at Each Section
0							125.0
1	2,600	10,520	410	2.47	25.7	0.30	125.30
2	2,400	14,580	555	1.78	26.3	0.14	125.44
3	1,800	10,300	450	2.53	22.9	0.25	125.69
4	2,000	15,420	610	1.69	25.3	0.11	125.80
5	2,500	13,230	595	1.97	22.2	0.22	126.02
6	2,200	15,350	540	1.69	28.4	0.11	126.13
7	900	8,410	380	3.09	22.1	0.19	126.32
8	1,600	12,530	588	2.08	21.3	0.16	126.48
9	3,900	10,890	605	2.39	18.0	0.61	127.09
10	1,500	11,360	650	2.29	17.5	0.22	127.31
11	2,900	12,160	855	2.14	14.2	0.46	127.77
12	2,600	6,470	420	4.02	15.4	1.35	129.12
13	2,400	9,090	515	2.86	17.7	0.55	129.67
14	3,000	8,940	625	2.91	14.3	0.88	130.55
15	2,100	7,400	465	3.51	15.9	0.80	131.35
16	1,200	7,880	495	3.30	15.9	0.41	131.76

131.76 is thus computed to be the elevation of the water surface at the dam eight miles upstream, during an ordinary spring freshet.

414. **Unsteady variable flow.** Where the volume of flow in open channels varies very much in short intervals of time, no method of solution will be offered; as the problem may be best solved by considering the flow as steady for the time being and making the calculations on this basis.

TRANSPORTATION OF SOLIDS BY MOVING WATER

415. Particles of solid matter heavier than water may be moved by a stream either by transporting them in suspension

or by rolling or drifting them along the stream bed. This does not refer to the moving of large stones by floating ice. Suspended matter is largely clay, fine sand, or loam; and drifting matter is usually coarse sand, gravel, or large pebbles. The shifting of solid particles in water implies erosion in some places and silting or building up in others. A stream may carry in suspension material which it has not power to erode, and can drift material which it can not carry. The one condition may occur continuously, the other only during freshets. The eroding and transporting power of a stream depends chiefly upon its velocity of flow, and the diameter and density of the particles of material through which it flows. Precise information on this subject is relatively meager, being chiefly Dubuat's experiments and Kennedy's experiments.

416. Dubuat's experiments. The following relations between the velocity of water and the movement of solids in water are those of Dubuat,* and are the result of a number of experiments

TABLE LVI

Material		Bottom Velocities at which		
		Transportation begins	Material is in Equilibrium	Silt[illegible] beg[illegible]
		——feet per second——		
†Clay; *s. g.* 2.64	Dark; suitable for pottery	0.35	0.27	
Sand; *s. g.* 3.36	Coarse, yellow . .	1.07	0.71	0.6[illegible]
Seine Gravel; *s. g.* 2.55	Size of anise seed .	0.53	0.35	0.[illegible]
	Size of pea or larger	0.71	0.62	0.5[illegible]
	Size of small brown bean	1.56	1.07	0.7[illegible]
Shingle; *s. g.* 2.61	Beach; rounded, one inch or more in diameter . . .	3.20	2.14	1.56
Flints; *s. g.* 2.25	Sharp; size of hen's egg	4.00	3.20	2.14

* *Principes d'Hydraulique*, etc., Dubuat. Paris, 1816. Nouvelle édition.

† The clay used deposited fine sand at bottom velocities of 0.53 and lower.

made by him in plank troughs. These troughs were of two sections, one rectangular having a bottom width of approximately 1.5 feet; the other of trapezoidal section with a bottom width of practically 0.5 foot and side slopes of about 3 to 4. This channel was probably not over 140 feet in length, and the operations were made in about the last 60 feet.

417. Kennedy's experiments. Kennedy* made a thorough study of the velocity, depth, and width in certain irrigation canals with reference to their effect on scouring and silting. The streams in question formed and maintained stable sections over many years though discharging large volumes of silt-bearing water. He found by plotting his results that the mean velocity at which silting is just prevented is a function of the depth and may be computed by the expression,

$$V_s = 0.84\, d^{0.64}.$$

V_s is the mean velocity at which with a given depth (d) silting is just prevented.

V_s is then the lowest velocity for which a channel in earth, or for a silt-bearing stream, should be designed. At a higher velocity erosion may occur; at a lower velocity water charged with silt will deposit part of its charge.

Sediment is kept in suspension by the vertical components of eddy currents, of which the force is in proportion to V^2. The silt-supporting power of a stream will be proportional to the width of its bed, other things being equal. While being thus supported the silt is moving forward at the velocity V; hence, the amount of silt suspended and transported will vary directly as the width of bed and as $(V^2 \times V^1) = V^3$, or allowing for the drifting silt which does not require support in the water, the silt-transporting power of the stream as deduced by Kennedy varies as the 2.5 power of the mean velocity of flow or $(V^{\frac{5}{2}})$.†

418. Design of earth channels modified by the transporting power of water. This transportation of materials by the water often make necessary lower velocities than other conditions

* Robert Greig Kennedy, *The Prevention of Silting in Irrigation Canals*, Proc. Inst. Civil Engrs., Vol. 119, p. 281.

† It is inferred that the canals mentioned were in moderately fine soil.

require for power or other canals in earth, in order to keep the cross section, once dug through soft material, of the form designed; on the contrary, heavy silt-bearing rivers may make necessary higher velocities at some points to prevent deposits, where such deposits would be a disadvantage.

TABLE LVII

Experiments on Open Channels

Numbers 1 to 17 are experiments by Darcy and Bazin taken from their *Recherches Hydrauliques*. The symbol *R. H.* followed by a number refers to the pages in that book; the symbol D. & B. followed by a number indicates their serial number for the experiments.

(1) **Semicircular**; $D = 4.101'$; lining, neat cement; slope of bottom $= .0015$; D. & B. 24; *R. H.* 426.

V	3.02	3.72	4.16	4.60	4.87	5.12	5.29	5.51	5.75	5.91	6.06	6.11
R	.366	.503	.605	.682	.750	.809	.867	.915	.949	.992	1.029	1.034
C	129	136	138	144	145	147	147	149	152	153	154	155

(2) **Semicircular**; $D = 4.101'$; lining, $\frac{2}{3}$ cement, $\frac{1}{3}$ very fine sand; slope of bottom $= .0015$; D. & B. 25; *R. H.* 429.

V	2.87	3.43	3.87	4.30	4.51	4.80	4.94	5.20	5.38	5.48	5.55	5.66
R	.379	.529	.635	.706	.787	.839	.900	.941	.984	1.007	1.021	1.038
C	121	122	125	132	131	135	134	138	140	141	142	144

(3) **Semicircular**; $D = 4.593'$; lining, planks; slope of bottom $= .0014$; D. & B. 26; *R. H.* 432.

V	2.61	3.23	3.71	4.04	4.25	4.51	4.64	4.87	5.00	5.18	5.29	5.45	5.54
R	.390	.537	.632	.717	.796	.856	.921	.964	1.015	1.054	1.096	1.129	1.148
C	108	114	121	123	123	126	125	128	128	130	130	132	134

(4) **Semicircular**; $D = 4.002'$; lining, small gravel (1 to 2 cm.) set in mortar $\frac{1}{3}$ cement; slope of bottom $= .0015$; D. & B. 27; *R. H.* 435.

V	2.17	2.50	2.69	2.93	3.05	3.22	3.33	3.54	3.73	3.95
R	.454	.546	.619	.681	.731	.784	.826	.900	.968	1.012
C	78	82	82	84	84	85	84	85	85	[illegible]8

(5) **Rectangular**; width, $5.943'$; lining, neat cement; slope of bottom $=$ .0049; D. & B. 2; *R. H.* 356.

V	3.34	4.39	5.04	5.68	6.51	6.83	7.12	7.41	7.63	7.86	8.07
R	.168	.251	.322	.375	.474	.518	.558	.595	.632	.665	.696
C	117	125	127	132	135	136	136	137	137	138	138

(6) **Rectangular**; width, 6.529′; lining, planks; slope of bottom = .0049; D. & B. 7; *R. H.* 373.

V	2.71	3.70	4.35	4.85	5.29	5.61	5.93	6.23	6.45	6.71	6.90	7.15
R	.188	.272	.342	.402	.453	.504	.547	.587	.628	.662	.698	.727
C	89	101	106	109	112	113	115	116	116	118	118	120

(7) **Rectangular**; width, 6.27′; lining, bricks laid flat; slope of bottom = .0049; D. & B. 3; *R. H.* 361.

V	2.75	3.66	4.18	4.72	5.10	5.33	5.68	6.01	6.15	6.47	6.60	6.72
R	.192	.284	.365	.424	.481	.540	.582	.620	.668	.697	.739	.779
C	90	98	99	104	105	104	106	109	107	111	110	109

(8) **Rectangular**; width, 6.01′; lining, small gravel (1 to 2 cm.) set in cement mortar; slope of bottom = .0049; D. & B. 4; *R. H.* 364.

V	2.16	2.95	3.40	3.84	4.14	4.43	4.64	4.88	5.12	5.26	5.43	5.57
R	.250	.357	.450	.520	.588	.644	.700	.746	.785	.832	.872	.910
C	62	71	73	76	77	79	79	81	83	82	83	83

(9) **Rectangular**; width, 6.106′; lining, gravel (3 to 4 cm.) set in cement mortar; slope of bottom = .0049; D. & B. 5; *R. H.* 367.

V	1.79	2.43	2.90	3.27	3.56	3.85	4.03	4.23	4.43	4.60	4.78	4.90
R	.291	.417	.510	.587	.656	.712	.772	.823	.867	.909	.946	.987
C	48	54	58	61	63	65	66	67	68	69	70	70

(10) **Trapezoidal**; bottom width, 3.281′, top width, 6.56; side batter, 1 to 1; lining, planks; slope of bottom = .0015; D. & B. 21; *R. H.*, 417.

V	2.39	2.93	3.35	3.62	3.85	4.03	4.20	4.39	4.51	4.64	4.76	4.87
R	.334	.485	.586	.673	.744	.809	.864	.911	.959	1.002	1.047	1.097
C	107	109	113	114	115	116	117	119	119	120	120	120

(11) **Triangular**; side batter, 1 to 1; lining, planks; slope of bottom = .0049; D. & B. 23, *R. H.* 423.

V	4.13	5.02	5.56	6.03	6.36	6.59	6.83	7.03	7.23	7.40	7.54	7.75
R	.327	.422	.494	.549	.597	.643	.683	.719	.752	.783	.814	.839
C	103	110	113	116	118	117	118	118	119	119	119	121

(12) and (13) **Trapezoidal**; nearly rectangular, bottom width, 5.91′; side batter, 1 to 10; lining, rubble masonry pointed with cement; side walls in better condition than bottom, which was broken in places, especially in the lower portions, having a slope of 0.037; furthermore, the bottom was covered with a light deposit of sticky mud. *R. H.* 446–447.

	D. & B. 32. Bottom slope = .101					D. & B. 33. Bottom slope = .037			
V	12.3	16.2	18.7	21.1	*V*	9.04	11.5	13.6	15.1
R	.324	.467	.580	.662	*R*	.424	.620	.745	.852
C	68	75	77	82	*C*	72	76	82	85

(14) and (15) al; bottom width, 5.91′ to 6.56′; side batter, 3 to
ining, dry maso .. er poor condition; two series on the same channe
R. H. 446–447.

D. & B. 8
Walls completel, v red with vegetable growths
Bottom slope = .0146

V	4.19	5.75	7.20	8.27	8.99
R	.856	1.087	1.383	1.586	1.694
C	38	46	51	54	57

D. & B. 35
Walls carefully cleaned
Bottom slope = .0142

V	5.66	7.36	8.94	10.1	11
R	.703	.930	1.227	1.394	1.4
C	57	64	68	72	7

(16) and (17) **Trapezoidal**; side batter about 2.3 to 1; lining, rough eart
R. H., 454–446; two parts of same channel

D. & B. 49
Bottom width, 6.44′
Bottom and slopes rough but quite regular and completely free of vegetable growth
Slope of bottom = .000367

V	.89	1.34	1.36	1.47
R	.96	1.32	1.57	1.78
C	57	70	69	66

D. & B. 50
Bottom width, 6.29′
B om and slopes rough with con- siderable vegetable growth
Slope of bottom = .00033

V	.82	1.26	1.30	1 41
R	1.05	1.42	1.65	1 85
C	45	62	56	57

(18) **Sudbury Aqueduct** of the Metropolitan W. W., Boston; horseshoe shape; 9.0′ wide × 7.7′ high (inside dimensions); hard brick lining, well laid. Experiments of Fteley & Stearns. *Trans. Am. Soc. C. E.*, vol. 12, pp. 114–117; the **aqueduct being new**. Bottom slope = .000189.

Values of *C* for various values of *R*

R	.5	.6	.7	.8	.9	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4
C	117	119	122	124	125	127	130	132	134	136	138	139	140

Limits of experimental velocities, .5 to 2.33.

The following formula will give results identical with the valu immediately preceding:

$$V = 127\, R^{0.62}\, S^{0.5}.$$

In a portion of the conduit (600 feet long) chosen on accou of its uniformity of section and grade, the surface was scrap clean; in this case the flow was represented by the formula:

$$V = 129\, R^{0.62}\, S^{0.5}.$$

In a portion of the conduit lined with a coating of neat cem mortar, the following formula very nearly represented the flov

$$V = 137\, R^{0.62}\, S^{0.5}.$$

In tunnel through rock (4614 feet long) in which the rock sides had been left ragged for 4362 feet without lining, but with smooth concrete floor, the rock excavation being at no place less than 2 feet wider than the brick-lined section, the following formula very nearly represented the flow:

$$V = 96\, R^{0.62}\, S^{0.5}.$$

After 20 years in service with a collection of one year's slime the flow was represented by:

$$V = 129\, (RS)^{\frac{1}{2}}.$$

One month later, after being cleaned:

$$V = 143\, (RS)^{\frac{1}{2}}.$$

(19) **Croton Aqueduct**, New York Waterworks; horseshoe shape, 13.6′ wide × 13.53′ high for about 125,000 feet, of circular form; 12.25′ diameter for about 35,000 feet, and for short distances somewhat varied; hard brick lining well laid; *Aqueduct Commissioners' Report*, 1895, p. 97. Slope of bottom = .00013257. Values of C for various values of R.

R	1.0	1.5	2.0	2.5	3.0	3.5	4.0
C	119	126	130	132	133	134	134

The flow was represented by the formula:

$$V = 124\, R^{0.56}\, S^{0.5}.$$

After $9\frac{1}{2}$ years' use a measurement reported by J. R. Freeman at

$V = 2.3$ gave $C = 105$ (in the Chezy formula).

(20) **Lake Katrine Aqueduct**, Glasgow, Scotland, Corporation Waterworks; horseshoe shape, straight, 5763 feet long; 10′ wide × 9′ high, lined with concrete for about .53 of its length, and unlined rock 12′ wide × 9′ high, with concrete bottom, for .47 of its length.

Experiments by A. F. Bruce, *Proc. Inst. of C. E.*, vol. 123, p. 414.

Bottom slope = .0001818

V	1.871	2.069	2.108	2.216	2.128	2.155	2.175	2.206	2.233
R	1.227	1.471	1.471	1.490	1.498	1.500	1.547	1.599	1.607
C	125	127	129	135	129	130	130	129	131
n	.0123	.0125	.0123	.0119	.0124	.0123	.0124	.0125	.0125

V	2.226	2.245	2.255	2.259
R	1.611	1.621	1.627	1.740
C	130	131	131	127
n	.0125	.0124	.0124	.0129

(21) **Brick Sewers**; Metropolitan Sewerage Board, Boston; hard lining. Experiments by R. E. Horton, *Trans. Am. Soc. C. E.*, vol. 46, Shape, basket-handle; 6′ wide × 6.67′ high; lining, brick, cement washe

Bottom slope = .0005

1896 after 10 months' use	V	1.99	2.46	2.82	3.13	3.44
	R	.688	.958	1.187	1.387	1.539
	C	107	112	115	118	124
	n	.0129	.0131	.0132	.0133	.0130

1897 after 2 years 2 months' use	V	2.97	3.16	1900 after 3 years 9 months' use	V	2.66	2.86
	R	1.54	1.65		R	1.342	1.508
	C	107	111		C	102	104
	n	.0149	.0147		n	.0151	.0152

Shape, circular, 9 feet in diameter; same lining. Slope of bottom = .00

1896	V	**1.58**	**2.21**	**2.70**	**3.03**	**3.48**	**3.73**	**4.18**
	R	**.619**	**.928**	**1.208**	**1.408**	**1.830**	**1.999**	**2.309**
	C	**110**	**126**	**134**	**139**	**141**	**145**	**150**
	n	**.0122**	**.0117**	**.0116**	**.0115**	**.0117**	**.0116**	**.0115**

1897	V	**2.55**	**2.90**	**3.06**	**3.18**	**1900**	V	**2.38**	**2.82**	**3.16**
	R	**1.280**	**1.560**	**1.762**	**1.771**		R	**1.120**	**1.606**	**1.952**
	C	**123**	**127**	**126**	**131**		C	**119**	**121**	**124**
	n	**.0126**	**.0127**	**.0129**	**.0126**		n	**.0130**	**.0132**	**.0133**

TABLE LVIII*

VALUES OF C BY KUTTER'S FORMULA

$s = .0010$

Mean Hydraulic Radius R	Coefficient of Rugosity, n													
	.009	.01	.011	.012	.013	.015	.017	.02	.025	.03	.035	.04	.045	.05
Feet														
.1	108	94	82	73	65	53	45	35	26	20	16	14	12	10
.2	130	113	100	89	80	66	56	45	34	26	21	18	16	14
.3	142	124	111	99	89	75	64	52	39	31	25	21	18	16
.4	151	132	118	106	96	80	69	56	42	34	28	24	20	18
.6	162	143	128	115	105	89	77	63	48	39	32	27	24	21
.8	170	150	135	122	112	95	82	68	52	42	35	30	26	23
1	175	156	140	127	117	99	86	71	55	45	38	33	28	25
1.5	185	166	150	136	125	108	94	78	61	50	43	37	33	29
2	191	171	155	141	130	112	98	83	65	54	46	40	35	31
3	199	180	163	149	138	119	105	89	71	59	51	44	39	35
3.28	201	181	165	151	139	121	107	91	72	60	52	45	40	36
4	205	184	168	154	143	124	110	93	75	63	54	48	42	38
6	211	190	174	160	149	130	115	99	80	68	59	52	46	42
8	215	195	178	165	153	134	119	103	84	71	62	55	50	45
10	218	198	181	167	155	137	122	105	87	74	64	58	52	47
15	222	202	186	171	160	141	127	110	91	78	68	62	56	51
20	226	205	188	174	163	144	129	113	94	80	72	64	59	54
30	229	209	192	178	167	147	133	116	97	84	75	67	62	58
40	231	211	194	180	169	150	135	119	99	87	77	70	65	60
60	233	213	197	183	171	153	138	121	102	89	81	73	67	63
80	235	215	198	184	173	154	139	122	104	91	82	75	70	65

* Computed; but checked from Trautwine's *Pocket Book*, and *The Flow of Water*, etc.

TABLE LVIII

Values of C by Kutter's Formula

$s = .0002$

Mean Hydraulic Radius R	Coefficient of Rugosity, n													
	.009	.01	.011	.012	.013	.015	.017	.02	.025	.03	.035	.04	.045	.05
Feet														
.1	99	86	75	66	59	48	41	32	24	18	15	13	11	9
.15	112	97	85	76	68	56	47	38	28	22	18	15	13	11
.2	121	105	92	83	74	61	52	42	31	24	20	17	14	13
.3	134	117	104	93	84	70	59	48	36	29	24	20	17	15
.4	143	126	112	101	91	76	65	53	40	32	26	22	19	17
.6	156	138	123	111	101	85	73	60	46	37	31	26	23	20
.8	164	146	131	118	108	91	79	65	50	41	34	29	25	22
1	171	152	137	124	113	96	83	69	54	44	37	31	27	24
1.5	182	163	147	134	123	105	92	77	60	49	41	36	32	28
2	189	170	154	140	129	111	97	82	64	54	45	39	35	31
3	200	179	163	149	137	119	105	89	72	59	51	45	40	36
3.28	201	183	165	151	139	121	107	91	72	60	52	45	40	36
4	205	185	168	155	143	125	111	94	76	63	55	48	43	38
6	213	193	176	163	151	132	117	100	82	69	60	53	47	43
8	218	198	181	168	155	136	122	105	86	73	64	57	51	46
10	222	202	185	171	159	140	125	108	89	76	66	59	53	49
15	228	207	190	176	164	145	130	114	94	81	72	64	58	[illegible]
20	231	209	194	181	168	149	134	117	98	85	75	67	62	[illegible]
30	236	215	198	184	172	153	139	122	103	89	79	72	66	61
40	237	218	201	187	175	156	142	125	105	92	82	75	68	64
60	242	221	204	190	179	160	145	128	109	95	86	78	73	[illegible]
80	243	222	206	193	180	162	147	130	111	98	88	81	75	70

TABLE LVIII

VALUES OF C BY KUTTER'S FORMULA

$s = .0001$

Mean Hydraulic Radius R	Coefficient of Rugosity, n													
	.009	.01	.011	.012	.013	.015	.017	.02	.025	.03	.035	.04	.045	.05
Feet														
.1	91	78	69	61	54	44	37	30	22	17	14	12	10	9
.15	103	90	79	70	63	51	43	35	26	20	17	14	12	11
.2	112	98	87	77	69	57	48	39	29	23	19	16	14	12
.3	125	110	98	88	79	65	56	45	34	27	22	19	16	14
.4	136	119	106	96	86	71	61	50	38	30	25	21	18	16
.6	150	132	118	106	96	81	70	57	44	35	29	25	22	19
.8	159	141	126	114	104	88	76	63	48	39	32	28	24	21
1	166	147	132	120	109	93	81	67	52	42	35	31	27	24
1.5	180	160	144	131	120	103	90	75	59	48	41	35	31	28
2	188	168	151	138	127	109	96	81	64	52	45	39	34	30
3	198	178	162	149	137	119	104	89	71	59	51	45	39	35
3.28	201	181	165	151	139	121	107	90	72	60	52	45	40	36
4	206	186	169	155	143	125	111	94	76	64	55	49	43	39
6	215	195	178	164	152	133	118	102	83	70	61	54	48	44
8	220	201	185	170	158	139	124	107	88	75	65	58	52	47
10	226	205	188	174	162	143	128	111	92	78	69	61	55	50
15	233	212	195	182	169	150	135	118	98	84	74	67	61	56
20	237	217	200	186	174	154	139	122	102	89	79	71	64	59
30	243	222	206	191	179	160	145	128	107	94	84	76	70	65
40	248	226	210	195	183	163	149	132	111	98	88	80	73	68
60	251	230	213	199	188	168	153	136	115	103	92	85	78	74
80	254	232	215	202	190	171	156	139	119	106	96	88	81	76

TABLE LIX

VALUES OF C BY BAZIN'S NEW FORMULA

MEAN HYDRAULIC RADIUS R FEET	CLASS I Very Smooth Surfaces; Neat Cement; Planed Wood $\Gamma = .109$	CLASS II Smooth Surfaces; Planks; Bricks; Ashlar $\Gamma = .290$	CLASS IIIa Rough Surfaces; Rubble Masonry $\Gamma = .833$	CLASS IIIb Canals with Mixed Linings; Very Regular Earth, or Paved with Stones $\Gamma = 1.54$	CLASS IV Earth Canals in Ordinary Condition $\Gamma = 2.35$	CLASS V Earth Canals in Bad Condition $\Gamma = 3.17$
.1	117	82	43	27	19	14
.2	127	96	55	35	25	19
.3	131	103	63	41	30	23
.4	135	108	68	46	33	26
.6	138	115	76	53	39	31
.8	141	119	82	58	43	35
1.0	142	122	86	62	47	38
1.50	145	127	94	70	54	44
2.0	146	131	99	75	59	49
3.0	148	135	106	83	67	56
4.0	150	138	111	89	72	61
6.0	151	141	118	97	80	69
8.0	152	143	122	102	86	74
10.0	152	145	125	106	90	79
15.0	153	147	130	113	98	86
20.0	154	148	133	117	103	92
30.0	155	150	137	123	110	100
40.0	155	151	139	127	115	105
50.0	155	151	141	129	118	109
60.0	155	152	142	131	121	112

Problems

1. Compute the mean hydraulic radii (a) for a circular channel 2 fee diameter when flowing at the following depths: ¼ foot; 1 foot; 1½ f and full; and (b) for a channel 2 feet square when flowing at the same dept

2. Compute the mean velocity and the mean hydraulic radius of trapezoidal channel 5 feet wide at the bottom, and having side slopes 2 to when running 1.96 feet deep and discharging 50 cubic feet per second.

3. A wooden flume 2500 feet long, 6 feet by 6 feet, made of plank la lengthwise; slope = .0002. (a) What is the discharge, when the me hydraulic radius is 1.41? (b) What velocity?

uld be the maximum discharge through a 20-inch vitrified on a slope of 0.0003? What will be discharge when R is

the discharge in cubic feet per second through a brick-wer, 18 inches in diameter, grade 1 in 200, when flowing ıches.

neter of circular sewer, if of concrete, would be needed on a to carry 1200 cubic feet per minute at its maximum flow?

neter of circular sewer, if of brick, would be required on a to carry 150 cubic feet per second, flowing ½ full?

grade should be given a 20-inch vitrified pipe sewer to ıbic feet per minute when flowing ½ full? (*b*) For the compute the maximum discharge.

the discharge through a rectangular channel; bottom of s of rubble masonry. Depth of water, 10 feet; mean width ope, 1 in 5000.

ctangular channel, lined with planks, 11 feet wide, maximum feet. When free from ice, the discharge is 7.48 million Find the mean velocity and how much the discharge is s formed on top, but the depth of the water remains the

canal with bottom 80 feet wide in gravel, side slope, 2 to 1. 2 feet in a mile. Depth of water, 7 feet. What will be the feet per second?

tunnel is made of hard brick, carefully pointed. It is ter and 7166 feet long. (*a*) When flowing full of sewage hydraulic grade line was .079 per 1000, and $n = .0195$. lischarge? (*b*) When carrying ¾ clear water, ¼ sewage; 1000, $n = .014$. What was the discharge?

to be built to discharge 1000 cubic feet per second. Com-ss section having a bottom slope of .0002: (*a*) in gravel, ectangular with dry rubble bottom and sides, and (*d*) rectan-te bottom and side walls. Determine whether, in (*b*), scouring

CHAPTER XVII

DYNAMIC ACTION OF FLOWING WATER

419. Definitions and nomenclature.

Absolute velocity is velocity relative to a point on the earth's surface.

Relative velocity is velocity relative to a point on some body stationary or moving; if the body is stationary, relative velocity is the same as absolute velocity.

Direction of forces and motion will be stated in degrees from 0° to 360° measured from the zero end of the X-axis.

For a stationary solid the X-axis will be horizontal; and angles will read contra-clockwise, as in trigonometry.

For motion in a straight line, the X-axis will be the line of motion of the solid; and the direction of its motion will be 0°.

For circular (or curvilinear) motion, the X-axis will be a tangent to the path of motion of the solid at the points where the water enters or leaves the solid; and the direction of its motion will be 0°.

In certain types of water motors, the water enters and leaves the vane at practically a constant distance from the center of rotation. In other types the points of entrance and exit are at different distances from the center of rotation; in this latter case there will be two sets of axes of reference.

Forces and motion are assumed to be in, or reduced to, a common plane, or parallel planes.

Forces will be expressed in pounds; velocities, in feet per second; areas, in square feet; heads, in feet; intensity of pressure, in pounds per square inch measured from atmospheric pressure; work, in foot pounds per second.

F = a constant force producing a change in velocity, direction, position, or pressure.

F_X and $F_Y = X$ and Y components of F.

$P =$ a constant force equal in magnitude to F, but opposite in direction $= F \cos 180° = F \times -1$.

P_X and $P_Y = X$ and Y components of P.

$V_0 =$ the component of the original velocity parallel to the line of action of F.

$V_1 =$ the component of the final velocity parallel to the line of action of F.

$\frac{\delta v}{\delta t} =$ the constant acceleration or retardation in feet per second per second parallel to the line of action of F.

$V_A =$ absolute mean velocity in a channel of approach.

$V_a =$ absolute mean velocity from guide vanes or nozzles; absolute entrance velocity.

$V_e =$ absolute mean velocity from a wheel vane or vessel; absolute exit velocity.

$v_\beta =$ relative mean velocity at entrance to a vane or vessel.

$v_\delta =$ relative mean velocity at exit from a vane or vessel.

$S =$ velocity of the point of application of a force.

$\alpha =$ angle of absolute velocity at entrance, direction of V_a.

$\beta =$ angle of relative velocity at entrance, direction of v_β.

$\delta =$ angle of relative velocity at exit, direction of v_δ.

$\epsilon =$ angle of absolute velocity at exit, direction of V_e.

a_A, a_a, a_β, and $a_\delta =$ cross-sectional areas of a stream or jet measured at right angles to the direction of flow; each area corresponding in the order named to the velocities V_A, V_a, v_β, and v_δ.

$E =$ Energy expended, work done, in foot pounds per second.

$W =$ Weight of a solid, or of volume of water, in pounds.

$w = 62.4$ pounds, weight of a cubic foot of water.

$Q =$ volume of flow in cubic feet per second.

$g =$ acceleration due to gravity in feet per second per second, $= 32.16$.

20. The force required to produce acceleration or retardation. onstant force F which, acting upon a body weighing W, will

cause an acceleration or retardation, δv, in an interval of time, δt, may be found as follows:

$$F=\frac{W}{g}\frac{\delta v}{\delta t}=\frac{W}{g}\frac{(V_1-V_0)}{t}. \tag{1}$$

Or $$F\delta t=\frac{W}{g}\delta v;\text{ and } Ft=\frac{W}{g}(V_1-V_0). \tag{2}$$

The impulse, therefore, which is the product of the force multiplied by the time during which it acts, equals the change in momentum in the same time. If the time be one second, Ft, the impulse, equals F, the force itself.

A stream of water, or other fluid, impinging upon a solid body which is stationary, or moving at a different velocity or in a different direction than the stream, will by the impact or collision have its velocity or direction changed; and in the process impart to the body itself a force or pressure, which is equal in magnitude, but opposite in direction, to the force (frequently called the reaction) which is exerted by the body upon the stream.

In hydraulic problems dealing chiefly with continuous streams, not isolated bodies of water, the exact interval of time in which the change in velocity or direction occurs can rarely be determined. It is only necessary, however, to consider the weight of the mass of water which acts, or is acted upon, in one second.

If Q is the rate of discharge of a stream in cubic feet per second: then W in equation (2) corresponding to a time (t) of one second equals wQ pounds per second.

Then equation (2) becomes $$F=\frac{wQ}{g}(V_1-V_0). \tag{3}$$

F is the constant force which must be exerted on the water to change the velocity of Q cubic feet per second from V_0 to V_1 feet per second.

V_0 and V_1 are the components of the original and final velocity in the line of action of the force F.

421. The components of F for any given set of axes. The horizontal or X components of the original and final velocity are:

for absolute velocities $V_0=V_a\cos\alpha$, and $V_1=V_\epsilon\cos\epsilon$; (4)

for relative velocities $V_0=v_\beta\cos\beta$, and $V_1=v_\delta\cos\delta$. (5)

The horizontal or X component of F, by (3) is:

$$F_X = \frac{wQ}{g}(V_\epsilon \cos \epsilon - V_a \cos \alpha), \text{ in terms of absolute velocity; } \quad (6)$$

$$\text{or } F_X = \frac{wQ}{g}(v_\delta \cos \delta - v_\beta \cos \beta), \text{ in terms of relative velocity.} \quad (7)$$

The vertical or Y components of the original and final velocities are:

for absolute velocities

$V_0 = V_a \sin \alpha$, and
$V_1 = V_\epsilon \sin \epsilon$; (8)

for relative velocities

$V_0 = v_\beta \sin \beta$, and
$V_1 = v_\delta \sin \delta$. (9)

The vertical or Y component of F, by (3) is:

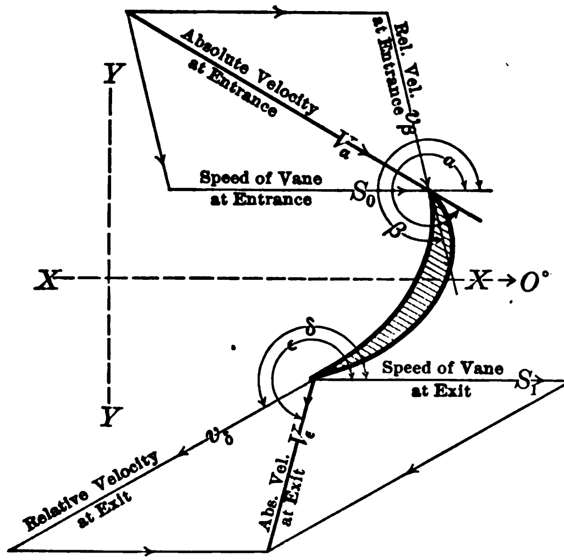


FIG. 142. — Velocity Diagrams for a Moving Vane.

$$F_Y = \frac{wQ}{g}(V_\epsilon \sin \epsilon - V_a \sin \alpha), \text{ in terms of absolute velocity; } \quad (10)$$

$$\text{or } F_Y = \frac{wQ}{g}(v_\delta \sin \delta - v_\beta \sin \beta), \text{ in terms of relative velocity.} \quad (11)$$

422. The force or pressure exerted on a body in consequence of changes in motion of moving water, in contact with a body, necessarily implies a resistance equal in magnitude, opposite in direction, and having a common point of application. The body must therefore have sufficient strength in its material to withstand the stresses set up by the force, and must supply in itself or transmit from other external forces resistance equal to the force exerted.

The force exerted upon moving water to change its velocity or direction sets up an equal and opposite force which is exerted on the body by the water; and its value,

$$P = F \cos 180° = F \times (-1) = \frac{wQ}{g}(V_0 - V_1). \quad (12)$$

423. The components of P for any given set of axes.

The horizontal component,

$$P_X = \frac{wQ}{g}\left(V_a \cos \alpha - V_e \cos \epsilon\right), \text{ in terms of absolute velocity; } \quad (13)$$

$$= \frac{wQ}{g}\left(v_\beta \cos \beta - v_\delta \cos \delta\right), \text{ in terms of relative velocity.} \quad (14)$$

The vertical component,

$$P_Y = \frac{wQ}{g}\left(V_a \sin \alpha - V_e \sin \epsilon\right), \text{ in terms of absolute velocity; } \quad (15)$$

$$= \frac{wQ}{g}\left(v_\beta \sin \beta - v_\delta \sin \delta\right), \text{ in terms of relative velocity.} \quad (16)$$

424. The equations above derived (1 to 16) are general, and applicable either to stationary or moving bodies.

Either absolute or relative velocities may be used in computing forces or work due to changes in flowing water while in contact with a body. *The difference between the components of original and final absolute velocities* with reference to a given axis *equals the difference between the components of original and final relative velocities;* since the component of the motion of the body with reference to the same axis has a constant and an equal effect on each difference. For convenience and uniformity final results will herein as far as possible be expressed in terms of absolute velocities.

Inspection of the equations (1) to (16) shows:

That the magnitude of the force F or P depends upon:

The weight of water delivered every second by the stream;

The change in velocity and direction at entrance and exit; viz. the algebraic difference between the components of original and final velocity parallel to the direction of F or P;

That the magnitude of F or P is independent of the radius of curvature of the path of the stream during contact with a body; except in so far as the smoothness and gradual curvature decrease friction losses, and in so far as curvature determines the direction of entrance and exit velocities; hence theoretically the path may be simply bent at an angle, not curved;

That the direction of the force F or P depends upon the algebraic sign (+ or −) of the algebraic difference between the components of original and final velocities with reference to the given axes.

If an X component force has a plus sign, its direction is 0°; if a minus sign, its direction is 180°; if a Y component force has a plus sign, its direction is 90°; if a minus sign, its direction is 270°. The direction of F or P may be computed from the magnitudes of its X and Y components.

The point of application of the force F or P must be somewhere on the solid between the entrance and exit points of the stream. If there is a well-defined place through which the resistance is transmitted, that place would be the point of application; otherwise it will usually be indeterminate.

425. The effect of a force. If a body is at rest and opposes a force due to changes in velocity or direction of impinging water with sufficient resistance to overcome the tendency to move the body, the force will merely *set up stresses* tending to deform the body; if, however, the resistance is insufficient to prevent motion, the force will not only set up stresses, but will also impart to the body an onward *motion.*

If the body is already in motion, the force due to a change in velocity or direction will in addition to setting up stresses which tend to deform the body, impart to it a *change of motion.*

426. The work done by moving water is due to changes in velocity or direction, or both. If a force is exerted against a resistance and their common point of application is moving, work is done; that is, energy is expended. If St is the distance traveled by the point of application during the period of time (t) of the exertion of a force F or P,

The work done $= FSt = PSt$ (foot pounds).

NOTE: t in most problems concerning the work done by flowing water is *one second;* in such cases, the distance $St = S$, and the work done $FSt = FS$.

427. Work done in changing velocity. If a stream in a contact with a constant resistance, which is moving at the rate of S feet per second, has its velocity changed from V_0 to V_1, the work done in one second against the stream equals

$$E = FS = \frac{wQ}{g}(V_1 - V_0)S; \qquad (17)$$

or the work done by the stream equals

$$E = PS = \frac{wQ}{g}(V_0 - V_1)S. \qquad (18)$$

In terms of kinetic energy,

$$E = FS = wQ\left(\frac{V_1^2}{2g} - \frac{V_0^2}{2g}\right), \text{ whence } F = \frac{wQ}{S}\left(\frac{V_1^2}{2g} - \frac{V_0^2}{2g}\right); \quad (19)$$

or, $$E = PS = wQ\left(\frac{V_0^2}{2g} - \frac{V_1^2}{2g}\right); \text{ whence } P = \frac{wQ}{S}\left(\frac{V_0^2}{2g} - \frac{V_1^2}{2g}\right). \quad (20)$$

If V_0 is greater than V_1, the moving water must have parted with or transferred some of its energy, and if it is transmitted to a suitable machine, this energy may be transformed into useful work, as in the case of water motors.

If V_1 is greater than V_0, energy must have been transferred to the water from some other source, as in the case of pumps.

428. Work done in changing elevation. If wQ pounds of water fall from a higher to a lower elevation against a constant moving resistance, or are raised from a lower to a higher elevation, work is performed; in the first instance by the force of gravity in the water, or in the second instance against the force of gravity. The work done in one second against the stream equals

$$E = FS = wQ(h_1 - h_0); \text{ whence } F = \frac{wQ}{S}(h_1 - h_0); \quad (21)$$

or the work done by the stream equals

$$E = PS = wQ(h_0 - h_1); \text{ whence } P = \frac{wQ}{S}(h_0 - h_1). \quad (22)$$

h_0 = the original elevation above a datum plane (in feet).
h_1 = the final elevation above the same datum plane (in feet).

429. Work done in changing intensity of pressure. If wQ pounds of water per second flowing in a closed vessel have its intensity of pressure increased or diminished against a constant moving resistance, the work done against the stream equals

$$E = FS = wQ\left(\frac{p_1}{\gamma} - \frac{p_0}{\gamma}\right); \text{ whence } F = \frac{wQ}{S}\left(\frac{p_1}{\gamma} - \frac{p_0}{\gamma}\right); \quad (23)$$

or the work done by the stream equals

$$E = PS = wQ\left(\frac{p_0}{\gamma} - \frac{p_1}{\gamma}\right); \text{ whence } P = \frac{wQ}{S}\left(\frac{p_0}{\gamma} - \frac{p_1}{\gamma}\right). \quad (24)$$

p_0 = original intensity of pressure;
p_1 = final intensity of pressure.

430. Equivalent expressions for work. The force exerted or the energy expended in consequence of changes in velocity, direc-

tion, elevation, or pressure of moving water may be expressed according to the conditions in which it exists in terms of velocity, head of water, or intensity of pressure, or in terms of two or more forms. All expressions may be freely interconverted to facilitate computation by the equivalent expressions in feet;

$$\frac{V^2}{2g} = h = \frac{p}{\gamma}.$$

If a stream divides upon entering a channel, or follows diverging paths through a channel upon its departure, for instance, in a complex waterwheel runner, unless the divergence is symmetrical, each subdivision of the stream must be computed independently. This makes the computations for work done by certain kinds of turbines very complicated.

EXAMPLES

The following examples will illustrate the manner of applying the principles expressed by equations (1) to (24) to vanes and channels of various forms.

431. Flow through a vessel. In a vessel such as shown in figure 143, water so entering that it will flow vertically is changed in direction and velocity and issues horizontally through an orifice. The forces necessary to cause these changes, and resulting therefrom, are to be computed. Neglect friction.

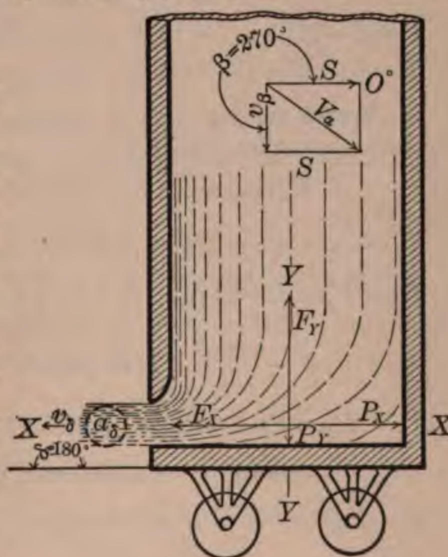


FIG. 143.

A. Vessel stationary. Let $v_\delta = 30$; $a_\beta = 10$; $a_\delta = 1$; $\beta = 270°$; $\delta = 180°$; $Q = a_\beta v_\beta = a_\delta v_\delta$. Since the vessel is held stationary, absolute and relative velocities are identical, $S = 0$; hence

$$V_a \cos \alpha = v_\beta \cos \beta; \quad V_\epsilon \cos \epsilon = v_\delta \cos \delta;$$

$$Q = 1 \times 30 = a_\beta v_\beta; \text{ and } v_\beta = \tfrac{30}{10} = 3; \text{ then}$$

$$F_X \text{ (equation 7)} = \frac{wQ}{g}(v_\delta \cos 180° - v_\beta \cos 270°)$$

$$= \frac{wa_\delta v_\delta}{g}[(v_\delta \times -1) - (v_\beta \times 0)] = -\frac{wa_\delta v_\delta^2}{g}$$

$$= -\frac{62.4 \times 1 \times 30^2}{32.16} = -1746$$

$$= 1746 \text{ pounds, direction } 180°.$$

$$P_X = F_X \cos 180° = (-1746) \times (-1) = +1746$$

$$= 1746 \text{ pounds, direction } 0°.$$

$$F_Y \text{ (equation 11)} = \frac{wQ}{g}(v_\delta \sin 180° - v_\beta \sin 270°)$$

$$= \frac{wa_\beta v_\beta}{g}[(v_\delta \times 0) - (v_\beta \times -1)] = +\frac{wa_\beta v_\beta^2}{g}$$

$$= +\frac{62.4 \times 10 \times 3^2}{32.16} = +175$$

$$= 175 \text{ pounds, direction } 90°.$$

$P = (+175) \times (-1) = -175 = 175$ pounds, direction 270°.

$E = 0$; since the vessel is stationary, no work is done.

B. **Vessel moving with a velocity $S = 10$ feet per second, 0°, against a constant resistance.**

$$F_X \text{ (equation 6)} = \frac{wQ}{g}(V_\epsilon \cos \epsilon - V_a \cos \alpha).$$

$$V_a \cos \alpha = v_\beta \cos 270° + S \cos 0°; \quad V_\epsilon \cos \epsilon = v_\delta \cos 180° + S \cos 0$$

$$\text{Then } F_X = \frac{wa_\delta v_\delta}{g}[(v_\delta \times -1) + (S \times 1) - (v_\beta \times 0) - (S \times 1)]$$

$$= -\frac{wa_\delta v_\delta^2}{g} = 1746 \text{ pounds, direction } 180°;$$

and $$P_X = F_X \cos 180 = (F_X \times -1) = +\frac{wa_\delta v_\delta^2}{g}$$

$$= 1746 \text{ pounds, direction } 0°.$$

$$E = P_X S = \frac{wa_\delta v_\delta^2}{g} S = 1746 \times 10 = 17{,}460 \text{ foot pounds per second}$$

$$F_Y \text{(equation 10)} = \frac{wQ}{g}(V_\epsilon \sin \epsilon - V_a \sin \alpha).$$

$$V_a \sin \alpha = v_\beta \sin 270° + S \sin 0°; \quad V_\epsilon \sin \epsilon = v_\delta \sin 180° + S \sin 0$$

$$n\ F_Y = \frac{wa_\beta v_\beta}{g}[(v_\delta \times 0) + (S \times 0) - (v_\beta \times -1) - (S \times 0)]$$

$$= + \frac{wa_\beta v_\beta^2}{g} = + \frac{62.4 \times 10 \times 3^2}{32.16} = 175 \text{ pounds, direction } 90°.$$

$$P_Y = F_Y \cos 180° = (F_Y \times -1) = - \frac{wa_\beta v_\beta^2}{g}$$

$$= 175 \text{ pounds ; direction } 270°.$$

$E = P_Y S = 0$; no work done in a vertical direction, since vessel can not or does not e vertically.

2. **A stream or jet of water nging upon a flat plate or without boundaries.** A jet ater of cross-sectional area ıaving a velocity (V_a, 0°), nges upon a flat plate set ıal to the direction of flow. me water to flow symmetri- over the plate and leave it direction parallel to the sur-

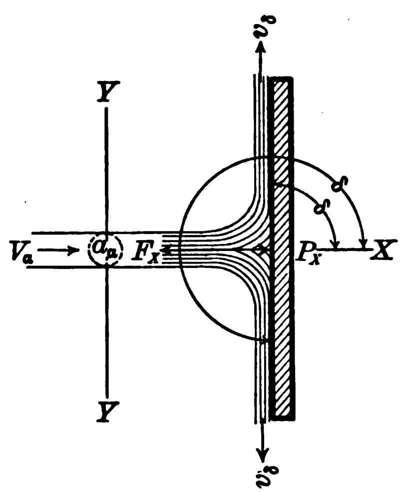


FIG. 144.—Flat Vane, Stationary.

ompute the X and Y forces ted upon the plate by the ɜr, and the work done. Neg- friction.

. **The vane is stationary.** See figure 144.

$_a = .5$; $V_a = 40$; $\alpha = 0°$; and as the stream is symmetrical, ay be taken as 90° for the upper $\frac{Q}{2}$, and as 270° for the er $\frac{Q}{2}$;

$Q = a_a V_a$; as the vane is stationary, $V_\epsilon = v_\delta = V_a$.

$$P_X \text{ (equation 13)} = \frac{wQ}{g}(V_a \cos \alpha - V_\epsilon \cos \epsilon).$$

$V_a \cos \alpha = V_a \cos 0°$; $V_\epsilon \cos \epsilon = v_\delta \cos \delta = V_a \cos 90°$.

$$n\ P_X = \frac{wa_a V_a}{g}[(V_a \times 1) - (V_a \times 0)] = + \frac{wa_a V_a^2}{g}$$

$$= + \frac{62.4 \times .5 \times 40^2}{32.16} = 1552 \text{ pounds, direction } 0°.$$

P_Y (equation 15) $= \frac{wQ}{g}(V_a \sin \alpha - V_\epsilon \sin \epsilon).$

$V_a \sin \alpha = V_a \sin 0°$; $V_\epsilon \sin \epsilon = V_a(\sin 90° + \sin 270°) = V_a \times 0.$

Then $P_Y = \frac{wa_a V_a}{g}[(V_a \times 0) - (V_a \times 0)] = 0.$

$E = 0$, since the vane is stationary.

B. **A single flat vane moving** against a resistance with a velocit (20, 0°) in a straight line; other data same as in A. See figu 145.

P_X (equation 13) $= \frac{wQ}{g}(V_a \cos \alpha - V_\epsilon \cos \epsilon).$

$v_\beta = V_a - S$; and as, neglecting losses on the vane, there sho

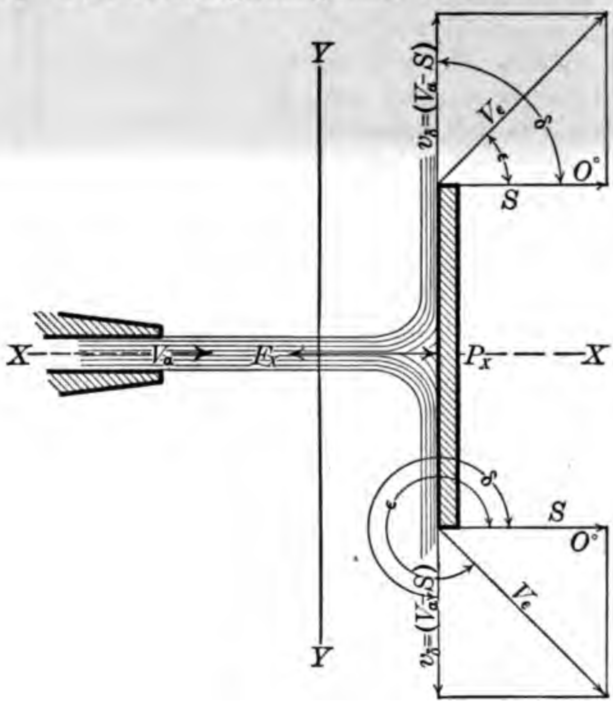


Fig. 145.—Flat Vane, Moving.

be no change in relative velocity in passing over the va v_δ also $= V_a - S.$

$Q = a_a(V_a - S)$; as the vane is moving, only part of the stream strikes it;

$$V_a \cos \alpha = V_a \cos 0° ; \; V_\epsilon \cos \epsilon = (V_a - S) \cos 90° + S.$$

$$\text{Then } P_X = \frac{wa_a(V_a - S)}{g}[(V_a \times +1) - (V_a - S) \times 0 - S]$$

$$= + \frac{wa_a(V_a - S)^2}{g} = + \frac{62.4 \times .5 \times (40 - 20)^2}{32.16}$$

$$= 388 \text{ pounds, direction } 0°.$$

$P_Y = 0$; because the resultant of the vertical components is zero.

$$E = P_X \times S = \frac{wa_a(V_a - S)^2}{g} S = 388 \times 20$$

$$= 7760 \text{ foot pounds per second.}$$

Maximum work for a single flat vane. To find the ratio $\frac{S}{V_a}$ which shall make E a maximum, place $\frac{\delta E}{\delta S} = 0$; and solve for $\frac{S}{V_a}$.

$$\frac{\delta E}{\delta S} = (V_a^2 - 4\, V_a S + 3\, S^2) = 0; \text{ hence } \frac{S}{V_a} = \frac{1}{3}. \qquad (25)$$

Substitute $S = \frac{V_a}{3}$ in the equation for E; then,

$$\text{Maximum work, } E = \frac{4}{27} \frac{wa_a V_a^3}{g} = \frac{8}{27} wQ \frac{V_a^2}{2g} = \frac{8}{27} W \frac{V_a^2}{2g}$$

$$= \frac{8}{27} \text{ of the total kinetic energy in stream.} \qquad (26)$$

C. **A series of flat vanes moving** against a resistance with a velocity of (20, 0°), as, for example, if the plates are set on the rim of a wheel. This is the case of an undershot wheel or an impulse wheel with flat vanes.

Here $Q = a_a V_a$ because with a series of vanes, succeeding vanes take whatever water the preceding ones have left behind (compare with a single vane); substitute this value of Q in the equations of B.

$$\text{Then } P_X = + \frac{wa_a V_a}{g}(V_a - S) = \frac{62.4 \times .5 \times 40 \times 20}{32.16}$$

$$= 776 \text{ pounds, direction } 0°.$$

$P_y = 0$, because the resultant of the vertical components is zero.

$$E = P_x\beta = \frac{wa_\alpha V_\alpha}{g}(V_\alpha - \beta)\beta$$

$$= 776 \times 20 = 15{,}520 \text{ foot-pounds per second.}$$

Maximum work for a series of flat vanes. To find the ratio $\frac{\beta}{V_\alpha}$ which shall make E a maximum:

$$\frac{dE}{d\beta} = V_\alpha^2 - 2V_\alpha\beta = 0, \text{ hence } \frac{\beta}{V_\alpha} = \frac{1}{2} \qquad (27)$$

Substitute $\beta = \frac{V_\alpha}{2}$ in the equation for E; then

$$\text{Maximum } E = \frac{wa_\alpha V_\alpha^3}{4g} = \frac{1}{2}wQ\frac{V_\alpha^2}{2g} = \frac{WV_\alpha^2}{4g}$$

$$= \tfrac{1}{2} \text{ of the total kinetic energy in stream.} \qquad (28)$$

406. A jet of water impinging on curved or bent vanes. Theoretically the effect is identical for vanes bent with a sharp angle or with a smooth curve; the latter offers less actual resistance to flow.

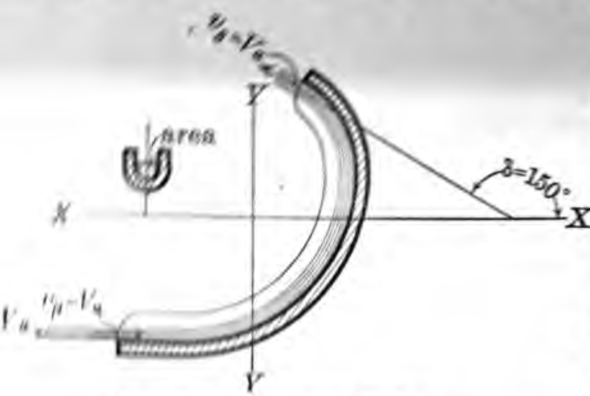


FIG. 146.—Curved Vane, Stationary.

A jet of water of area $a_\alpha = .5$ with a velocity $V_\alpha = 40$, 0°, impinges on a curved vane having boundaries (see figure 146). Compute the X and Y forces exerted on the vane by the water.

A. The vane is stationary.

$$\delta = \epsilon = 150°;\ a_\alpha = a_\beta = a_\delta = .5;\ v_\delta = V_\epsilon = V_\alpha.$$

$$P_x \text{ (equation 13)} = \frac{wQ}{g}(V_\alpha \cos\alpha - V_\epsilon \cos\epsilon)$$

$$= \frac{wa_\alpha V_\alpha^2}{g}(\cos 0° - \cos 150°)$$

$$= \frac{62.4 \times .5 \times 40^2}{32.16}[1-(-.866)] = +2896$$

$= 2896$ pounds, direction 0°.

$$P_Y \text{ (equation 15)} = \frac{wQ}{g}(V_a \sin \alpha - V_\epsilon \sin \epsilon)$$

$$= \frac{wa_a V_a^2}{g}(\sin 0° - \sin 150°)$$

$$= \frac{62.4 \times .5 \times 40^2}{32.16}(0-(+.5)) = -776$$

$= 776$ pounds, direction 270°.

B. A series of curved vanes moving against a resistance with a **velocity** of 20, 0° (see figure 147).

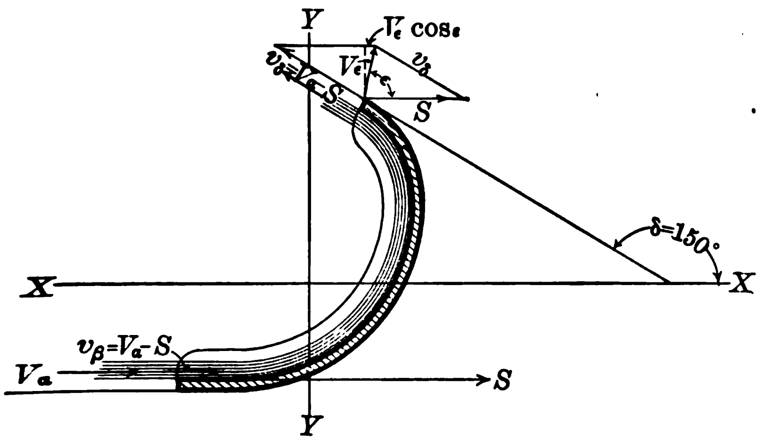


FIG. 147. — Curved Vane, Moving.

$$P_X \text{ (equation 13)} = \frac{wQ}{g}(V_a \cos \alpha - V_\epsilon \cos \epsilon).$$

$$Q = a_a V_a; \quad V_a \cos \alpha = 40 \cos 0°.$$

$$V_\epsilon \cos \epsilon = v_\delta \cos \delta + S = (V_a - S) \cos 150° + S.$$

Then
$$P_X = \frac{62.4 \times .5 \times 40(40-20)}{32.16}[1-(-.866)]$$

$= +1448 = 1448$ pounds, direction 0°.

$$P_Y \text{ (equation 15)} = \frac{wQ}{g}(V_a \sin \alpha - V_\epsilon \sin \epsilon).$$

$$V_a \sin \alpha = 0;$$

$$V_\epsilon \sin \epsilon = (V_a - S) \sin 150° = (40-20)(+.5).$$

Then $$P_Y = \frac{62.4 \times .5 \times 40}{32.16}(20 \times -.5) = -388$$

$$= 388 \text{ pounds, direction } 270°.$$

$$E = P_X S = 1448 \times 20 = 28{,}960 \text{ foot pounds per second.}$$

Maximum efficiency for any series of vanes where the points of entrance and exit are moving in the same direction occurs theoretically (equation 27) when $S = \frac{V_a}{2}$, hence at maximum efficiency

$$E = \frac{wa_a V_a}{g}\left(V_a - \frac{V_a}{2}\right) \times \frac{V}{2}(\cos \alpha - \cos \epsilon). \quad (29)$$

Hence when $\alpha = 0°$, and $\epsilon = 180°$, $(\cos \alpha - \cos \epsilon) = 1 - (-1) = 2$ the maximum $E = \frac{wa_a V_a}{g}\left(\frac{V_a^2}{4}\right) \times 2 = \frac{wQ}{2g} V_a^2$, which is equal to $\frac{WV_a^2}{2g}$ the total kinetic energy contained in the jet.

434. The runner of a waterwheel is a series of vanes, or buckets which are fixed to a shafting set in bearings. If the vanes are so constructed as to deflect the streams of water entering the buckets through 180°, measured in each bucket from the direction of motion of the bucket, theoretically all the available energy of the water may be used. Actually there is some loss in friction; and the total angle can not be made 180° without causing the jet from one vane to strike on the back of the following vane; moreover the angles must also be modified to suit the type of wheel.

In the American impulse wheel, buckets are used which deflect the water through about 170°.

If the distance from the center of rotation of the buckets at entrance (r_0) is different from the distance at exit (r_1), as in many forms of water motors, the fundamental equation for energy may be derived by a consideration of angular momentum and angular impulse.

435. The angular momentum relative to a fixed point of a body having a motion of translation is found by multiplying (*a*) its momentum by (*b*) the perpendicular distance from the fixed point to the line of motion of the body's center of gravity at any instant.

436. The angular impulse is the product of (*a*) a force (F or P), by (*b*) the perpendicular distance (r) from a fixed point to the

e of action of the force, and by (*c*) the time during which it ts (t).

437. The variation in angular momentum in a given time is equal the angular impulse producing that variation, that is,

$$Fr\delta t = \frac{wQ}{g}\delta v\delta r. \tag{30}$$

438. The couple acting. If wQ pounds of water per second undergo a change in angular momentum in passing through a wheel, the angular impulse becomes the couple acting on the wheel,

$$Pr = \frac{wQ}{g}(V_0r_0 - V_1r_1). \tag{31}$$

439. The work done. If the body is moving against a constant resistance with an angular velocity, ω radians per second, then

$$E = P\omega r = PS = \frac{wQ}{g}(V_0\omega r_0 - V_1\omega r_1). \tag{32}$$

The equation (32) is the fundamental equation for work done on moving vanes by changes in velocity and direction of the water flowing on it. Perhaps a more satisfactory way, as showing also the effects of channels and wheel case, is to derive the equation of work by the aid of Bernoulli's theorem, as will be done in Chapter XVIII.

440. Water hammer. If the velocity of water flowing in a closed channel should be suddenly diminished, the energy thus taken from the water will be used partly in compressing the water itself, and in deforming the pipe walls, and may even rupture the pipe. The force thus set up is called water hammer. In water pipes or penstocks the velocity of the water is frequently reduced, and water hammer set up by the sudden closing of a gate near the discharge end of the pipe. The intensity of the water hammer set up will depend primarily upon the volume of water contained in the pipe, and the rate of change in velocity, the latter depending on the rapidity of closing the gate.

From equation (12) it would appear that this pressure should amount to $\frac{wQ}{g}(V_0 - V_1)$; but Church* and others have shown that in order to make a reasonable determination of the intensity of

* *Hydraulic Motors*, pp. 203–214.

pressu[illegible]ed by the sudden retardation of a volume of flowing wate[illegible]ecessary to consider not only the change in velocity in a g[illegible]ie, but also the compressing of the water in the pipe, the distend[illegible]g of the pipe walls in length and in circumference, and th[illegible]h of the pipe.

441. [illegible]**h's formula for water hammer in pipes,** due to instantaneous sto[illegible]age of velocity,

$$p = V\left(\frac{\gamma EE't'}{g(t'E' + DE)}\right)^{\frac{1}{2}}. \qquad (33)$$

$p =$ the intensity of inte[illegible]essure in pounds per square inch due to the in[illegible]neous destruction of all the velocity of flow; [illegible] in addition to hydrostatic pressure, existing [illegible] the water begins to change velocity.

$V =$ the mean velocity of [illegible] feet per second.

$\gamma = \frac{62.4}{144} = .4333.$

$E = 294{,}000$, the bulk modulus of water.

$E' =$ modulus of elasticity in tension of the pipe material.

$t' =$ the thickness of the pipe walls in inches.

$D =$ the internal diameter of the pipe in inches.

In certain experiments on water hammer by Joukowsky,* which Church considered in making up his formula, it was shown that if the time of closing a gate is less than the time required for a wave of compression (which would have about the same velocity as a sound wave) to travel from the gate to the reservoir and back again to the gate, the intensity of water hammer will be about the same as if the gate were instantly closed. Church proposed the following formula for determining the velocity of the wave of compression:

$$V_s = \left(\frac{gEE't'}{\gamma(t'E' + DE)}\right)^{\frac{1}{2}}. \qquad (34)$$

$V_s =$ velocity in feet per second of a wave of compression in the pipe.

* For a résumé of Joukowsky's experiments, see *Proc. Am. Waterworks Association*, 1904.

ie time (T_s) required for a wave of compression to travel
› the length of the pipe, that is, make the round trip, is

$$T_s = \frac{2L}{V_s} \text{ in seconds.} \tag{35}$$

the time of closing the gate in seconds (T) is greater than
ie intensity of pressure due to water hammer will diminish as
ws:

$$\frac{p_T}{p} = \frac{T_s}{T}. \tag{36}$$

p_T = intensity of water hammer pressure if the time of closing is T (greater than T_s).

2. **In computing the thickness of pipes** (see Chapter II) the in-
ty of water hammer pressure should be added to the hydro-
› pressure; unless the possibility of water hammer can be
uded by slow-moving gates, by-passes, standpipes, or surge
rs, relief valves, or expansion chambers. Flat or deformed
ces in cylindrical pipes or flumes are more likely to rupture
true cylindrical forms.

ample. Given a steel-riveted pipe $\frac{1}{4}$ inch thick, 48 inches in
eter, and 10,000 feet long, containing water flowing at a
ity of 6 feet per second. If a gate at the discharge end were
ntly closed, compute the intensity of water hammer pressure.
E' for steel as 28,000,000.
ie intensity of pressure due to instantaneous closure,

$$p = 6\left(\frac{.4333 \times 294000 \times 28000000 \times .25}{32.16(.25 \times 28000000 + 48 \times 294000)}\right)^{\frac{1}{2}}$$
$$= 217 \text{ pounds per square inch.}$$

ısume the time of closing to be 30 seconds, compute the in-
ty of pressure due to water hammer.
ie velocity of a wave of compression,

$$V_s = \left(\frac{32.16 \times 294000 \times 28000000 \times .25}{.4333(.25 \times 28000000 + 48 \times 294000)}\right)^{\frac{1}{2}}$$
$$= 2690 \text{ feet per second.}$$

e time required for a wave of compression to travel the round

$$T_s = \frac{2 \times 10000}{2690} = 7.43 \text{ seconds.}$$

The intensity of pressure if the gate were closed in 30 seconds instead of instantly.

$$p_T = \frac{7.43}{30} \times 217 = 56 \text{ pounds per square inch.}$$

If the hydrostatic pressure is 100 pounds per square inch, the pipe should be designed for (100 + 217) or (100 + 56) according to the time of closing the gate.

Church's formula, which agrees well with Joukowsky's experiments in which the time of closure was .03 second, seems preferable for very sudden closure; but the following formula of Gibson is to be preferred for uniform and more gradual closure.

443. Gibson's formula for water hammer in pipes due to the uniform gradual closing of a valve. In a recent book* A. H. Gibson published the results of experiments on, and the development of formulas for, water hammer. The experiments were made with a valve set on a piece of $2\frac{1}{2}$-inch wrought-iron pipe (8 feet long) set in a cast-iron pipe $3\frac{3}{4}$ inches in diameter and about 550 feet long. The larger pipe led off from an open tank about 105 feet higher than the valve. The experiments were made in the laboratory of Manchester (England) University. Gibson proposed, as the result of his experiments, the following formula:

$$p = \frac{\gamma}{g}\left[\left(\frac{La_1}{Ta}\right)^2 + \left(\frac{La_1}{Ta}\right)\left(2gh + \left[\frac{La_1}{Ta}\right]^2\right)^{\frac{1}{2}}\right] \qquad (37)$$

p = rise of pressure due to closing the gate in pounds per square inch in excess of that due to h.

L = length of pipe in feet.

T = time of uniform closing of gate.

a = pipe area.

a_1 = effective area of the stream (at its vena contracta) through the gate at the time closing begins.

h = the difference in head in feet between the two sides of the gate when there is no flow.

This formula is applicable when T is greater than $\frac{4L}{V_s}$.

* A. H. Gibson, *Water Hammer in Hydraulic Pipe Lines*; 60 pages; Van Nostrand Co., 1908.

Example. Given a pipe 550 feet long; ratio when closing begins, $\frac{a}{a_1} = 8.6$; time of closing gate 2 seconds; difference in static head on the two sides of the gate when closed is 104 feet.

$$p = \frac{.4333}{32.16}\left[\left(\frac{550}{2 \times 8.6}\right)^2 + \frac{550}{2 \times 8.6}\left(64.32 \times 104 + \left[\frac{550}{2 \times 8.6}\right]^2\right)^{\frac{1}{2}}\right]$$

$= 52$ pounds per square inch.

Problems

1. A pipe bent in the form of a semicircle having a radius of 10 feet discharges 5 cubic feet of water per second with a velocity of 25 feet per second. The pipe is suspended by two strings so that the two ends are in the same horizontal plane and water is supplied in a vertical direction and without shock. What is the tension in the strings due only to the change in direction of the water?

2. Water is flowing at a velocity of 10 feet per second through a pipe 2 feet in diameter, which is bent through a central angle of 120 degrees. Compute the force exerted parallel to either tangent and the force exerted perpendicular to either tangent, by the change in direction of the water.

3. A stream of water delivering 897.6 gallons per minute at a velocity of 40 feet per second strikes an indefinite plane: (*a*) normal, (*b*) at 30 degrees with the normal. Compute the normal pressure on the plane.

4. A stream of water 100 square inches in area issues from the stern of a ship with a velocity relative to that of the vessel of 50 feet per second, the ship moving at the rate of 10 miles per hour in the opposite direction. Find the force exerted on the ship by the jet.

5. A stream with a transverse section of 6 square inches delivers 2 cubic feet per second against a single flat vane, in a direction normal to its surface. Find the force acting on the vane, if the water after striking glides over it parallel to the surface:

(*a*) If the vane is stationary.

(*b*) If the vane is moving at a velocity that will give the maximum work.

(*c*) Compute the work done on the vane in (*b*).

6. A nozzle $1\frac{1}{4}$ inches in diameter, of which the coefficient of discharge is .98, delivers water under an effective head of 169 feet, against a series of flat vanes fixed on the circumference of a wheel at a point 10 feet from the center of rotation. (*a*) Determine the velocity of the vanes that will give maximum efficiency. (*b*) What will be the angular velocity of the wheel?

7. A street sprinkler is running on the tracks of a street railway at the rate of 15 miles per hour. From the front 500 streams issue, each $\frac{1}{8}$ inch effective diameter. What is the retarding force in pounds when the head upon the openings has an average of 7 feet? Neglect friction. Solve (*a*) when streams flow parallel with track; (*b*) when openings make a complete semicircle.

8. The jet from a nozzle strikes a series of curved vanes in a direction tangent to the rim of the wheel on which they are set, and at a point 1.5 feet from the axle. The lips of the vanes where the water enters are parallel to the jet and are bent through 165° from this line. The velocity of the jet is 172 feet per second and the wheel is revolving at the rate of 548 revolutions per minute. $Q = 3.75$ cubic feet per second.

Compute (*a*) the constant force exerted on the vanes, and (*b*) the horse power developed.

9. A stream of 500 cubic feet per second falls through 60 feet. Compute the horse power available.

10. At the entrance to a turbine the intensity of pressure is 20 pounds per square inch above atmospheric; and at the exit from the runner 13 pounds below atmospheric. How much horse power is available when 300 cubic feet per second is going through the wheel?

11. Water discharging from a nozzle impinges on a wheel with an absolute velocity of 200 feet per second, and leaves the wheel with an absolute velocity of 10 feet. $Q = 10$. What horse power has been used in the wheel?

12. A penstock of riveted steel 8 feet in diameter and 1000 feet long leads from an open reservoir to a turbine. Assume, that when the velocity is 7 feet per second and the pressure at the turbine is 20 pounds per square inch, the gate at the lower end is instantly closed.

(*a*) Compute the pressure due to water hammer, and the necessary thickness of the pipe.

(*b*) If the pipe were 10,000 feet long, make a similar computation.

(*c*) If the time of closure in each case is 3 seconds.

CHAPTER XVIII

IMPULSE WHEELS — TURBINES — CENTRIFUGAL PUMPS

444. The essentials of a water-power plant. A water-power plant presupposes a supply of water falling through some height. Natural streams are subject to considerable variation in the volume of flow, and in consequence to fluctuations in the height of the fall. The flow and the fall for which the plant is to be designed must be based on a study of the seasonal fluctuations in flow, the effect of storage, and variations in the depth of the stream when carrying different quantities of water.

In order to utilize the energy available in a waterfall some sort of water motor or motors with the necessary appurtenances must be provided.

A water-power plant (see figures 148 and 152) comprises, in addition to the water wheels:

A dam to localize the available fall, and to create a mill pond to equalize the flow over short periods of time. The still water elevation in this pond is called the *head-water*.

Channels to conduct the water from the head-water to the wheels; if open channels, they are called *power canals or flumes*; channels under pressure, *penstocks*.

Head gates for controlling the flow in the channels, with racks and screens to prevent drift getting into the wheel; and oftentimes sluice gates for getting rid of refuse and ice and for drawing off the water.

Gates for controlling the flow into the wheels, often called *speed gates*, usually connected to a hand regulator or an automatic governor, which controls the speed of the wheel by varying the supply of water as the load increases or diminishes.

A tight metal case or open pit for the speed gate, guides, runner, and shafting; these comprise *the setting*.

A channel or raceway for conducting the water away from the wheel, which may be set at the bottom of the fall or for conven-

ience and economy of setting may include a draft tube. The still-water elevation about the wheel, or at the downstream end of a draft tube, is called the *tail-water.*

445. The earlier waterwheels were of three general types: (*a*) undershot, having flat radial vanes, occasionally with curved vanes; (*b*) overshot, containing a series of buckets designed to

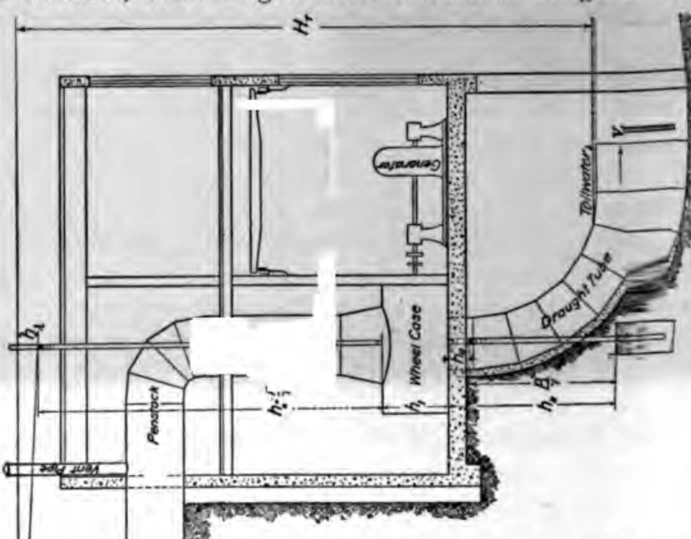
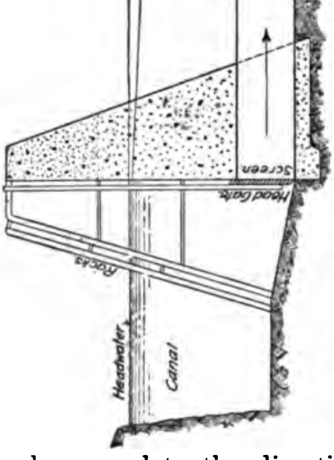



FIG. 148.—Outline of a Modern Water-power Plant.

take the water at the highest possible point, and convey it to the lowest possible discharge point with minimum spilling; (*c*) breast wheels, designed to receive the water near or below the axis of rotation, and deliver it as low as possible with minimum spilling.

Undershot or paddle wheels are in general of low efficiency, their buckets usually being merely a series of flat vanes nearly normal to the direction of flow and working partly submerged. By reference to Chapter XVII, equation (28) it will be seen that the maximum theoretic efficiency for a series of flat mov-

ing vanes is but .5; the actual efficiency is rarely over .35. Certain undershot wheels with curved vanes have given considerably higher efficiency.

Overshot and breast wheels have been built which gave nearly as high efficiency as the best modern turbines. The work in such wheels is nearly all done by the weight of the water falling in, or in contact with, the wheel bucket from head-water to tail-water. The size, therefore, depends on the fall; the diameter of an overshot wheel must be nearly equal to the total fall acting on the wheel, and the diameter of a breast wheel nearly or more than double the fall. Both of these types, while they may be efficient, are cumbersome and costly.

446. Modern water motors, in which the work is performed chiefly by changes in the velocity and direction of the water, may be classified in many ways: according to the general direction of flow; radial (parallel to the radii of the wheels), or axial (parallel to the axis); inward (toward) or outward (away from) the center of rotation; or after the names of inventors or adapters of certain types. A more scientific method of classification is based on whether changes in velocities and directions occur in air at atmospheric pressure, or in closed vanes or buckets at pressures above or below atmospheric pressure. On this basis two general classes of modern wheels are recognized, viz.:

1. **Impulse** (action) wheels in which all changes in velocity or direction take place in air at atmospheric pressure; the water leaves one or more stationary nozzles or guides, with a velocity due to the total effective head at the orifice of the nozzle, impinges upon and flows over the moving wheel buckets with such changes in direction and velocity as are imposed on the stream by the curvature of the buckets and their speed; but the buckets are essentially open channels. Impulse wheels are usually classified as: (*a*) **tangential** or **American impulse wheels,** and (*b*) **impulse turbines, European types.**

This latter type has been, at times, *arranged so that it will also run submerged* (an obsolescent practice), which is done by limiting the area of the bucket apertures to such dimensions that they will run entirely full of water; the wheel is *then called a limit turbine.*

2. **Reaction turbines**, so called, in which the guide vanes o
guides, wheel vanes or buckets, and the channels leading to and
from the wheel form a continuous, though devious, channe
from head-water to tail-water, which is sealed from the air, an
when properly working at full gate opening should have all it
parts filled with water flowing under pressure. In the reactio
turbine water leaves the guides with a velocity due only to part
the effective head, and enters the buckets under pressure. Th
buckets, being then in the nature of moving nozzles, receive th
water from the guides and discharge it with velocities due to th
kinetic energy therein contained and to differences of pressure b
tween the inside and the outside of the buckets. The changes
pressure which occur from point to point in the passage of t
water through the wheel are important only in so far as they brin
about changes in velocity and direction at the critical poin
namely, at entrance to and exit from the moving buckets.
order that a reaction wheel may work to the best advantage, t
various passages should have the required cross-sectional area
the direction of flow to correspond with the velocities determin
by the other factors.

447. European practice, based on theory and testing. By
aid of Bernoulli's theorem, the expressions for the work done
changes in velocity or direction of flowing water may be
duced; from which, together with the knowledge of the act
performance of full-sized wheels in tests and in use, satisfact
rules for proportioning or designing wheels have been develop
European practice has grown chiefly along such lines, placi
great emphasis on theory, and as a result has produced wheels
great variety of form and capacity, adapted to a wide range
heads, and giving high efficiency. To a large extent each wh
or unit has been designed to fit the particular location.

448. American practice based on empirical design and testi
The practice of waterwheel design in the United States has be
largely developed by inventors or waterwheel builders, who ha
for improvement or adaptation, embodied in new types ideas
tained from previous types and tested out their wheels (f
size) to prove their merits; and by changing and retesting ha
persisted until the desired ends were attained. On the part

American wheel makers there has been a persistent and successful effort to reduce the diameter and cost of wheels for given power and at the same time to increase the power, speed, and efficiency. In the matter of efficiency at full gate, the gain over early wheels has been slight; but in the matter of low cost, adaptability, and part gate efficiency, the improvement has been very great. The most marked achievement of the American wheel makers has been the production of stock wheels (as compared with special design) for very high heads, 200 feet or more, and for low heads, 50 feet or less, especially under 20 feet. The first type is embodied in the tangential impulse wheels of the form shown by figures 152, 154, 155, and 157; the second in wheels having runners resembling that shown in figures 161, 168, 169, 170, 171, and 172.

The American impulse wheel is a development from a cheap and originally inefficient wheel, used in Western mining operations, to a highly efficient machine, and is distinctly an American product.

The first outward flow reaction turbine, built in America by Ellwood Morris (1843), followed the designs of Fourneyron; and Boyden's wheel (1844) (see figure 167) was very similar. The Francis turbine (1849) was (see figure 163) stated by Francis to be, in its essential features, similar to a wheel proposed by Poncelet (1826). The Swain wheel (see figure 168) with combined inward and downward flow, which is the prototype of the modern low head American turbine, was an original American wheel; this and those that followed were the outcome of the ideas of practical wheelwrights who had little knowledge of the principles of theoretic mechanics.

449. Increasing interchange of practice. At the present time there is a growing interchange of practice between American and European builders. Low head American turbines receive high recognition in Europe, and their use is increasing; a large portion of the recent impulse wheels are of the tangential or American type, and nearly all new turbines are of the inward downward flow type, or so called "Francis" turbines; but they resemble the Swain wheel more than the Francis. In America there is an increasing tendency in building turbines to follow European

practice in the design of turbines for special places, particularly large units.

450. The design of waterwheels is a highly specialized branch of hydraulic engineering, but growing in importance; and even if the available data to show the relation between design and the results of testing provided sufficient basis for a proper treatment of this subject, it is too intricate for a book on elementary hydraulics. Nor at the present time is a hydraulic engineer ordinarily required to design wheels, but simply to select wheels, specify the work to be done, and design and construct the channels and waterways leading to the wheels, and away from them.

A determination of the changes in velocity, direction, pressure, and elevation, which the water undergoes in passing through a wheel and its appurtenances, is essential to the proper design of a water-power plant. In the present state of practice, suitable, efficient runners are moderately easy to get; but not infrequently runners, which show high efficiency under favorable tests, as at Holyoke, will, when tested in place, show marked falling off in efficiency, largely, if not wholly, due to incorrect proportioning of penstocks, wheel cases, and draft tubes. The design of these parts requires chiefly a thorough application of the principles of flow in channels, and through orifices or nozzles, and the principle of work. The same principles apply to the flow through the wheel itself, but on account of the somewhat intricate shapes of waterwheels, and the fact that some parts are standing still and others moving, the computations are more involved, and require further explanation.

451. Nomenclature. The following symbols will be used in the formulas for water motors; so far as they were used in the chapter just preceding they are identical:

V_A = absolute mean velocity of water in the channel of approach, just upstream from the speed gates.

V_a = absolute mean velocity of water from guide vanes or nozzles; absolute entrance velocity.

V_e = absolute mean velocity or water from wheel vanes; absolute exit velocity.

V_d = absolute mean velocity of water at exit from a draft tube.

v_β = relative (to the vane) mean velocity of water at entrance to the wheel vanes.

v_δ = relative (to the vane) mean velocity of water at exit from the wheel vanes.

α = angle of absolute velocity at entrance; direction of V_a.

β = angle of relative velocity at entrance; direction of v_β.

δ = angle of relative velocity at exit; direction of v_δ.

ϵ = angle of absolute velocity at exit; direction of V_ϵ.

NOTE. — Direction is measured from the tangents to the wheel rims: 0°, ɪing in every case the direction of motion at the points of entrance and exit.

a_A, a_α, a_β, a_δ, and a_d = the cross-sectional areas of a stream or jet measured at right angles to the direction of flow; each area corresponding in the order named to the velocities V_A, V_α, v_β, v_δ, and V_d.

H_T = total head available, which is the difference in elevation between surfaces of head water and tail water.

H = the effective head acting on wheel; namely H_T minus losses due to conducting the water to and from the wheel.

p_0 = intensity of pressure at the base of nozzles or guides, corrected for the elevation of the nozzle orifice.

p_β = intensity of pressure at the entrance to wheel vanes; it may be more than atmospheric.

p_1 = intensity of pressure at the exit from wheel vanes; it may be less than atmospheric.

h_0 = fall from head water level to discharge orifices of the guides, minus h_λ.

h_1 = fall from discharge orifices of the guides to discharge orifices of the runner.

h_2 = fall from discharge orifices of the runner to tail-water level.

h_λ = head lost in the channel of approach.

h_d = head lost in conducting the water away from the wheel; in reaction turbine the loss in the draught tube.

r_0 = radius of the runner, where water enters, in feet.

r_1 = radius of the runner, where water leaves, in feet.

N = revolutions per minute of a wheel.

S_0 = absolute velocity of a vane at the point where the water enters it; $S_0 = \frac{\pi r_0 N}{30}$. The direction of S_0 is 0°, being tangent to the runner at this point.

S_1 = absolute velocity of a vane at the point where the water leaves it; $S_1 = \frac{\pi r_1 N}{30}$. The direction of S_1 is 0°, being tangent to the runner at this point.

C_0, C_1 = coefficients of velocity from nozzles or vanes.

E = the work done by the water in the wheel as computed by formulas, foot pounds per second.

E_B = the work done by the water in the wheel measured by a friction brake on the wheel shaft, foot pounds per second.

K_h = the computed hydraulic efficiency of the wheel = $\frac{E}{wQH}$.

K_B = the actual measured efficiency of the wheel = $\frac{E_B}{wQH}$.

In wheel tests the loss in draught tube can not always be separated from losses in the wheel; in such cases, $K_B = \frac{E_B}{wQ(H + h_d)}$.

K_p = total computed efficiency of the wheel plant = $\frac{E}{wQH_T}$.

452. Impulse wheels or turbines have one or more stationary nozzles, or guides, which receive the water from the channel of approach, and direct the stream in the proper direction, and with suitable velocity against the buckets, which are a set of nozzles or channels set in the circumference of a wheel, which make up the runner and which revolve past the guide orifices. This runner is attached to a shaft which in turn is connected by suitable mechanical devices to machinery. If the *dimensions and speed of a wheel*, the *absolute entrance velocity*, and the *relative exit velocity* are known the work may be theoretically determined.

453. Absolute entrance velocity. The velocity from the guides is determined as for any nozzle by the sum of the energy available at the orifices of the guides, and may be called the absolute velocity at entrance, V_a.

454. Relative exit velocity. The velocity from the bucket orifices may be considered a nozzle velocity determined by the sum of the energy available at the orifices of the runner, and is called the relative (to the bucket) velocity at exit, v_δ.

455. Assumptions made in deriving formulas. The equations which follow are based upon the following assumptions:

(*a*) That the wheel is working against a steady moving resistance just sufficient to maintain the desired speed.

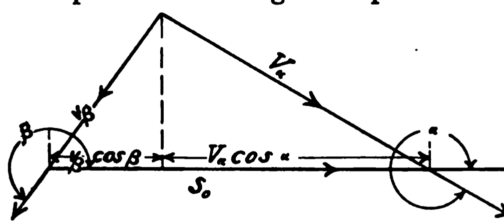


FIG. 149. — Diagram of Entrance Velocities.

(*b*) That the water enters the wheel without violent changes in direction or velocity, to secure which condition the resultant of v_β and S_0 must equal V_a, as shown graphically in figure 149.

Then $$V_a^2 = v_\beta^2 + S_0^2 + 2\, S_0 v_\beta \cos\beta, \tag{1}$$

and $$V_a \cos\alpha = v_\beta \cos\beta + S_0. \tag{2}$$

(*c*) That the water leaves the bucket at a minimum practical absolute velocity of exit, V_ϵ; because the energy due to this velocity is irrecoverably lost. V_ϵ should be sufficient in magnitude

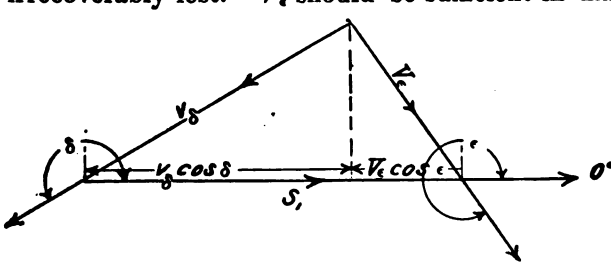


FIG. 150. — Diagram of Velocities at Exit.

to flow clear of the vanes. V_ϵ is the resultant of v_δ and S_1, as shown graphically in figure 150.

Then, $$V_\epsilon^2 = v_\delta^2 + S_1^2 + 2\,S_1 v_\delta \cos\delta, \tag{3}$$

and $$V_\epsilon \cos\epsilon = v_\delta \cos\delta + S_1. \tag{4}$$

The best practical working condition is usually assumed to exist when $V_\epsilon \cos\epsilon = 0$.

(*d*) That in an impulse wheel all the buckets must be open to atmospheric pressure, and have greater cross-sectional area considered as channels than the velocities require.

(*e*) That in a reaction wheel all guides and buckets must be sealed from atmospheric pressure; and in all parts the cross-sectional areas of the channel must equal those required by the velocities at these points due to changes in pressure and motion, viz.:

$$a_\alpha = \frac{Q}{V_\alpha} : a_\beta = \frac{Q}{v_\beta} : a_\delta = \frac{Q}{v_\delta}. \tag{5}$$

(*f*) That changes in velocity or direction in the interior of all channels shall be gradual, and all surfaces in contact with flowing water be smooth, in order to reduce friction losses.

(*g*) That these assumed conditions can hold only when a wheel is running at its full capacity and most efficient speed.

456. General formulas for impulse wheels and turbines. By the application of Bernoulli's theorem, general equations, for the flow

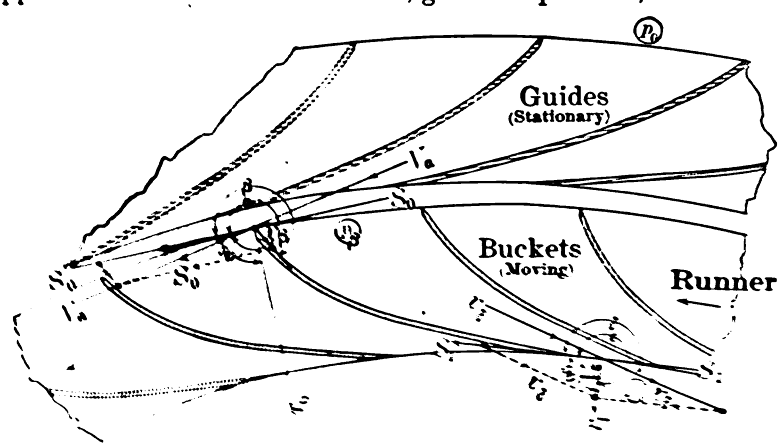


FIG. [illegible] — Guides and Buckets of Any Water Wheel.
(Actually part of Francis's Turbine.)

through, and the work done in any wheel due to changes in velocity and direction may be derived as follows — neglecting

friction from the entrance to the guides and the exit from the buckets, but considering the friction losses in conducting water to and from the wheel. Figure 151 may represent a part of any set of guides and a part of any runner.

By the fundamental formula for nozzles (§ 198), the theoretic velocity of the water on leaving the guides,

$$V_a = \left[2g\left(H - \frac{p_\beta}{\gamma}\right)\right]^{\frac{1}{2}}, \tag{6}$$

where $H = h_0 + h_1 + h_2 - h_d = H_T - h_\lambda - h_d$. (See figure 148.)

The total energy in foot pounds delivered each second to the wheel, if measured at the guide orifices

$$= wQ\left(\frac{V_a^2}{2g} + \frac{p_\beta}{\gamma} + \frac{p_a}{\gamma}\right). \tag{7}$$

The energy in foot pounds carried away each second by the water from the exit orifices of the buckets

$$= wQ\left(\frac{V_e^2}{2g} + \frac{p_a}{\gamma}\right). \tag{8}$$

Then E, which is the difference between the energy delivered to and the energy carried away, is the work done each second by the water.

$$E = wQ\left(\frac{V_a^2}{2g} + \frac{p_\beta}{\gamma} - \frac{V_e^2}{2g}\right) \tag{9}$$

Equivalent values for the terms of equation (9) may be determined as follows:

On the assumption that the water enters without shock.

From (1) $\qquad V_a^2 = v_\beta^2 + S_0^2 + 2\, v_\beta S_0 \cos\beta.$

Dividing by $2g$, $\qquad \dfrac{V_a^2}{2g} = \dfrac{v_\beta^2}{2g} + \dfrac{S_0^2}{2g} + \dfrac{2\, v_\beta S_0 \cos\beta}{2g}. \qquad (10)$

If there are no losses in the buckets, the kinetic energy $\left(\frac{v_\delta^2}{2g}\right)$ due to the relative velocity (v_δ) from the bucket orifices should equal the energy put into the buckets, plus the gain or minus the loss in kinetic energy of rotation in passing from entrance to exit of the buckets, or by Bernoulli's theorem:

$$\frac{v_\delta^2}{2g} = \frac{v_\beta^2}{2g} - \frac{S_0^2}{2g} + \frac{S_1^2}{2g} + \frac{p_\beta}{\gamma}. \tag{11}$$

Then from (11), $\qquad \dfrac{p_\beta}{\gamma} = \dfrac{v_\delta^2}{2g} - \dfrac{v_\beta^2}{2g} + \dfrac{S_0^2}{2g} - \dfrac{S_1^2}{2g}. \qquad (12)$

The absolute exit velocity V_e is the resultant of v_δ and S_1.

By (3), $V_e^2 = v_\delta^2 + S_1^2 + 2 v_\delta S_1 \cos \delta$.

Dividing by $2g$, $$\frac{V_e^2}{2g} = \frac{v_\delta^2}{2g} + \frac{S_1^2}{2g} + \frac{2 v_\delta S_1 \cos \delta}{2g} \qquad (13)$$

Insert in equation (9) the values thus found for V_a (10), for p_B (12), for V_e (13); then the work done by the wheel,

$$E = \frac{wQ}{2g}(v_\beta^2 + S_0^2 + 2 v_\beta S_0 \cos \beta + v_\delta^2 - v_\beta^2 + S_0^2 - S_1^2 - v_\delta^2 - S_1^2 - 2 v_\delta S_1 \cos \delta).$$

$$E = \frac{wQ}{g}[(v_\beta \cos \beta + S_0) S_0 - (v_\delta \cos \delta + S_1) S_1]. \qquad (14)$$

By (2), $v_\beta \cos \beta + S_0 = V_a \cos \alpha$;

and by (4), $v_\delta \cos \delta + S_1 = V_e \cos \epsilon$.

Then

$$E = \frac{wQ}{g}(V_a \cos \alpha \, S_0 - V_e \cos \epsilon \, S_1), \text{ foot pounds per second.} \qquad (15)$$

But $S_0 = \omega r_0$; and $S_1 = \omega r_1$. Also when $r_0 = r_1$, $S_0 = S_1$.

Equation (15) then is neglecting friction in the runner, a fundamental expression for the work done in the wheel by changes in velocity and direction of the water, and is identical with the equation for energy in equation (32), Chapter XVII.

Neglecting friction, the wheel efficiency $= \dfrac{E}{wQH}$. (16)

Tangential Impulse Wheels

457. A tangential impulse wheel is frequently called an American * impulse wheel, but is also made in Europe; its name comes from the fact that the stream from a nozzle discharging into the air strikes the wheel in a direction tangent to the rim of the runner at entrance.

Figure 152 is a diagrammatic outline of a water-power plant with tangential impulse wheel. A pipe leads from the reservoir to the wheel, to which is attached a nozzle or nozzles.

* Some of the trade names of these wheels in the United States are "Pelton," "Doble," and "Escher-Wyss." (The latter made by Allis-Chalmers Co.)

The nozzles are fitted with devices for regulating the volume of flow to suit variations in the demand for power; and are so set

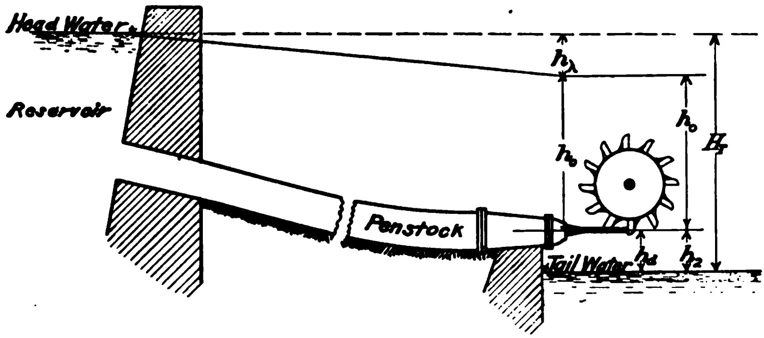


FIG. 152. — Water-power Plant with Tangential Impulse Wheel.

that the stream will be ordinarily tangent to the wheel runner at entrance. Smooth cone nozzles are often used. The form of nozzle shown in figure 153 is known as the Doble regulating nozzle; the flow may be varied by moving the needle into or away from the orifice without changing direction.

Figures 154 and 155 show two American impulse wheels in their settings. Figure 156, on Plate II, is a tangential wheel of European make.

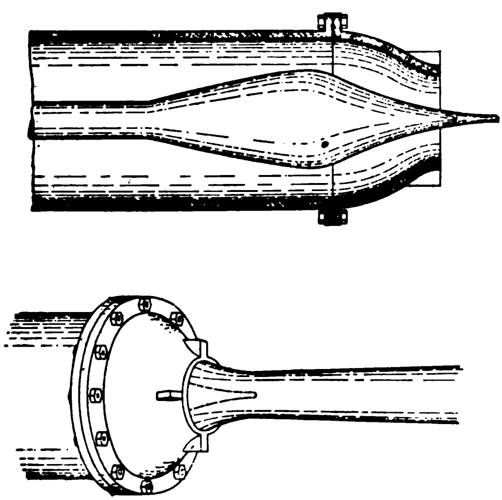


FIG. 153. — Doble Needle regulating Nozzle for Impulse Wheels.

The wheel is usually of **iron, steel**, or bronze, having **buckets** on the rim which are so shaped as to receive and deflect the water as nearly as possible through 180°, and make it fall from the wheel with a minimum residual velocity (V_e).

The buckets have an angle at entrance parallel to the direction of motion, hence $\alpha = \beta = 0$; and the angle at exit (δ) is usually

about 160° to 170°, or just enough to allow the water discharging from one bucket to clear the succeeding one. See figure 157.

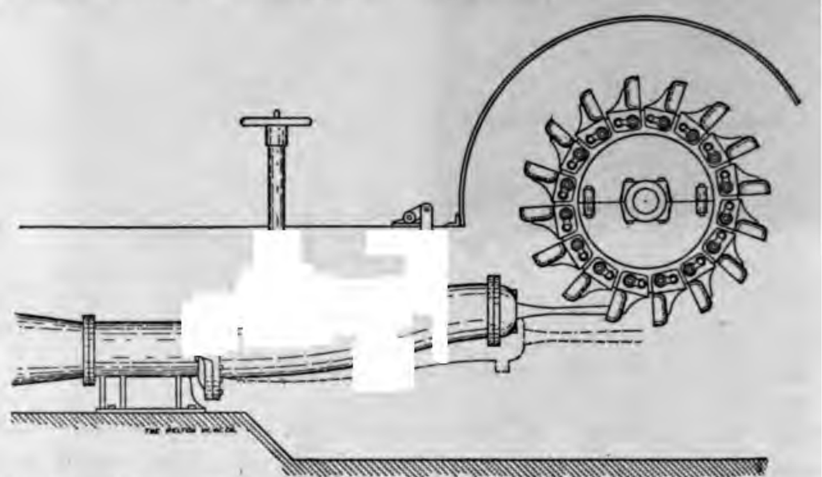


FIG. 154. — A Pelton Wheel; with Needle regulating Nozzle, and Device for deflecti[ng] the Jet as the Load Varies.

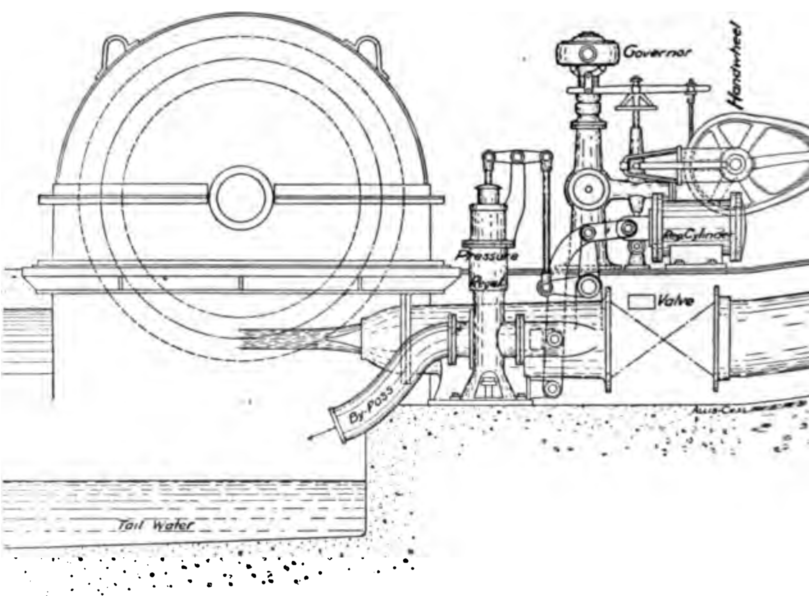


FIG. 155. — Another Type of Tangential Impulse Wheel.

458. Computation for a tangential impulse wheel, including hydraulic friction in the wheel. Figure 157 shows a section of a bucket.

The work done, formula (15), $E = \frac{wQ}{g}(V_a \cos \alpha S_0 - V_e \cos \epsilon S_1)$.

Since $r_0 = r_1$, $S_0 = S_1 = S = \omega r = \frac{\pi r N}{30}$, $d =$ nozzle diameter.

$C_0 =$ coefficient of nozzle velocity from the guides (in this case only one). $C_1 =$ coefficient of nozzle velocity from the buckets.

To find absolute entrance velocity V_a, the effective head on the nozzle orifice must be determined.

Atmospheric pressure $\left(\frac{p_a}{\gamma}\right)$ is on both head-and-tail water, and on the nozzle orifice.

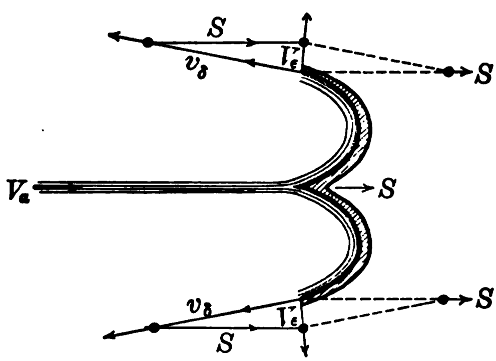


FIG. 157.—Bucket of a Tangential Wheel; Section in a Horizontal Plane through the Axis of the Jet.

The total head (H_T) is made up as follows:

(*a*) Pipe losses (neglecting minor losses)

$$= \frac{fLV_A^2}{D\,2g}, \text{ or } \left(\text{since } V_A^2 = V_a^2 \frac{d^4}{D^4}\right) = f\frac{LV_a^2}{D^5\,2g}d^4.$$

(*b*) Nozzle losses (see § 184)

$$= \frac{V_a^2}{2g}\left(\frac{1}{C_0^2} - 1\right) = \frac{V_a^2}{2gC_0^2} - \frac{V_a^2}{2g}.$$

(*c*) Kinetic energy of water discharging from nozzle

$$= \frac{V_a^2}{2g}.$$

(*e*) Distance (vertically) from nozzle orifice to tailrace

$$= h_2.$$

By Bernoulli's theorem, adding (*a*), (*b*), (*c*), and (*e*),

$$H_T + \frac{p_a}{\gamma} = \frac{p_a}{\gamma} + \frac{fLV_a^2 d^4}{D^5\,2g} + \frac{V_a^2}{2gC_0^2} + h_2.$$

Therefore,

$$V_a = 8.02\left[\frac{H_T - h_2}{\frac{fLd^4}{D^5}+\frac{1}{C_0^2}}\right]^{\frac{1}{2}}; \text{ or simply } V_a = C_0 8.02\,(H)^{\frac{1}{2}}; \quad (17)$$

And $\qquad V_a \cos\alpha = V_a \cos 0^\circ = V_a.$

The pipe should be so designed and the wheel so set as to make V_a as great as possible, consistent with most economical first cost and yearly maintenance of the pipe line. The ratio $\frac{d}{D}$ which theoretically makes E a maximum may be determined by

$$d = \left(\frac{D^5}{2fLC_0^2}\right)^{\frac{1}{4}};$$ but this is only a mathematical deduction.

To find $V_\epsilon \cos\epsilon$.

On the assumption that the water enters the wheel without shock, by diagram of velocities, figure 149,

$$v_\beta^2 = V_a^2 + S^2 - 2\,V_a S \cos\alpha. \quad (18)$$

From (18), since $\cos\alpha = +1$,

$$v_\beta^2 = V_a^2 - 2\,V_a S + S^2.$$

Then, $$v_\beta = V_a - S. \quad (19)$$

From (11), since $\frac{p_\beta}{\gamma} = \frac{p_a}{\gamma}$; and $S_0 = S_1$, and considering the losses in the bucket as nozzle or orifice losses (§ 184), by Bernoulli's theorem

$$\frac{v_\delta^2}{2g} + \frac{p_a}{\gamma} = \frac{v_\beta^2}{2g} + \frac{p_a}{\gamma} - \frac{v_\delta^2}{2g}\left(\frac{1}{C_1^2} - 1\right).$$

Then, also by (19), $\qquad v_\delta = C_1 v_\beta = C_1(V_a - S). \quad (20)$

By (4), (19), and (20)

$$V_\epsilon \cos\epsilon = v_\delta \cos\delta + S = C_1(V_a - S)\cos\delta + S. \quad (21)$$

Then, by (15), $$E = \frac{wQ}{g}(V_a \cos\alpha - V_\epsilon \cos\epsilon)\,S$$

$$= \frac{wQ}{g}(V_a - S)(1 - C_1\cos\delta)S. \quad (22)$$

NOTE. When $S = \frac{V_a}{2}$, the condition of maximum efficiency. (See (29), Chapter XVII.)

$$E = \frac{wQ}{4g}V_a^2(1 - C_1\cos\delta). \quad (23)$$

K_h, the hydraulic efficiency, $= \dfrac{(V_a - S)(1 - C_1 \cos \delta)S}{gH}$. (24)

K_p, the plant efficiency, $= \dfrac{(V_a - S)(1 - C_1 \cos \delta)S}{gH_T}$. (25)

459. *Example.* Given a tangential impulse wheel; total fall, 300 feet; diameter pipe, 2 feet; length of pipe 10,000 feet; diameter of nozzle $\frac{1}{3}$ foot; nozzle velocity coefficient, .96; nozzle orifice, 5 feet higher than tail-water; $N = 203.1$; diameter of wheel at the place where jet strikes, 6 feet; angle of guides at exit, $\delta = 170°$; friction factor, f, for pipe $= .0207$; $C_1 = .90$; $\frac{d}{D} = \frac{1}{6}$. The

$$\text{total head } 300 = .0207 \times \frac{10000}{2^5} \times \frac{V_a^2}{2g} \times \left(\frac{1}{3}\right)^4 + \frac{V_a^2}{2g \times .96^2} + 5.0.$$

$$\text{Hence by (17), } V_a = 8.02\left(\frac{300 - 5}{.0799 + 1.085}\right)^{\frac{1}{2}} = 8.02(253.3)^{\frac{1}{2}}$$

$$= 127.6 \text{ feet per second.}$$

$$S = \frac{\pi \times 3 \times 203.1}{30}$$

$$= 63.8 \left(\text{which} = \frac{V_a}{2}\right) \text{ feet per second.}$$

$$\text{Cos } 170° = -.9848;\ C_1 = .90;\ 1 - C_1 \cos 170° = 1.8863.$$

$$Q = a_A V_A = \frac{3.1416 \times 127.6}{36}$$

$$= 11.14 \text{ cubic feet per second.}$$

$$\text{By (22),} \qquad E = \frac{62.4 \times 11.14}{32.16}(127.6 - 63.8)(1.8863)(63.8)$$

$$= 165{,}850 \text{ foot pounds per second.}$$

$$\text{Horse power} = \frac{165850}{550} = 301.6.$$

The friction head in the pipe is 20.2 feet, and the distance from the nozzle orifice to tail-water is 5 feet. Therefore, the effective head $(H) = 300 - 25.2 = 274.8$.

Then K_h, the hydraulic efficiency of wheel and nozzle,

$$= \frac{(63.8)^2 \times 1.8863}{32.16 \times 274.8} = .868 = 86.8\ \%.$$

K_p, the plant efficiency, $= \dfrac{(63.8)^2 \times 1.8863}{32.16 \times 300} = .796 = 79.6\ \%.$

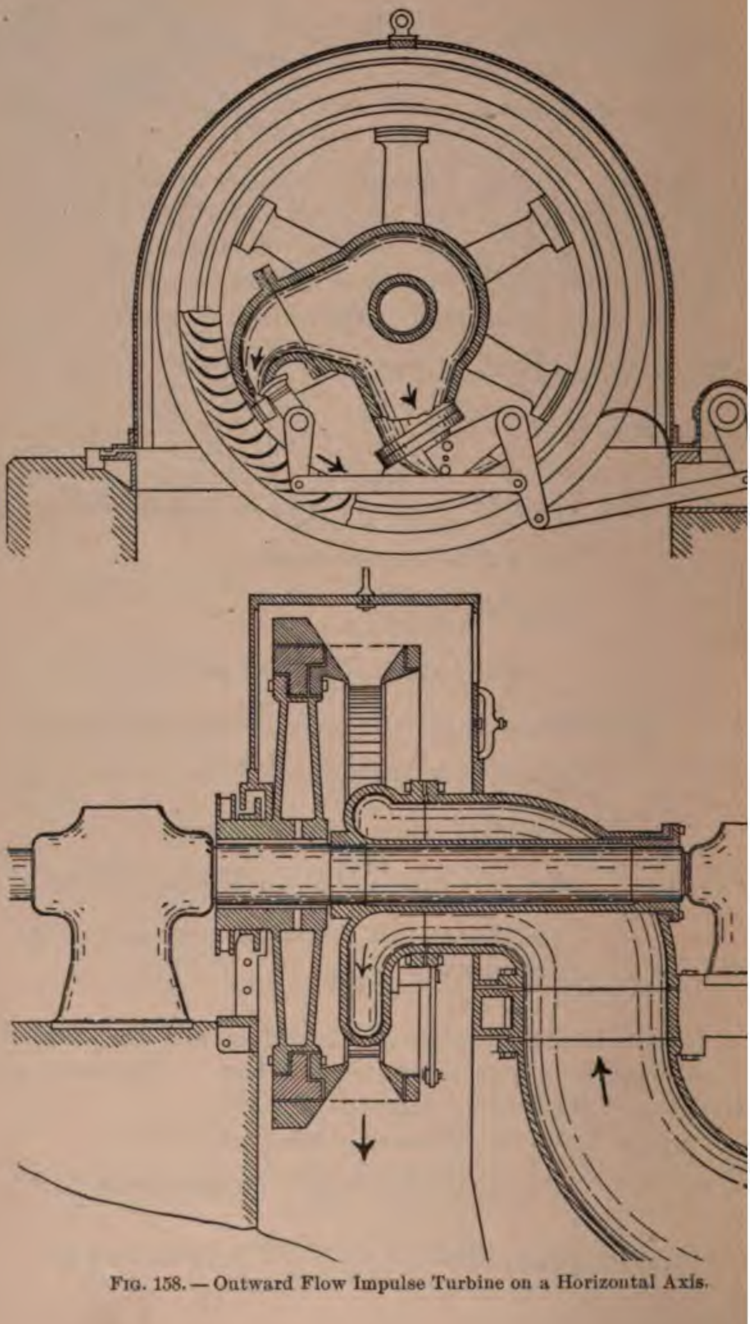


FIG. 158. — Outward Flow Impulse Turbine on a Horizontal Axis.

IMPULSE TURBINES

460. The essential principles of an impulse turbine are the same as for a tangential impulse wheel; differing from the latter in the form of nozzles, and the direction of the jet with respect to motion, in the shape of the wheel, and in the general appearance of the buckets, which somewhat resemble in shape those of simple reaction turbines. All the work is done under atmospheric pressure, only a few buckets at a time are in action, and these are not filled.

Pipe losses and the head lost from the wheel to tail-water are computed as for a tangential wheel.

The computations for work and efficiencies are, however, more intricate than those for a tangential wheel, owing to the fact that the radius of the wheel at entrance may be greater or less than the radius at exit, also the angles (α and β) at entrance are not 0°; and also there may be a gain or loss due to a small fall through the wheel.

Figure 158 shows one type of impulse turbines.

Computations for an impulse turbine, including hydraulic friction in the wheel. (See figure 159.)

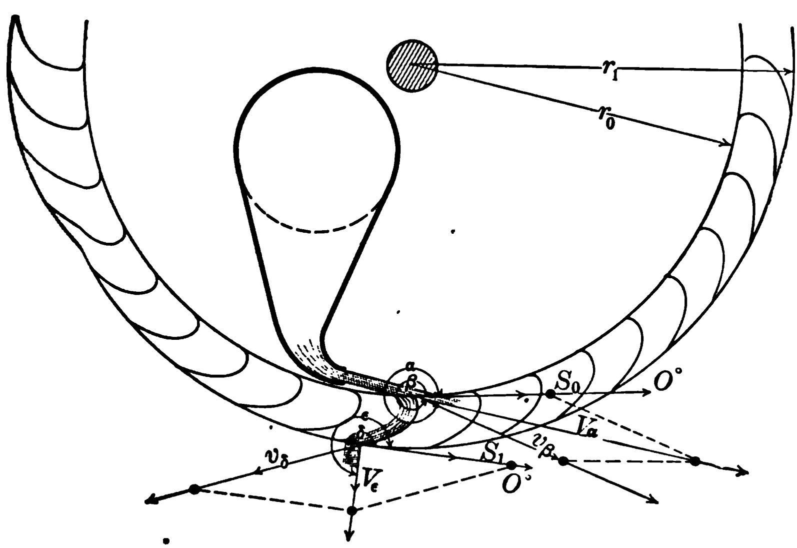


FIG. 159.—Nozzle and Half the Runner of an Outward Flow Impulse Turbine.

By (17),
$$V_a = C_0\, 8.02(H)^{\frac{1}{2}}.$$
$$C_0 = \text{about } .95.$$

On the assumption that the water enters the wheel without shock,
by diagram of velocities (figure 149),
$$v_\beta^2 = V_a + S_0^2 - 2\,V_a S_0 \cos\alpha.$$
$$S_0 = \frac{\pi r_0 N}{30};\quad S_1 = \frac{\pi r_1 N}{30}.$$

By Bernoulli's theorem,
$$\frac{v_\delta^2}{2g} + \frac{p_a}{\gamma} = \frac{v_\beta^2}{2g} + \frac{S_1^2}{2g} - \frac{S_0^2}{2g} + h_1 - \frac{v_\delta^2}{2g}\left(\frac{1}{C_1^2} - 1\right) + \frac{p_a}{\gamma}. \qquad (26)$$

Then
$$v_\delta = C_1(v_\beta^2 + S_1^2 - S_0^2 + 2gh_1)^{\frac{1}{2}}.$$

C_1, the coefficient of velocity from wheel vanes = about .95. (27)

By (4), $V_e \cos\epsilon = v_\delta \cos\delta + S_1$.

By (14),
$$E = \frac{wQ}{g}[V_a \cos\alpha\, S_0 - (v_\delta \cos\delta + S_1)S_1]. \qquad (28)$$
$$K_h = \frac{V_a \cos\alpha\, S_0 - (v_\delta \cos\delta + S_1)S_1}{gH}. \qquad (29)$$
$$K_p = \frac{V_a \cos\alpha\, S_0 - (v_\delta \cos\delta + S_1)S_1}{gH_T}.$$

461. *Example.* Given an outward flow impulse turbine (Girard wheel). See figure 159.

Effective head at nozzle orifice, 576 feet; nozzle coefficient of velocity, $C_0 = .95$; fall through the wheel, $h_1 = .65$; fall from the wheel to tail-water = 8.35 feet;

$r_0 = 4.1$ feet; $r_1 = 4.75$ feet;

$N = 200$ r. p. m.; $Q = 16$ cubic feet per second;

$\alpha = 347°\ 30'$; $\beta = 336°\ 53'$;

$\delta = 202°\ 8'$; coefficient of velocity from the buckets

$C_1 = .95.$

By (15),
$$E = \frac{wQ}{g}(V_a \cos\alpha\, S_0 - V_e \cos\epsilon\, S_1).$$
$$V_a = .95 \times 8.02(576)^{\frac{1}{2}} = 182.9 \text{ feet per second.}$$
$$\cos\alpha = \cos 347°\ 30' = +\,0.9763.$$
$$S_0 = \frac{\pi \times 4.10 \times 200}{30} = 85.87;$$
$$S_1 = \frac{\pi \times 4.75 \times 200}{30} = 99.48 \text{ feet per second.}$$

By (A), $v_\beta = (182.9^2 + 85.9^2 - 2 \times 182.9 \times 85.9 \times .9763)^{\frac{1}{2}}$
$= 100.8$ feet per second.

By (26), $v_3 = .95(100.8^2 + 99.5^2 - 85.9^2 + .65 \times 64.32)^{\frac{1}{2}}$
$= 107.2$ feet per second.

Then $V_e \cos \epsilon = 107.2 \cos (202° \, 8') + 99.5 = 0.$ See § 455 (*c*).

By (27), $E = \frac{62.4 \times 16}{32.16}(182.9 \times .9763 \times 85.9 - 0)$
$= 476{,}000$ foot pounds per second.

$$\text{Horse power} = \frac{476000}{550} = 865.$$

$$E_h, \text{ the hydraulic efficiency,} = \frac{476000}{576.65 \times 16 \times 62.4} = 0.827 = 82.7\,\%.$$

$$E_p, \text{ the plant efficiency,} = \frac{476000}{585 \times 16 \times 62.4} = 0.815 = 81.5\,\%.$$

Reaction Turbines

462. The work done in reaction turbines is due to changes in locity and direction of flowing water as in impulse wheels ; but

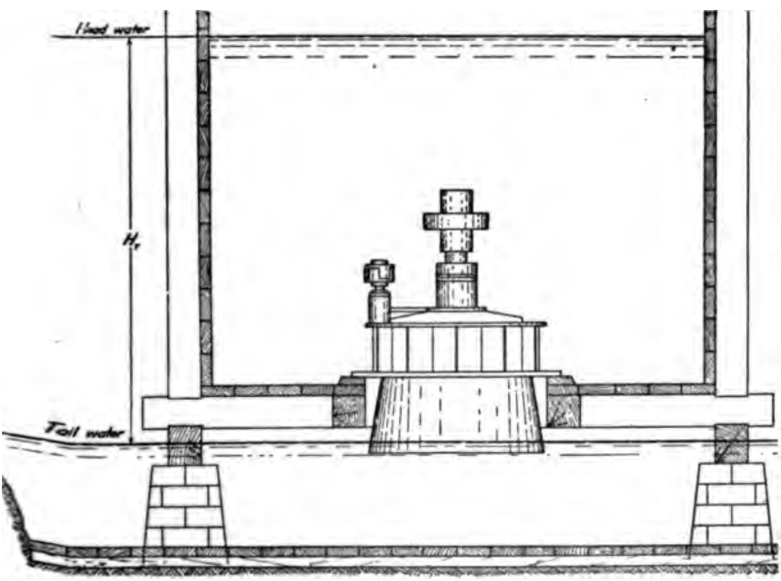


Fig. 160. — A Reaction Turbine set in an Open Flume.
(A simple but good setting.)

in reaction turbines these changes take place in a chamber s
from the air, and filled with the flowing water.

463. The setting. The wheel is set either in a pit or flum
sealed on top by a column of water (see figure 160); or
wheel case of sheet metal to which the water is conducted
pipe (penstock) tightly connected to the wheel case (see f
171).

Figure 148 shows the arrangement of a pair of reaction
bines on a horizontal shaft directly connected to an el
generator.

Figure 161 *a* is a vertical section of a single reaction turbi
a spiral case, and directly connected on a vertical shaft
electric generator; figure 161 *b* is a horizontal part plan an
tion of the same wheel.

464. The draft tube. Inward, downward, or mixed flow
bines may be built to discharge into a pipe, called a draft
which is tightly attached to the wheel case and has its outle
submerged (sealed). Outward discharge turbines usuall
charge under water, or through a compound tube called
fuser.

The function of a draft tube is to conduct the water away
the wheel, and to act as an air pump, which rarefies the air
exit from the wheel; it is comparable to a condenser of a
engine. If properly proportioned the draft tube enables the
to produce as much work as if it were set below tail-wate
in fact more; because if the wheel discharges into the water
the tail-water surface, the loss of head, caused by disturban
apt to be greater than the draft tube losses. In addition, i
means of setting the wheel economically, and where it can r
be inspected or repaired.

The pressure p_1 at the exit from the buckets should theoret
be equal to $p_a - h_2\gamma$, but is usually somewhat more or less thar

The draft tube should not be laid on a flat slope, as air
to rise through the water and decrease the rarefaction
should preferably have a shape which gradually enlarges t
the discharge end. The dimensions should be fixed with refe
to the average use of water.

The height of the draft tube above tail-water is limited, th

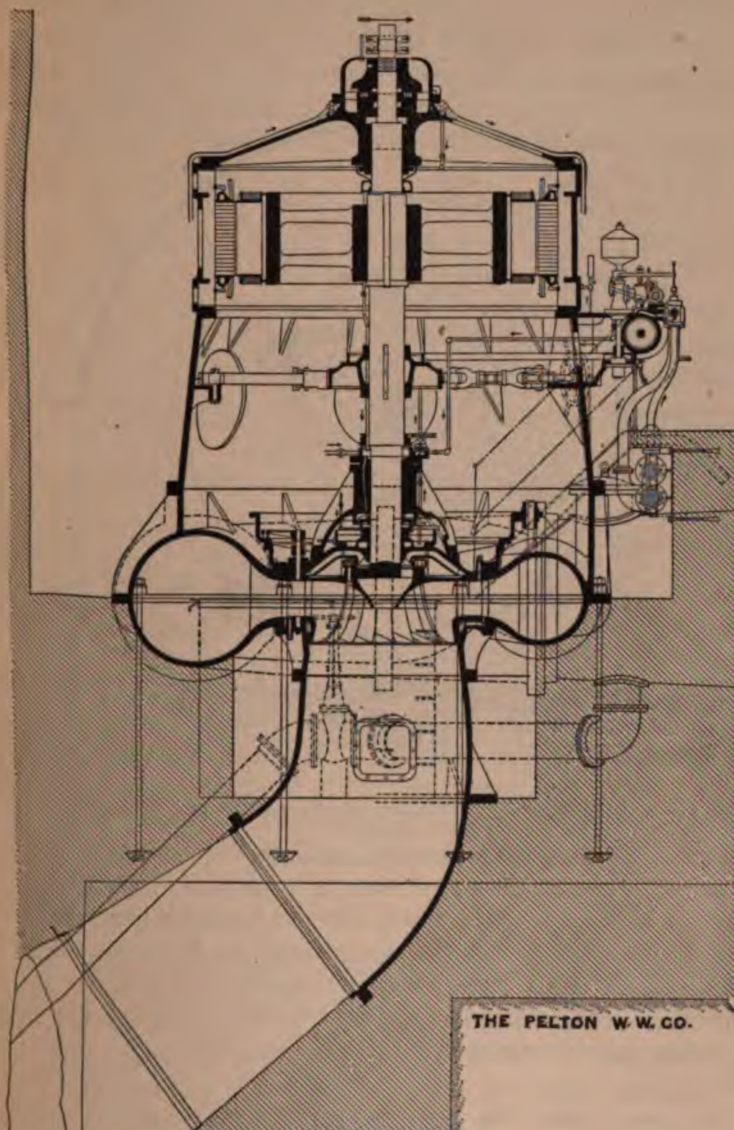


FIG. 161 *a*. — A Reaction Turbine in a Spiral Case, and with a gradually enlarging Draft Tube; directly connected with an Electric Generator.

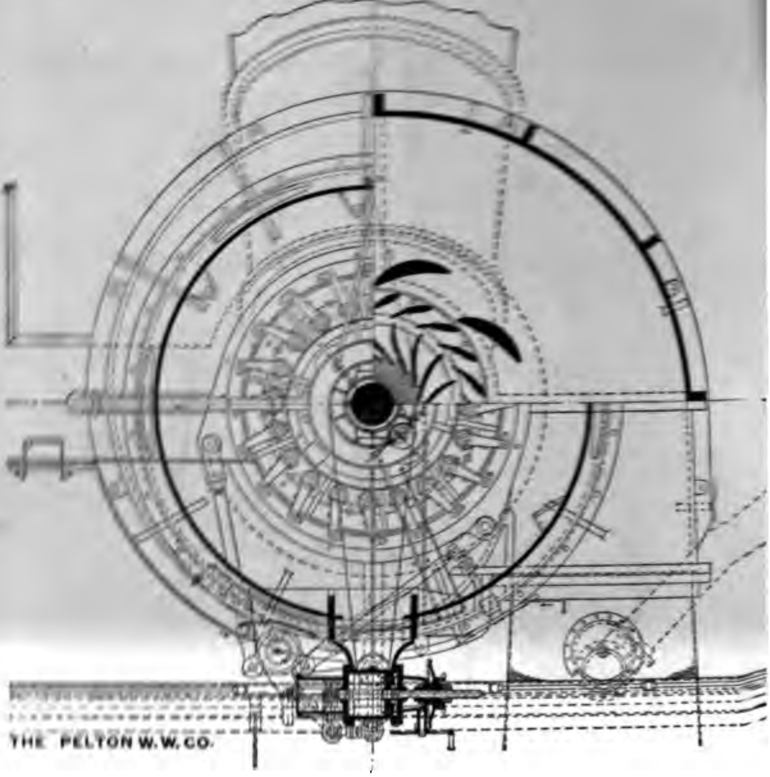


FIG. 161 b. Reaction Turbine; Horizontal Part Plan and Section of the Wheel sho[wn]
in 161 *a*.
(This is a very modern wheel.)

ically, to 34 feet; actually it will have to be much less. Mülle[r]* gives approximately the following relations between diamet[er] and height of draft tube, based on a study of existing pla[nts] which show that the height (h_2) must be diminished as the dia[m]eter is increased to about the following proportions:

Diameters,	feet	4	5	6	7	8	9	10	12	14
Heights,	feet	24	22	19	17	15	14	13	11	10

From these heights $\left(\frac{V_d^2}{2g} + \text{losses in the tube}\right)$ must be s[ub]tracted in fixing the allowable height h_2.

* Die Francis-Turbinen, p. 24.

465. The effective head. The effective head on the wheel is: **the pressure head plus the velocity head at the base of the guide orifices** added to **the fall through the wheel, less the pressure head at the runner orifices,** or the top of the draft tube if there is one. If p_1 is atmospheric pressure, the whole drop h_2 from the runner will be lost; if $\frac{p_1}{\gamma} = -h_2$, none of h_2 is lost.

Then the effective head, $H = \frac{p_0}{\gamma} + \frac{V_A^2}{2g} + h_1 - \frac{p_1}{\gamma}$.

If then from the total head H_T be subtracted the losses in conducting the water to the wheel and away from it, the remainder is the effective head under which the wheel works, or again, in other terms,

$$H = H_T - h_\lambda - h_d.$$

Theoretically the head due to velocity of approach $\left(\frac{V_A^2}{2g}\right)$ is as useful as pressure head; actually high velocity of approach at the entrance to the guides is detrimental to good working.

In the wheel itself there are other losses which will be considered independently.

466. The pressure heads at entrance to the guides, and at exit from the runner. The pressure head, p_0, at the entrance to the guides may be measured by a gauge set in the penstock at the base of the guides; and p_1 by another gauge (vacuum) set in the top of the draft tube; the algebraic difference of the two pressures corrected for any difference in elevation between the gauges will be a measure of the effective head under which a wheel is actually working.

467. The pressure head at the entrance to the runner. As the intensity of pressure changes from point to point in the passage of the water through the wheel, the pressure p_β at the entrance to the runner has a value lying somewhere between p_0 and p_1, which depends upon the form of the guides and the speed of wheel. When, as in the figure 162, $\beta = 2\alpha$, it can be shown that p_β = atmospheric pressure, and the wheel becomes an impulse wheel.

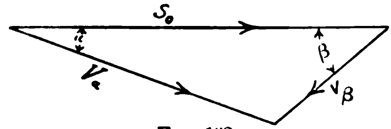


FIG. 162.

468. The entrance velocity not due to the whole effective head. The absolute velocity V_a from the guides is computed as for any nozzle; but as the intensity of the pressure, p_β, into which the water is discharged from the guides of a reaction wheel is greater than atmospheric, only part of the effective head H is used in producing V_a.

469. Reaction coefficient. The proportion of H which is used in producing V_a in a reaction turbine is usually about .5, but whatever its value, it may be represented by κ, the so-called "*reaction coefficient.*" If the reaction coefficient becomes 1.0, p_β is atmospheric pressure, and the wheel ceases to be a reaction turbine and becomes an impulse wheel.

The reaction coefficient, $\kappa = \dfrac{V_a^2}{2gC_0^2H}$.

In designing wheels the reaction coefficient is assumed to suit the requirements. If κ is known, then $V_a = C_0\, 8.02\, (\kappa H)^{\frac{1}{2}}$.

470. Velocity at exit. The water leaving the guides enters the buckets partly in the form of kinetic energy, $\left(\dfrac{V_a^2}{2g}\right)$; and partly in the form of pressure head, $\left(\dfrac{p_\beta}{\gamma}\right)$. The sum of this energy is diminished in passing from the guides to the buckets by leakage through the clearance space, and by eddy currents. Treating the buckets as nozzles, the velocity (v_δ) relative to the edge of the runner orifices will be due to the sum of energy transmitted in the water from the guides plus the algebraic difference between the kinetic energy of rotation, due to the speed of the rims at entrance (S_0) and exit (S_1), and the fall through the wheel if any, less losses in the wheel vanes. That is,

$$\frac{v_\delta^2}{2g} = \frac{V_a^2}{2g} + \frac{p_\beta}{\gamma} - \frac{S_0^2}{2g} + \frac{S_1^2}{2g} + h_1 - \text{clearance losses} - \text{bucket losses}.$$

471. Computations for a reaction turbine, including hydraulic friction in the wheel.

$$V_a = C_0 \times 8.02\,(\kappa H)^{\frac{1}{2}}; \text{ also, } V_a = \frac{Q}{a_a}. \qquad (30)$$

C_0 has a value about .95 to .97.

The guide losses in terms of nozzle velocity

$$= \frac{V_a^2}{2g}\left(\frac{1}{C_0^2} - 1\right) = (.11 \text{ to } .06)\frac{V_a^2}{2g}. \tag{31}$$

The losses at the clearance $= (.08 \text{ to } .06)\,\frac{V_a^2}{2g}$ (assumed). (32)

$$\text{Therefore } \frac{p_\beta}{\gamma} = H - \frac{V_a^2}{2g} - \frac{V_a^2}{2g}\left(\frac{1}{C_0^2} - 1\right) - (.08 \text{ to } .06)\frac{V_a^2}{2g}. \tag{33}$$

$$S_0 = \frac{\pi r_0 N}{30};\quad S_1 = \frac{\pi r_1 N}{30};$$

On the assumption that the water enters the runner without shock, (see diagram, figure 149),

$$v_\beta^2 = V_a^2 + S_0^2 - 2\,V_a S_0 \cos\alpha. \tag{A}$$

By Bernoulli's theorem,

$$\frac{v_\delta^2}{2g} = \frac{v_\beta^2}{2g} - \frac{S_0^2}{2g} + \frac{S_1^2}{2g} + \frac{p_\beta}{\gamma} + h_1 - \frac{v_\delta^2}{2g}\left(\frac{1}{C_1^2} - 1\right). \tag{34}$$

C_1, the coefficient of velocity from the runner orifice, may be taken as .94 to .96.

From (34), $$v_\delta = C_1\left[v_\beta^2 + S_1^2 - S_0^2 + 2\,g\left(\frac{p_\beta}{\gamma} + h_1\right)\right]^{\frac{1}{2}}. \tag{35}$$

By (4), $V_\epsilon \cos\epsilon = v_\delta \cos\delta + S_1$.

Hence as in (14),

$$E = \frac{wQ}{g}\left[V_a \cos\alpha\, S_0 - (v_\delta \cos\delta + S_1)\, S_1\right];$$

and as in (15), $$E = \frac{wQ}{g}(V_a \cos\alpha\, S_0 - V_\epsilon \cos\epsilon\, S_1). \tag{15}$$

The formulas for efficiency are identical with those for an impulse turbine (28) and (29).

472. *Example.* **An inward flow reaction turbine,** with 40 guides and 40 buckets, has the following dimensions:

$r_0 = 4.669$ feet; $r_1 = 3.994$ feet;

$\alpha = 349°$; $\delta = 198°$;

$a_a = 40 \times 1.007 \times .1467 = 5.909$; $a_\delta = 40 \times 1.23 \times .1384 = 6.809$.

This wheel is J. B. Francis's "center vent wheel with curved buckets," and shown in figures 151, 163, and 164. In a series of tests made by him on this wheel the highest efficiency was ob-

tained at a speed of $N = 38.1$, under a head of 13.36 feet, and the discharge through the wheel measured by a weir was 113.1 cubic feet per second.

Figure 163 shows a vertical and horizontal section of the Francis turbine used in this example.

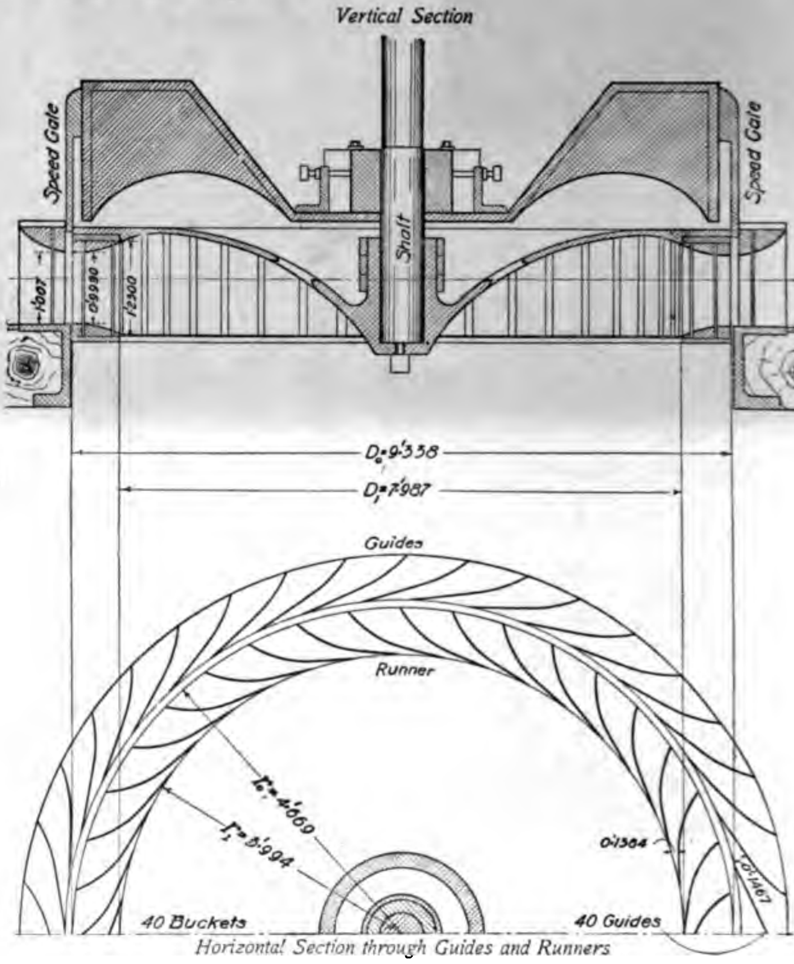


FIG. 163. — J. B. Francis's Inward Flow Reaction Turbine.

Figure 164 shows the dimensions by which the areas of a single guide orifice and a single bucket orifice are determined for the wheel shown in figure 163.

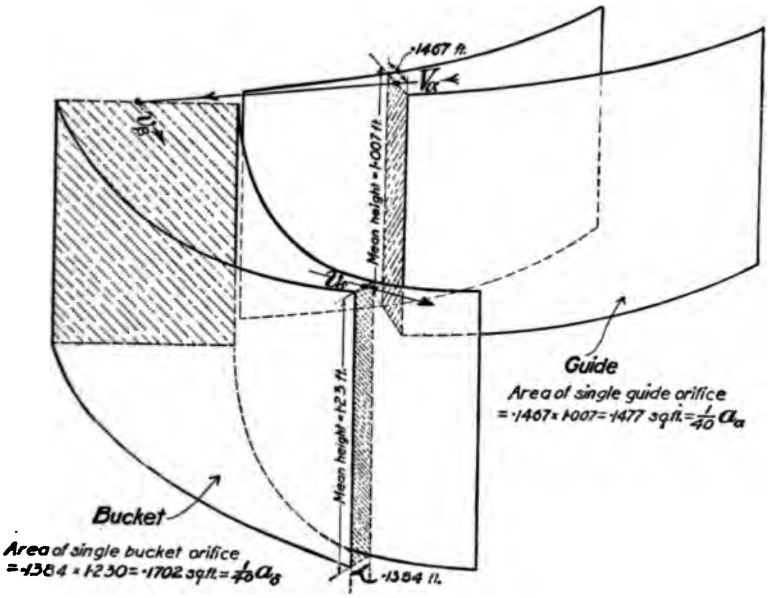


Fig. 164. — Showing the Dimensions which determine the Guide and Bucket Orifices in a Simple Form of Turbine. (This is the Francis Wheel.)

Computations based on the data given are as follows:

By (14) and (15), $E = \dfrac{wQ}{g}[V_a \cos \alpha S_0 - (v_\delta \cos \delta + S_1)S_1]$.

$V_a = \dfrac{Q}{a_a} = \dfrac{113.1}{5.909} = 19.14.$ $\qquad v_\delta = \dfrac{Q}{a_\delta} = \dfrac{113.1}{6.809} = 16.61.$

$S_0 = \dfrac{\pi \times 4.669 \times 38.1}{30} = 18.63;$ $\quad S_1 = \dfrac{\pi \times 3.994 \times 38.1}{30} = 15.93.$

$$V_a \cos \alpha S_0 = 19.14 \times .9816 \times 18.63 = +350.$$

$$v_\delta \cos \delta S_1 = 16.61 \times (-.9511) \times 15.93 = -251.7.$$

$$S_1^2 = 15.93^2 = +253.8.$$

By (14), $\quad E = \dfrac{62.4 \times 113.1}{32.16}[350 - (\overset{+347.9}{-251.7} + 253.8)]$

$$= 76340 \text{ foot pounds per second.}$$

$\text{H.P.} = \dfrac{76340}{550} = 138.8.$ $\quad K_h = \dfrac{347.9}{32.16 \times 13.36} = .81 = 81$ per cent.

Francis's test results were;

$E_B = 75143$; H.P. $= 136.6$; $K_B = 79.7$ per cent.

The penstock was large and very short and there was no draft-tube; the total head and the effective head were practically identical; and Francis's efficiency included the effect of machine friction.

Example. **Given an outward flow reaction turbine,** working under the following conditions: effective head = 130 feet; $Q = 44$[illegible] cubic feet per second; $N = 233\frac{1}{2}$; $r_0 = 2.625$ feet; $r_1 = 3.1$[illegible] feet; $C_0 = .95$; $C_1 = .95$; $\kappa = .48$; $\alpha = 19° 6'$; $\beta = 110° 4$[illegible] $\delta = 166° 43'$. $a_a = 7.45$ square feet; $a_\delta = 6.03$ square feet.

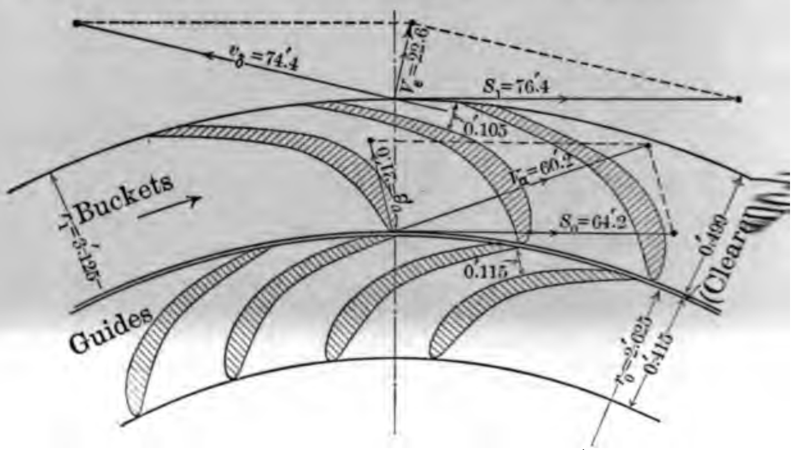


Fig. 165. — Outward Flow Reaction Turbine.
(86 guides and 82 buckets.)

Figure 165 shows part of the guides and buckets; figure 166 is a part plan, and sections of this wheel.

The absolute velocity at entrance,

By (30), $$V_a = .95(2\,g \times .48 \times 130)^{\frac{1}{2}} = 60.2.$$

Also $$V_a = \frac{Q}{a_a} = \frac{448.5}{7.45} = 60.2 \text{ feet per second.}$$

The losses in the guides by (31),

$$= \frac{V_a^2}{2\,g}\left(\frac{1}{.95^2} - 1\right) = .108\,\frac{V_a^2}{2\,g}, \text{ feet.}$$

The losses in the clearance (assumed) $= .070\,\dfrac{V_a^2}{2\,g}$ feet.

Then the pressure head at entrance to the runner, by (33);

$$\frac{p_\beta}{\gamma} = 130 - \frac{60.2^2}{2g} - .108\,\frac{60.2^2}{2g} - .07\,\frac{60.2^2}{2g} = 63.6 \text{ feet.}$$

$$S_0 = \frac{\pi \times 2.625 \times 233.5}{30} = 64.2\,; \quad S_1 = 64.2\,\frac{3.125}{2.625} = 76.4 \text{ feet per second.}$$

The velocity relative to the runner at entrance, by (A)

$$v_\beta = [60.2^2 + 64.2^2 - 2 \times 60.2 \times 64.2 \times \cos(19^\circ\ 6')]^{\frac{1}{2}}$$
$$= 21.0 \text{ feet per second.}$$

Then the velocity relative to the runner at exit, by (35);

$$v_\delta = .95[21^2 + 76.4^2 - 64.2^2 + 64.32 \times 63.6]^{\frac{1}{2}}$$
$$= 75.0 \text{ feet per second,}$$

Also $\quad v_\delta = \frac{Q}{a_\delta} = \frac{448.5}{6.03} = 74.4$; using this latter value of v_δ;

Then $V_e \cos \epsilon = 76.4 + 74.4 \cos 166^\circ\ 43' = 76.4 - 72.4 = +4.0.$

By (14), $E = \frac{62.4 \times 448.5}{32.16}[60.2(\cos 19^\circ\ 6') \overset{(+3346)}{\times 64.2} - 4 \times 76.4]$

$$= 2{,}912{,}000 \text{ foot pounds.}$$

Horse power $= \frac{2912000}{550} = 5295.$

Hydraulic efficiency, $K_h = \frac{3346}{32.16 \times 130} = .80 = 80$ per cent.

473. Illustrations of reaction turbines. Figure 166 is an outward flow turbine, the one used in the example immediately preceding.

Figure 167 (Plate I) shows a Boyden wheel, the earliest turbine of the outward flow type used in America. (Compare this with the wheel in figure 166.) Figure 167 *a* shows a set of guides, and a runner. Figure 167 *b* shows an outside view of another Boyden wheel set in place. It shows also a Boyden diffuser, which is merely a flaring outlet built around the entire wheel at exit from the runner. The crosshatching and dimensions show how areas of guide orifices (a_a) and of runner or bucket orifices (a_δ) are determined.

Figure 168 (Plate II) shows two views of a Swain runner, the earliest type of the upward and downward flow (mixed flow) type. Crosshatching shows the area of a single runner orifice.

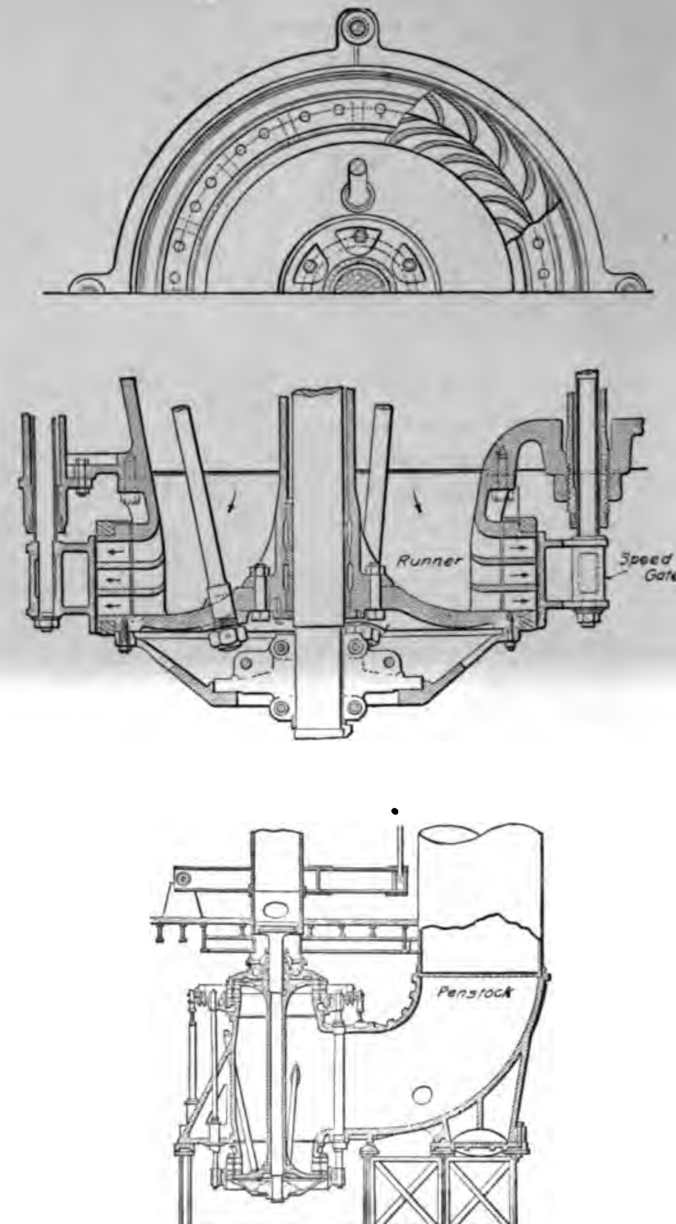


Fig. 166. — Outward Flow Reaction Turbine (a very modern wheel).

The guides and gate for this wheel are similar to those shown in figures 169 and 170.

Figure 169 *a* shows the inside of a wheel case, and the guides of a pair of modern turbines (54 inches diameter). Figure 169 *b* shows the inside of one of the draft tubes and the bottom of the runner of one of the pair of wheels of which the guides are shown in figure 169 *a*. The crosshatching on 169 *a* shows a guide orifice area, on 169 *b* a runner orifice area. (See Plate III.)

Figure 170 *a* shows part of the wheel case for a pair of 33-inch turbines. In this setting each wheel discharges through a short quarter turn into a common draft tube. Figure 170 *c* shows more of the case, and the guides of the wheels which were set with figure 170 *a*. Figure 170 *b* shows the pair of runners with their shafting, which were set into the case shown in 170 *a* and *c*. Figure 170 *d* shows one of these runners with an orifice area (a_5) marked. (See Plates IV and V.)

474. From the fundamental principles, formulas or equations for proportioning turbines may be deduced. By the aid of these equations, starting with a known head (H), the required speed (usually determined by the character of the installation), a reasonable assumption of efficiency based on tests, and data gathered from practice and tests as to diameters, speed factors, reaction coefficients, shape of guides and buckets, wheels may be designed. This subject is too detailed and difficult to be further treated in a book on elementary hydraulics; and the mere elaboration of rational formulas without test data has little practical usefulness.

The runners of the turbines selected for illustration in figures 163, 164, 165, 166, and 167 represent very well some of the simple types of turbines; and the examples illustrate the application of the basic formulas to turbines. The runners of the types represented in figures 168, 169, 170, and 172 are more complex; and the water through the bucket orifices cannot be treated as a single stream, but as an aggregate of streams, each of which has its own path through the runner and each of which must be independently computed; the computations, while more difficult, because much more detailed, are in principle the same as for simple types.

475. Summary of the losses of energy in turbines. In addition to the hydraulic losses due to friction and change of velocity and

direction in the penstock, wheel case, wheel, and draught tube, leakage and energy carried away, there are certain losses due to the machine friction. The total losses have been variously estimated in terms of percentage of the energy available. They may be stated approximately as follows:

Hydraulic losses	8 to 15 %
Leakage between guides and runner	2 to 3 %
Energy carried away by water	3 to 7 %
Friction of runner and shafting	2 to 5 %
	15 to 30 %

This means an efficiency of 85 per cent to 70 per cent.

476. Commercial efficiency. The efficiency must be clearly defined in any case, but in general the ordinary trade use of the term means the ratio of the brake horse power with wheel counterbalanced as measured on the shafting to the horse power due to the discharge of the wheel for the total net fall measured very close to the wheel. See figure 148. All losses between the points where the head is measured are therefore charged against the wheel. In general commercial usage,

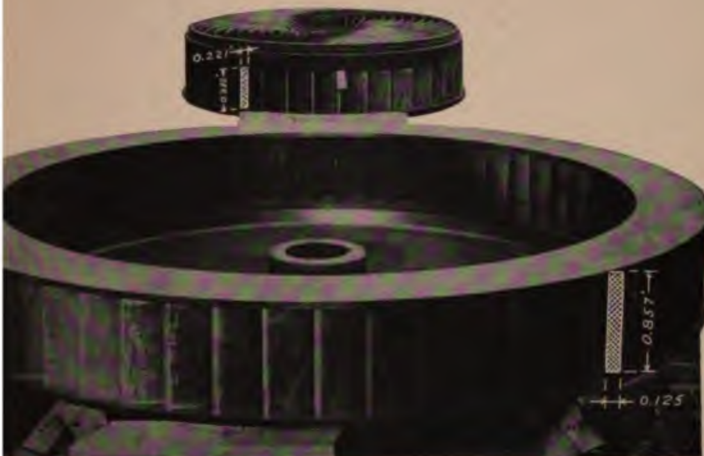
$$\text{Efficiency} = \frac{\text{Brake horse power} \times 550}{wQ\,(H - h_\lambda)}. \tag{36}$$

An efficiency of 75 per cent for a good modern wheel, if properly set up and run, is a safe basis of estimate; 80 per cent is usually specified for tests, and efficiency as high as 85 per cent is occasionally attained in tests where the conditions are favorable; but the losses due to defective settings may entirely offset any particular merit in a wheel.

Testing Water Wheels

477. Water wheels may be tested either in a testing station or flume or after being set up in place and ready to run. Testing in place, if feasible, is a more trustworthy guide as to the actual performance, because such a test includes the effects of the wheel settings, which are a very important factor in the output of work.

Testing wheels in place has not until recent years been feasible in most plants: because of the expense and difficulty of setting up a friction brake to measure the work; and because a weir,



7 a.—Runner (lower figure) and Guides (upper figure) of a Boyden Wheel, an Outward Flow Reaction Turbine.

b.—Boyden Wheel in Place (similar to 167 a, but not the same wheel). Out-iew of the Runner, looking toward the Center through a "Boyden Diffuser."

PLATE I

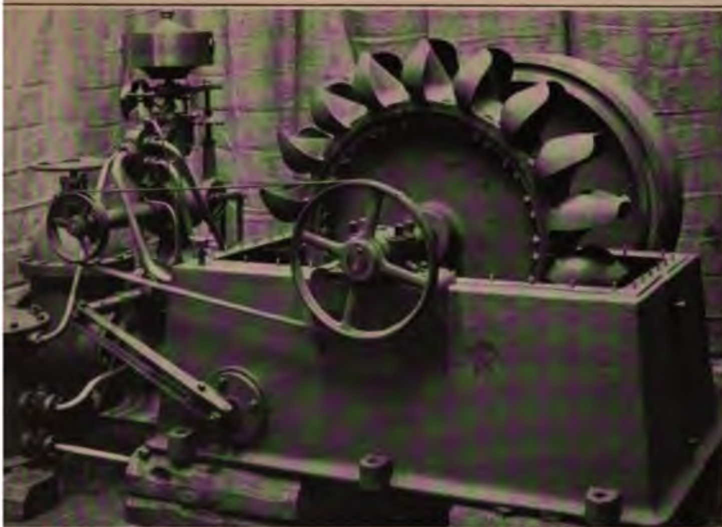


FIG. 156. — A Tangential Impulse Wheel ; European "Pelton Wheel" of Special Design.

into the Entrance Orifices of the Runner.

Looking into the Exit Orifices of the Runner.

168. — Runner of Swain Wheel ; Inward-downward Flow Reaction Turbine.

FIG. 172. — A 48-inch Runner. The tests of this wheel are worked out in §§ 479 to 494. The hatching shows the area (a_2) of an exit orifice.

PLATE II

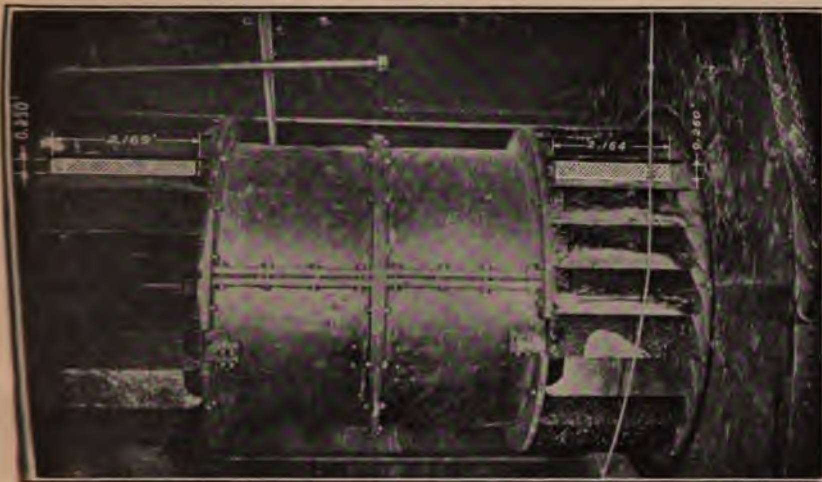


FIG. 169 *a*. — Inside of Wheel Case, showing the Entrances to Guides of Both Wheels.

FIG. 169 *b*. — Inside of Quarter Turn of Draft Tube (partly removed), showing Exit Orifices of One Runner.

Modern Reaction Turbines (mixed flow); a Pair of 54-inch Turbines set on the Same Shafting, each Wheel having a Separate Draft Tube.

PLATE III

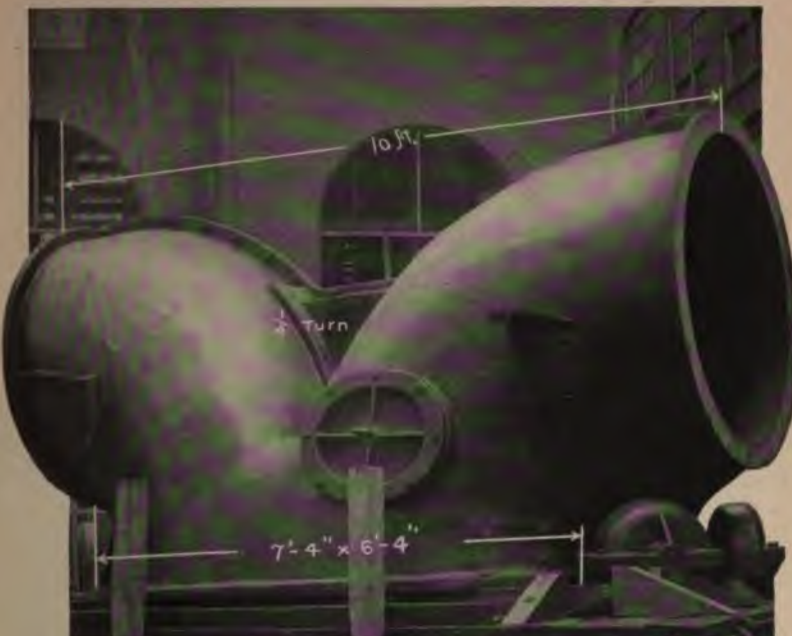


Fig. 170 *a*. — Casting of Quarter Turns for the Pair of 33-inch Wheels shown in Figure 170 *b*.

Fig. 170 *b*. — Modern Reaction Turbines (mixed flow); a Pair of 33-inch Runners on the Same Shafting, both Wheels discharging into a Common Draft Tube, each through a Quarter Turn.

PLATE IV

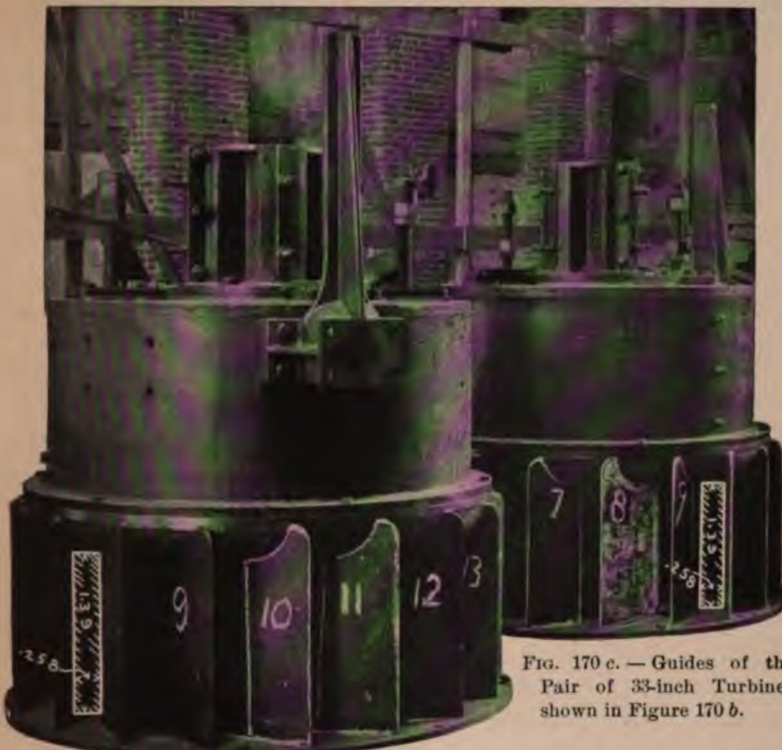


Fig. 170 c. — Guides of the Pair of 33-inch Turbines shown in Figure 170 *b*.

Fig. 170 *d*. — One of the 33-inch Runners of the Pair shown in Figure 170 *b*, showing the Area (a_3), of One Exit Orifice.

PLATE V

which has been the only satisfactory means of measuring the discharge, can not be operated without a considerable diminution in the head acting on the wheel, which may interfere with the ordinary running of the plant. In modern hydroelectric plants, a new electric generator in which the losses have been previously determined by tests in the maker's shops furnishes a measure of the brake horse power. The Alden dynamometer,* which can be readily transported and set up, furnishes an accurate and convenient brake for measuring the horse power of wheels in place. The space occupied by this brake is usually no greater than that required for the generator, which is operated by the wheels. With these methods of measuring power, and with the well-established use of current meters, rod floats, and the increasing use of the pitometer, wheels of practically any capacity can be accurately tested in place both for power and for consumption of water.

478. The Holyoke testing flume.† The only public testing flume in the United States is one owned and operated by the Holyoke (Mass.) Water Power Company. This testing flume, in brief, comprises: a wheel pit 20 feet square into which the wheel is set for testing; gauges for measuring the head acting on the wheel; Prony brakes for measuring the work done by the wheel, which can be set for either vertical or horizontal shafts; and a weir similar to Francis's original weir, but with some modifications in the apparatus for measuring the head. The capacity of the weir is about 230 cubic feet per second; small wheels can be tested for heads from 4 to 18 feet, and large wheels from 11 to 14 feet. Turbines, on vertical shafts, up to 300 *HP* have been tested. In this plant several thousand wheels have been tested; and on these tests, the design of low-head American turbines is largely based.

479. A Holyoke test and computations. The necessary observations comprising a water wheel test, and the common deductions from a test, are indicated by the following copy of a Holyoke test, and certain computations (in italics) which an engineer would usually make to adapt the test to the conditions under which the wheel is to run. See Table LX.

* Description by C. M. Allen, *Engineering Record*, March 31, 1906, and February 9, 1907. † R. H. Thurston, *Trans. Am. Soc. M. E.*, Vol. 8, p. 359.

TABLE LX

Testing Flume at the Holyoke Water Power Company, Holyoke, Mass.

Report of tests of a 48" turbine wheel with cylinder gate, conical draft tube

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
	Proportional Part of											
No. of Experiment	Full Opening of Speed Gate	Full Discharge of the Wheel	Head acting on Wheel In Feet (H)	Duration of Experiment In min.	Revolutions of the Wheel per Minute N	Quantity of Water discharged by Wheel Cubic Feet per Second Q	Power developed by the Wheel HP	Efficiency of the Wheel In %	*Speed Factor* SF	*Quantity of Water at 21 Feet Head* Q_{21}	*Horse Power at 21 Feet Head* HP_{21}	*Coefficient of Discharge for Runner Orifices* C_1
9	1.000	0.996	15.50	2	95.00	208.38	281.64	76.89	*.630*	*242.6*	*444.9*	*.506*
8	1.000	0.994	15.48	3	105.33	207.78	290.97	79.87	*.699*	*242.1*	*459.9*	*.508*
7	1.000	0.994	15.44	3	112.00	207.63	294.30	80.95	*.745*	*242.2*	*466.9*	*.509*
6	1.000	0.995	15.41	3	115.00	207.48	294.44	81.20	*.766*	*242.3*	*468.3*	*.509*
5	1.000	0.999	15.40	3	119.67	208.23	298.33	82.03	*.797*	*243.2*	*475.2*	*.508*
4	1.000	0.997	15.39	3	123.00	207.93	298.34	82.21	*.819*	*242.9*	*475.5*	*.507*
11	1.000	0.999	15.36	4	122.75	208.08	297.74	82.14	*.818*	*243.3*	*476.0*	*.508*
10	1.000	1.000	15.42	3	127.33	208.68	300.27	82.25*	*.848*	*243.3*	*477.1*	*.508*
3	1.000	0.993	15.42	3	128.67	207.18	294.76	81.36	*.857*	*242.0*	*468.4*	*.505*
2	1.000	0.955	15.62	4	135.50	200.47	273.89	77.12	*.896*	*232.4*	*427.0*	*.485*
1	1.000	0.729	16.70	3	181.33	158.32						
18	0.780	0.892	15.85	2	103.00	188.71	270.65	79.79	*.676*	*217.2*	*412.8*	*.454*
17	0.780	0.891	15.91	3	107.00	188.85	273.95	80.40	*.701*	*217.0*	*415.3*	*.453*
16	0.780	0.889	15.90	3	110.00	188.27	274.22	80.78	*.721*	*216.4*	*410.4*	*.452*
15	0.780	0.886	15.92	3	113.67	187.84	275.71	81.30	*.745*	*215.8*	*417.7*	*.450*
19	0.780	0.884	15.79	3	115.67	186.68	272.77	81.60*	*.761*	*215.3*	*418.3*	*.449*
14	0.780	0.874	16.08	2	119.00	186.24	272.01	80.27	*.776*	*212.9*	*406.8*	*.445*

13	0.780	0.850	16.10	3	122.00	183.07	263.04	78.09	.794	299.2	392.0	.437
12	0.780	0.822	16.34	2	130.50	176.65	246.10	75.21	.844	200.4	358.3	.418
27	0.638	0.783	16.28	4	92.50	167.85	227.48	73.86	.599	190.7	333.2	.398
26	0.638	0.785	16.28	3	98.33	168.23	235.10	75.72	.636	191.2	344.5	.399
25	0.638	0.783	16.21	3	102.00	167.53	237.10	76.98	.662	190.7	349.7	.398
24	0.638	0.780	16.24	2	105.00	167.12	237.00	77.00	.681	190.0	348.5	.397
23	0.638	0.778	15.92	3	108.09	164.90	232.85	78.21*	.707	189.4	352.8	.395
22	0.638	0.763	16.12	3	114.00	162.70	230.43	77.47	.742	185.7	342.6	.388
21	0.638	0.748	16.28	3	120.33	160.36	227.01	76.67	.779	182.2	332.5	.380
20	0.638	0.734	16.41	4	127.25	157.91	214.34	72.94	.821	178.7	310.3	.373
34	0.500	0.655	16.74	3	89.67	142.51	181.25	66.99	.573	159.7	254.7	.333
33	0.500	0.655	16.75	3	97.33	142.38	186.90	69.10	.621	159.5	262.3	.333
32	0.500	0.649	16.87	4	102.00	141.72	188.99	69.70	.649	158.2	262.5	.330
31	0.500	0.645	16.90	2	106.00	140.80	189.26	70.13*	.674	157.1	262.2	.328
30	0.500	0.638	17.11	4	112.50	140.15	189.50	69.68	.711	155.3	257.8	.324
35	0.500	0.633	16.84	4	115.25	138.06	182.48	69.21	.734	154.2	254.1	.322
29	0.500	0.619	17.19	4	123.50	136.37	174.74	65.73	.778	150.7	236.0	.315
28	0.500	0.602	17.13	3	133.00	132.37	152.34	59.24	.840	146.6	206.8	.306
42	0.370	0.525	16.95	4	85.25	114.89	132.11	59.82	.541	127.9	182.2	.267
41	0.370	0.524	16.94	3	92.67	114.64	134.24	60.95*	.588	127.6	185.3	.266
40	0.370	0.520	16.85	3	97.67	113.42	131.61	60.72	.622	126.6	183.1	.264
39	0.370	0.513	16.96	4	103.75	112.21	129.32	59.92	.658	124.9	178.2	.261
38	0.370	0.507	17.09	4	109.50	111.36	125.42	58.11	.692	123.5	170.8	.258
37	0.370	0.495	17.15	4	120.00	108.83	113.19	53.48	.757	120.4	153.4	.251
36	0.370	0.473	17.31	3	132.33	104.67	89.16	43.39	.832	115.3	119.1	.241

NOTE. For experiment No. 1, jacket loose. During the above experiments, the weight of the dynamometer, and of that portion of the shaft which was above the lowest coupling, was suspended by ball bearing. With the flume empty, a strain of 16 lbs. applied at a distance of 3.5 feet from the center of the shaft sufficed to start the wheel.

* Highest efficiency for each gate opening.

The wheel referred to in this test is one of a pair which shown in their wheel case in figure 171.

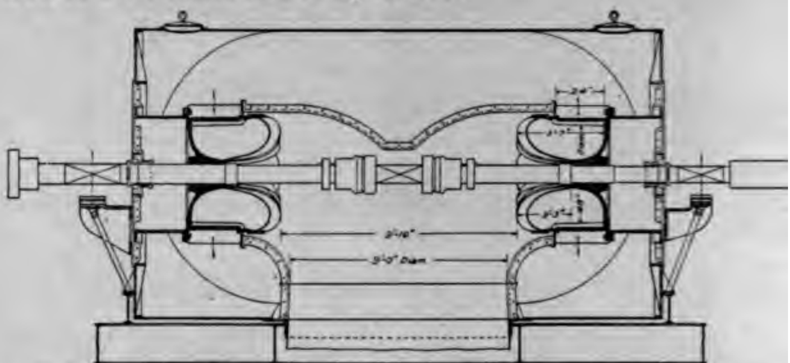


FIG. 171.—Modern Reaction Turbines ; a Pair of 48-inch Wheels, discharging int Draft Tube.

NOTE. The test of one of these wheels is given in Table LX. The discharge gram, Fig. 174, is for this pair.

A view of the runner is shown in figure 172 (Plate II), the cro hatching showing how the areas (a_8) of the runner orifices we determined. See § 483.

480. The items in the original report of the test are follow

COLUMN 1: The number of the experiment or run.

COLUMN 2: The proportional part of the full opening of spe gate at which the gate is set during each run.

COLUMN 3: The proportional discharge for each run compare with the maximum full gate discharge at about t point of highest efficiency, experiment 10, reduce to any. common head by formula (40); here it the ratio of each discharge in column 11 to t discharge for experiment 10 in the same colum

COLUMN 4: The head acting on the wheel, being the differen in elevation between the water surface just abo the wheel, and the water surface in the chann below the wheel.

COLUMN 5: The duration of each run in minutes.

COLUMN 6: The number of revolutions in each run divided the duration of each run, or the revolutions minute.

COLUMN 7: The discharge in cubic feet per second for each experiment, measured on a weir of known length.

COLUMN 8: The horse power developed on the wheel shaft during each run from computed measurements of the load on a Prony brake, the radius of the brake arm, and the number of revolutions.

COLUMN 9: The percentage efficiency of the wheel for each run, being the brake horse power (col. 8) divided by the theoretical horse power due to weight of water falling each second (col. 7) through the head during the experiment (col. 4). Thus for experiment 10 the efficiency $= \dfrac{300.27 \times 550}{208.68 \times 15.42 \times 62.4}$

$= 82.25$ per cent.

481. The probable performance of a wheel at other heads and speeds than those of a test may be predicted with reasonable assurance from a test provided that the setting for use is as favorable to efficiency as that of the test, and provided the divergence of head and speed in test and use are not too great. The method of making such comparisons is to consider the relation of efficiency, horse power, and discharge to the "speed factor."

482. The "speed factor" (SF) or "relative velocity," as often designated, is a ratio of the speed of the rim of the runner (at a point midway between the upper and lower limits of the bucket entrance orifices), to the theoretic velocity due to the head acting on the wheel; and is figured for the entrance of the runner simply for convenience of comparison.

The "speed factor," or "relative velocity"

$$SF = \frac{\pi DN}{60 \times (2gH)^{\frac{1}{2}}} = \frac{.006529\, DN}{H^{\frac{1}{2}}}. \tag{37}$$

$D =$ diameter in feet of a circle marking the edges of the entrance orifices of the wheel buckets.

Column 10 gives the speed factors computed for the actual experiment No. 9, the wheel runner being assumed to be 48 inches (4 feet) in diameter at the middle of the entrance rim.

For experiment 9, $SF = \dfrac{\pi \times 4 \times 95}{60 \times 8.02 \times 15.5^{\frac{1}{2}}} = .630.$

483. Assumptions made. The computations of the results of a test are based upon the following assumptions:

(1) For a constant proportional gate opening and a constant speed factor, the efficiency should be constant. (38)

(2) For a constant proportional gate opening and a constant speed factor, the discharge in cubic feet per second should be proportional to the square root of the head acting upon the wheel.

In the case of most of the trade wheels with cylinder gates, this gate is about an inch thick and the reduction in area through the guides is confined simply to the area under the gate.

The outlets of the buckets may be regarded as nozzle orifices, the area of which may be determined by direct calibration, being the sum of the cross-sectional areas of all the orifices at right angles to the average direction of flow at the exit. In the figures of turbines shown the measurements needed to determine these areas are indicated.

In a turbine the area (a_δ) of the runner orifices is constant, but the entrance of water to them and discharge from them is modified by the structure, form, and proportional opening of the gate, by the speed of the runner past the guide orifices, and by the dimensions and angle of the guides. If the coefficient of discharge C_δ of the runner is made to depend on these factors, it may be assumed constant for a given speed factor and gate opening.

Then for a given gate opening and a given speed factor,

$$\frac{Q_0}{Q_1} = \frac{C_\delta a_\delta (2\,gH_0)^{\frac{1}{2}}}{C_\delta a_\delta (2\,gH_1)^{\frac{1}{2}}} = \left(\frac{H_0}{H_1}\right)^{\frac{1}{2}}, \qquad (39)$$

Q_0 being the discharge for a head H_0, determined by experiment; and

Q_1 being the discharge for any other head (H_1) at the same speed factor and gate opening.

The area (a_a) of the guide orifices, and the area (a_δ) of the runner orifices in the wheel tested were measured as follows:

Number of guides, 20.

Mean width of each guide, .330 foot.

Mean height, 1.915 feet.

Then a_a, the area of the guide orifices, $= 20 \times .330 \times 1.915 =$ 12.64 square feet.

Number of buckets, 14.

Mean width of buckets normal to stream flow, .245 foot.

Mean length of the edge of bucket orifice, 3.80 feet.

Then a_8, the area of the bucket orifices, $= 14 \times .245 \times 3.8 = 13.034$ square feet.

Then for any experiment,

$$C_8 \text{ for the runner} = \frac{Q}{13.034 \times 8.02\, H^{\frac{1}{2}}}. \tag{40}$$

For example, for experiment 10,

$$C_8 = \frac{208.68}{13.034 \times 8.02 \times 15.42^{\frac{1}{2}}} = .508.$$

In column 13 of the test, the values of C_8 given are thus computed.

(3) For a constant proportional gate opening and a constant speed factor the brake horse power of a wheel is proportional to the three halves power of the head acting upon it.

The energy put into the wheel in any experiment $= wQH_0$ foot pounds per second.

The energy delivered at the shaft which is measured by the brake is given in column 8 for each experiment, $= HP_0$.

The efficiency as measured by the brake, $K_B = \frac{HP_0 \times 550}{wQH_0}$.

If the assumption holds that for a constant gate opening and speed factor the efficiency is constant and the discharge varies as $H^{\frac{1}{2}}$, then

For a given speed factor and gate opening,

$$\frac{HP_0}{HP_1} = \frac{K_B w C_8 a_8 (2\,gH_0)^{\frac{1}{2}} H_0}{K_B w C_8 a_8 (2\,gH_1)^{\frac{1}{2}} H_1} = \frac{H_0^{\frac{3}{2}}}{H_1^{\frac{3}{2}}}. \tag{41}$$

HP_0 = measured horse power by experiment for a head H_0; and

HP_1 = the horse power for any head (H_1) at the same speed factor and gate opening.

434. The wheel tested was one of a pair rated as 48 inches in diameter (see figures 171 and 172), and intended to run at a speed of 133 revolutions per minute under a head of 21 feet, which was fixed for the best average speed at part gate.

For these conditions,

$$\text{The speed factor} = \frac{.02612\,N}{H^{\frac{1}{2}}} = \frac{.02612 \times 133}{21^{\frac{1}{2}}} = .758.$$

[illegible]ion of efficiency to proportional gate opening for a speed fac[illegible].—In figure 173 *A* are drawn curves, one for each of the [illegible] openings tested (full gate, .78, .638, .5, and .37 gates); the c[illegible]ates of the points through which the curves are drawn are the speed factors (column 10) and the corresponding efficiencies (column 9) as determined by the test. From these curves for a speed [illegible]actor of .758 (corresponding to $N = 133$, and $H = 21$) the following relation between gate opening and efficiency is found:

G[illegible] opening	1.0	.78	.638	.50	.37
P[illegible]	[illegible]	[illegible]	77.5	67.0	53.5

From [illegible]se [illegible] of efficiency for other speed factors due to any cor[illegible]eed and heads may within the limits of the exp[illegible] in a similar manner, the speed factor being computed [illegible]37).

486. Variation in maxim[illegible]ncy with speed factor and gate opening. Maximum efficien[illegible]l gates is not attained at the speed factor .758, but at th[illegible] factors shown in the following table.

The following figures interpolated from columns 9 and 10, or from the curves, show the relation of gate opening and maximum efficiency to different heads and speeds:

Gate opening	1.0	.78	.638	.50	.37
Maximum per cent efficiency	82.3	81.6	78.2	70.1	60.9
Speed factor for maximum efficiency	.848	.761	.707	.674	.588

For a constant head (21 feet), the speed for maximum efficiency is as follows:

Required N for $H = 21$ feet	149	133	124	118	103

For a constant speed ($N = 133$), the head for maximum efficiency is as follows:

Required H for $N = 133$	16.7	20.7	24.1	26.6	34.9

487. Efficiency at full gate for various heads when $N = 133$.

Heads	16	20	24	28	32	36
Speed factors	.868	.777	.709	.656	.614	.579
Per cent efficiency	74	76	78	80	81	80

488. Variation in efficiency with changes in speed and gate. Inspection of the tables and curves shows that between .78 gate and full gate opening at moderate speeds for a considerable range in speed, the efficiency varies but little with either change

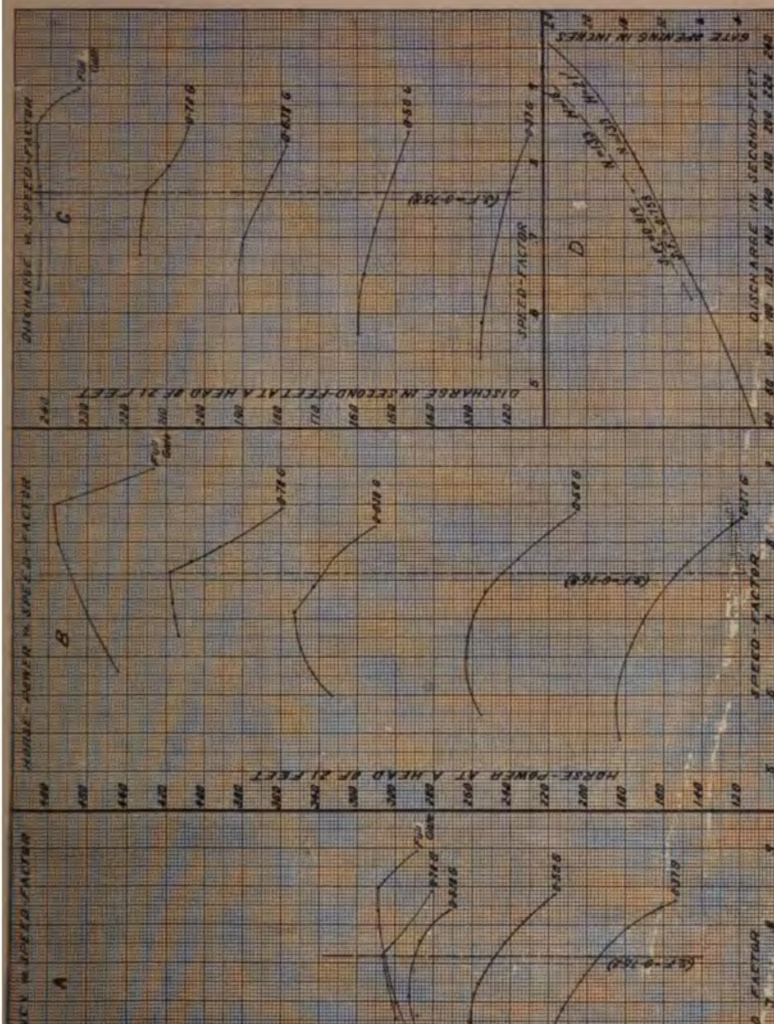


FIG. 173. — Tests of a 48-inch Turbine reduced to a Head of 21 Feet.

in gate or speed factor; but at high speeds the changes are much more marked; for smaller gate openings and at high and low speed factors the falling off in efficiency is rapid.

489. The relation of discharge to proportional gate opening for a speed factor of .758. — In figure 173 *C* are drawn curves for each of the five gate openings tested; the coördinates of the points through which the curves are drawn are the speed factors (column 10) and the corresponding discharges (column 11) for a uniform head of 21 feet, which were computed by formula (39) from the values in column 7; thus, for experiment 10:

$$\frac{Q_{21}}{208.68} = \left(\frac{21}{15.42}\right)^{\frac{1}{2}}; \; Q_{21} = 243.3.$$

From these curves are then taken the discharge for the five gate openings at a speed factor of .758 (corresponding to $N = 133$ and $H = 21$) as follows:

Gate opening	1.0	.78	.638	.50	.37
Discharge at 21 feet head	242.2	215.4	184.3	152.2	120.4

From these same curves the discharge for any other speed and the same head may be taken by entering the curves with the proper speed factor; or for any head and speed by means of the speed factor, the discharge at 21 feet may be taken, and reduced by formula (39) to the discharge for any given head and speed.

490. The relation of horse power to proportional gate opening for a speed factor of .758. — In figure 173 *B* are drawn curves for each of the five gate openings tested; the coördinates of the points through which the curves are drawn are the speed factors (column 10) and the corresponding horse power (column 12) for a head of 21 feet, which were computed by formula (40). Thus for experiment 10

$$\frac{HP_{21}}{300.27} = \left(\frac{21}{15.42}\right)^{\frac{3}{2}}; \; HP_{21} = 477.1.$$

From the curves are then taken the horse power for the five gate openings at a speed factor of .758 (corresponding to $N = 133$ and $H = 21$).

Gate opening	1.0	.78	.638	.50	.37
HP at 21 feet head	467.2	418.0	338.3	244.2	153.3

From these same curves the horse power for any other speed

and the same head may be taken by entering the curves with the proper speed factor, or for any head and speed the horse power at 21 feet may be taken, and reduced by formula (41) to the horse power for the given head and speed.

Turbines as Water Meters

491. The construction of discharge diagram. In figure 173 *D*, the discharge of the wheel for a speed factor .758 corresponding to a head of 21 feet at a speed of 133 revolutions per minute, is shown diagrammatically by drawing a curve through five points of which the ordinates are the proportional gate openings reduced to inches, and the abscissas discharges for a speed factor of .758. This is merely a plotting of the brief table (§ 489) which shows the relation of discharge to gate opening for a speed factor of .758. Such a curve (diagram *D*) is then a discharge curve for the wheel tested when running under a head of 21 feet and at a speed of 133 revolutions per minute. In this case the experiments were not carried below .37 gate; and additional experiments at lower gates would be necessary to complete the curve. It may be extended, however, in this case through the origin without serious error to zero.

If two or more wheels of the same capacity are to be run on the same shaft, their combined discharge may be shown in one curve by making the abscissas the sum of the discharges of all the wheels at the given gate openings with any slight modifications due to the setting.

In any water-power plant the head is likely to vary; for textile and hydroelectric plants, or where close regulation of speed is required, the speed must be practically constant. Under these conditions a diagram may be constructed for a wheel or any number of wheels on the same shaft for a range of heads and gate openings; and from this diagram, with actual observations of the heads and gate openings at different times, the compounding discharge may be read from the diagram.

The diagram is constructed as follows:

Compute the speed factor for each head. For example:

$$\text{If } H = 18, \quad \text{and } N = 133; \quad SF = \frac{.02612 \times 133}{18^{\frac{1}{2}}} = .819.$$

Enter the discharge curves for head of 21 feet (diagram 173 *C*) with a speed factor of .819; the discharge for each of the five gate openings is seen from the curves to be 243.3, 206.2, 178.5, 148, and 116.

Multiply each of these discharges by $\left(\frac{18}{21}\right)^{\frac{1}{2}}$ which gives discharges of 225.3, 190.9, 165.2, 137.0, 107.3 for the five different gate openings.

Plot the discharges thus computed against the corresponding gate openings and there results a new discharge curve for the desired head (in this case 18 feet). In a similar manner each additional head is computed and plotted. The dotted line on figure 173 *D* is for a head of 18 feet.

492. A complete set of curves for various heads for the wheel tested combined with the other wheel of the same pair which was also tested, and which is set on the same shaft in the same wheel case, is given in figure 174 (Plate VI). The discharge is practically double (not exactly) the discharges already computed; but each wheel was tested and computed separately and the results combined. This diagram is one made up for daily use in determining the water used by the wheel.

493. Discharge curves when the speed varies as well as the head. If the speeds vary as well as the heads, a similar curve is made; but the curves, though plotted as gate openings against discharge, should he labeled each with a speed factor instead of a head. The observer must determine the speed, as well as the head and gate opening; compute the speed factor and enter the curve by this instead of the head. A revolution counter set on the shaft will give the total revolutions for any period between observations, from which an average speed quite accurate enough may be computed.

494. The use of water wheels as water meters for measuring water in water-power plants is very common and must continue to be the chief means available; and by many comparisons, with simultaneous measurements by weirs, rod floats, or current meters, has been shown to be accurate where all the conditions of test and setting are properly considered.

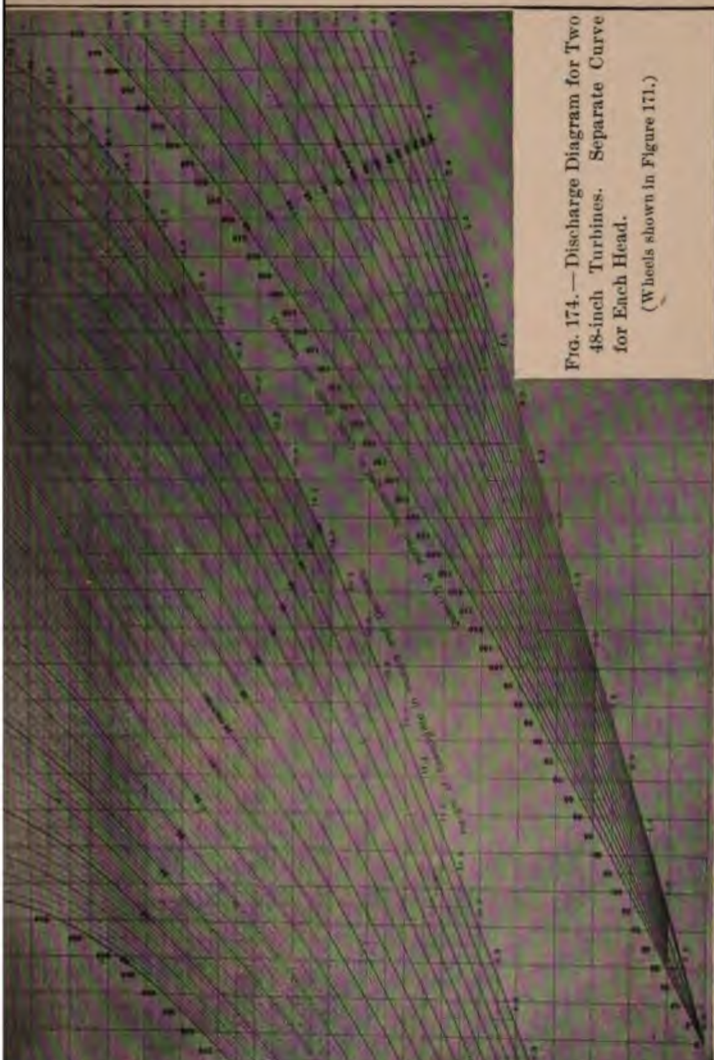


FIG. 174.—Discharge Diagram for Two 48-inch Turbines. Separate Curve for Each Head.

(Wheels shown in Figure 171.)

diagram was constructed for permanent office use from Holyoke tests, by hods explained in §§ 479 to 494. The wheel considered in the discussions is his pair.

PLATE VI

The nozzles of impulse wheels furnish an even simpler and more exact measure of the use of water because the conditions of use are better understood and they can be easily calibrated. By referring to Chapter XI it will be seen that nozzle measurement requires simply the calibration of the pipe and nozzle orifice diameters, and the measurement of the head at the base of the nozzle. The computation of discharge should be made by the common nozzle formula; for cone nozzles Freeman's coefficients are applicable; for nozzles such as are shown in figure 153 the coefficient must be determined by experiment.*

Centrifugal Pumps

495. A centrifugal pump is an apparatus for raising water by receiving it at a low velocity on the interior edge of a set of moving impeller vanes, not unlike turbine buckets, and discharg-

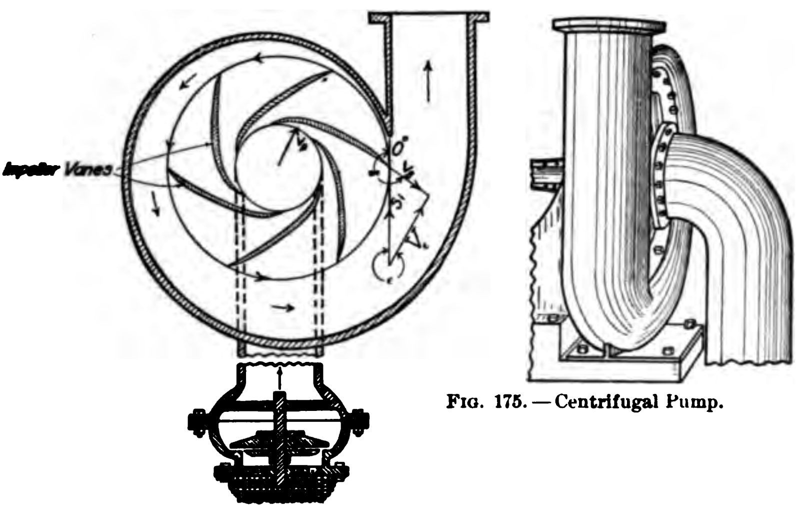


Fig. 175. — Centrifugal Pump.

ing it from the outer edge with kinetic energy sufficient to raise it to a desired height; and then by means of a gradually enlarging spiral passage transforming the kinetic energy into pressure energy. The impeller vanes are rigidly attached to a shaft which is rotated by some form of prime mover or motor, and set inside an air-tight metal case. A suction pipe leads into the center part

* See *Investigations of the Doble Regulating Nozzle*, by H. C. Growell and G. C. D. Leith, Bulletin No. 6, Abner Doble Co.

of the case, and should be fitted with a check or foot valve; and a discharge pipe leads from the outer part of the casing to the place of delivery. The pump may work submerged or may be set some height above the water surface of the supply basin, less than 33.9 feet, rarely over 25 feet, and usually much less. Figure 175 shows a diagrammatic section and an outline of a centrifugal pump.

496. Formulas. The work done by the pump in increasing the velocity of the water may be computed by inverting the formula for the work done in a turbine by the water.

Making $E = -E$ in (15), since the energy must be supplied by some source outside the water,

Formula (15) becomes,

$$-E = \frac{wQ}{g}(V_a \cos \alpha S_0 - V_e \cos \epsilon S_1);$$

or designating the energy delivered to the pump as E_e,

$$E_e = \frac{wQ}{g}(V_e \cos \epsilon S_1 - V_a \cos \alpha S_0).$$

The work done each second (E_e) by the pump must be equal to lifting a given weight of water wQ pounds per second to a total equivalent height (H_T), comprising the actual lift (h), the friction head (h_f) in the suction and discharge pipes, the hydraulic losses in the impeller vanes and casing, and the mechanical losses.

Or, $\quad H_T = h + h_f +$ hydraulic losses in pump + machine friction. (I)

Then, $E_e = wQH_T = \dfrac{wQ}{g}(V_e \cos \epsilon S_1 - V_a \cos \alpha S_0)$. (J)

From (J), $\quad V_e \cos \epsilon S_1 - V_a \cos \alpha S_0 = gH_T$. (K)

A centrifugal pump, like a reaction turbine, should run with all its parts full of water; and for a given discharge the areas of the impeller orifices or the interior channels will determine the velocities.

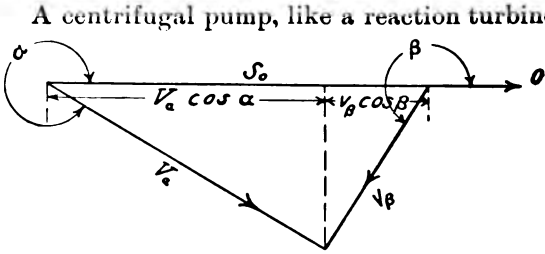


FIG. 176. — Diagram of Velocities at Entrance.

Hence the shape areas and direction of vanes or interior channels should be such as to correspond to the velocities produced

the motion of the impeller. Changes in area or direction ıld be gradual, and the interior surfaces smooth.

he water should enter the impeller vanes without violent ıge in direction. That is, v_β should be determined ı (1),

$$v_\beta^2 = V_a^2 + S_0^2 - 2V_a S_0 \cos \alpha.$$

hese conditions are shown by the velocity diagram, figure 176. he absolute velocity (V_ϵ) at exit is the resultant of v_δ and S_1, s shown by velocity diaɑ, figure 177. ɔr conditions economy the lute velocity ntrance (V_a) ld have no

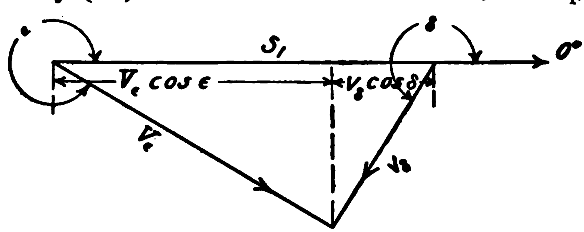


FIG. 177.—Diagram of Velocities at Exit.

ential component; that is, $V_a \cos \alpha$ should be 0. To secure condition β should be radial (90° or 270°).

$V_a \cos \alpha = 0$, then equation (K) becomes,

$$V_\epsilon \cos \epsilon S_1 = gH_T; \text{ or} \tag{L}$$

ı $V_\epsilon \cos \epsilon = v_\delta \cos \delta + S_1$; (cos δ has here a negative value)

ɭuation (L) becomes $S_1^2 + S_1 v_\delta \cos \delta = gH_T$. (42)

om equation (42) and the principles of the flow of water in d channels, with a knowledge, derived from testing, of ›roper form and proportions of the passages through which vater enters and leaves the impellers, and a knowledge of the ər shape of the impeller vanes themselves, the relation of ı, and δ to a given head may be determined.

7. **Single stage centrifugal pumps** which contain one set of llers in a single case are built for lifts as high as 150 feet. ɑigher lifts the speed becomes excessive for a single impeller.

8. **Multistage centrifugal pumps** contain a number of sets of llers mounted on a single shaft, each operating in a separate ıber, arranged to pass the water through the different sets of llers in succession, increasing the intensity of pressure in chamber. Such pumps have been made for lifts as high as feet while keeping the speed within practical bounds.

499. The efficiency of centrifugal pumps is measured by the ratio of the weight of water raised per second through the actual lift (h) plus friction head (h_f) in the pipe, to the energy delivered to the shaft, viz.:

$$K_c = \frac{wQ(h+h_f)}{HP \times 550}.$$

Actual efficiencies of centrifugal pumps vary from less than .50 to occasionally about .85.

Problems

1. A tangential impulse wheel has a diameter of 5.6 feet; its speed is 300 revolutions per minute and its buckets deflect the water through 165. The head on the center of the nozzle is 620 feet; the nozzle orifice has a diameter of $6\frac{3}{8}$ inches. Assuming that the coefficient of velocity from the nozzle is .97 and the coefficient of velocity from the bucket is .88; compute (a) the absolute entrance and exit velocities; (b) the work done in foot pounds per second; (c) the horse power; and (d) the hydraulic efficiency.

2. A penstock 2000 feet long and 42 inches in diameter made of riveted steel with cylinder joints leads from a reservoir to two Pelton wheels. The fall from the reservoir to the center of the nozzle orifice is 858 feet and the nozzle orifices are 20 feet above tail-water. If 35 cubic feet of water per second are to be delivered to each wheel and the coefficient of velocity through the nozzle is .98, determine (a) the loss of head in the pipe; (b) the total head at the base of the nozzle.

The wheels are 8.67 feet in diameter, the buckets deflect the water through 166°; take C_1, as .90, and assume that the wheels are running at 235 revolutions per minute. Compute (a) the relative velocities at the entrance and the exit of the runners; (b) the absolute velocity at exit and its direction; (c) the work done; (d) the hydraulic efficiency of the nozzle and the wheel combined; (e) the efficiency of the plant.

3. Given an outward flow impulse turbine. The effective head on the nozzle orifice is 394 feet; diameter at entrance is 5.5 feet; diameter at exit is 6.5 feet, fall through the wheel is .5 foot; the direction of the water at entrance is 342°; the direction of the runner orifices at exit is 194°; the speed is 262 revolutions per minute; the nozzle orifices have a total area of .1337 square foot; the nozzle coefficient of velocity is .95; the coefficient of velocity from the runner is .96.

Compute (a) the velocity from the nozzle orifice; (b) the velocity relative to the runner at entrance and its direction; (c) the relative and absolute velocities at exit from the runner; (d) the work done; and (e) the hydraulic efficiency of the nozzle and wheel combined.

4. The Francis turbine shown in figures 151 and 163 discharged 112.5 cubic feet per second when running at 42.5 revolutions per minute under a head

13.4 feet. Compute (*a*) the absolute velocities at entrance and exit; (*b*) the direction of the absolute exit velocity; (*c*) the speed factor; (*d*) the reaction coefficient; (*e*) the work; and (*f*) the efficiency.

5. A reaction turbine having a downward axial flow discharged 64.4 cubic feet per second when running at 90 revolutions per minute.

The dimensions of the wheel were as follows: mean diameter, 4.17 feet; mean area of each guide vane, .127 square foot; guide vane angle, 342° 15′; number of guide vanes, 24; mean area of each bucket, .136 square foot; bucket angle, 199°; number of buckets, 24. Effective head, 15 feet.

Compute (*a*) the absolute entrance velocity; (*b*) the relative velocity of exit from the buckets; (*c*) the absolute velocity from the buckets, and its direction; (*d*) the work done in foot pounds per second; (*e*) the horse power; and (*f*) the hydraulic efficiency.

6. The following is the report of the test of a 42-inch turbine:

Number of the Experiment	Proportional Part of the Full Opening of the Speed Gate	Proportional Part of the Full Discharge of the Wheel; being the Discharge at Full Gate when giving Best Efficiency	Head acting on the Wheel	Duration of the Experiment	Revolutions of the Wheel	Quantity of Water discharged by the Wheel	Power developed by the Wheel	Efficiency of the Wheel
	per cent	per cent	feet	min.	per min.	cubic feet per sec.	*HP*	per cent
35	1.000	1.007	13.25	4	102.00	130.78	155.77	78.26
34	1.000	1.006	13.18	4	106.00	130.27	155.38	79.79
33	1.000	1.000	13.14	4	109.00	129.30	153.83	79.83
37	1.000	0.999	13.41	5	112.90	130.50	157.79	79.50
29	0.800	0.906	16.58	4	108.75	131.65	197.22	79.67
28	0.800	0.903	16.52	3	113.00	130.85	196.45	80.13
27	0.800	0.899	16.45	4	118.00	130.07	195.49	80.56
26	0.800	0.892	16.39	4	123.00	128.75	192.87	80.59
25	0.800	0.886	16.36	4	127.00	127.85	188.75	79.57
22	0.656	0.797	16.55	3	107.00	115.67	167.78	77.28
21	0.656	0.794	16.55	4	112.00	115.17	167.99	77.71
20	0.656	0.790	16.51	4	116.75	114.45	167.15	78.00
19	0.656	0.785	16.51	4	121.00	113.83	164.99	77.41
14	0.518	0.674	16.74	4	106.25	98.43	136.18	72.88
13	0.518	0.670	16.76	4	111.75	97.86	136.38	73.32
12	0.518	0.664	16.81	4	116.75	97.17	134.52	72.61
11	0.518	0.657	16.81	4	122.00	96.04	131.42	71.77
7	0.396	0.546	16.97	4	102.00	80.20	101.53	65.78
6	0.396	0.544	16.89	3	108.00	79.77	100.87	66.02
5	0.396	0.540	16.90	5	114.20	79.23	99.66	65.63

Assuming that the wheel is to work under a head of 16 feet at 122 revolutions per minute, compute for the gate openings given; (*a*) the efficiency; (*b*) the discharge; (*c*) the horse power.

Compute (*d*) the maximum efficiency for each opening with a head of 16 feet, and the speeds at this maximum efficiency; (*e*) the efficiency at heads of 13, 17, 20, 21, and 25 feet, at a speed of 122 revolutions per minute.

7. An inward flow reaction turbine working under a head of 131 feet discharges 35 cubic feet per second, when running at 566 revolutions per minute. The outer diameter of the runner is 2 feet, the inner diameter, 1.2 feet, α is 344°, β is 270°, δ is 212° 43′. Taking the reaction coefficient as .50; C_{g}, .95; C_{1}, .95; and the loss of clearance, $.072\dfrac{V_a^2}{2g}$; compute (*a*) V_a; v_β; v_δ; (*b*) the intensity of pressure at the entrance to runner; (*c*) the total effective area of the streams of water on leaving the guides and buckets; (*d*) the work done; (*e*) the horse power; (*f*) the efficiency.

TABLE LXI

H = head in feet of water = 1.1330 H_g = 2.3077 p.
H_g = head in inches of mercury = .88264 H = 2.0369 p.
p = pressure in pounds per square inch = .43333 H = .49095 H_g.
V = velocity in feet per second due to a head of H feet = $8.02(H)^{\frac{1}{2}} = 8.5365(H_g)^{\frac{1}{2}}$ = $12.183\ (p)^{\frac{1}{2}}$.
Q = discharge in cubic feet per second of a stream having an actual area of one square inch and a mean velocity of V feet per second = $\frac{V}{144}$.
G = discharge in gallons per minute of a stream having an actual area of one square inch and a mean velocity of V feet per second = $\frac{V \times 1728 \times 60}{144 \times 231}$.
$(2g)^{\frac{1}{2}} = 8.02$. Weight of one cubic foot of water, 62.4 pounds. One gallon = 231 cubic inches. $g = 32.16$.

H	H_g	p	V	Q	G
.001600	.001412	.0006933	.3208	.002225	1
.01555	.01372	.006737	1.000	.006944	3.117
.06219	.05489	.02695	2.000	.01389	6.234
.1399	.1235	.06063	3.000	.02083	9.351
.2488	.2196	.1078	4.000	.02778	12.47
.3887	.3431	.1684	5.000	.03472	15.58
.5597	.4940	.2425	6.000	.04167	18.70
.7618	.6724	.3301	7.000	.04861	21.82
.9950	.8782	.4311	8.000	.05556	24.94
1.000	.8826	.4333	8.02	.05569	25.00
1.133	1.000	.4909	8.537	.05928	26.61
1.259	1.111	.5457	9.000	.06250	28.05
1.555	1.372	.6737	10.00	.06944	31.17
1.881	1.660	.8151	11.00	.07639	34.29
2.000	1.765	.8667	11.34	.07875	35.35
2.239	1.976	.9701	12.00	.08333	37.40
2.266	2.000	.9819	12.07	.08384	37.63
2.308	2.037	1.000	12.18	.08461	37.97
2.627	2.319	1.138	13.00	.09028	40.52
3.000	2.648	1.300	13.89	.09646	43.30
3.047	2.690	1.320	14.00	.09722	43.64
3.399	3.000	1.473	14.79	.1027	46.09
3.497	3.087	1.516	15.00	.1042	46.75
3.980	3.513	1.725	16.00	.1111	49.87
4.000	3.531	1.733	16.04	.1114	50.00
4.493	3.966	1.947	17.00	.1181	52.99
4.532	4.000	1.964	17.07	.1186	53.21
4.615	4.074	2.000	17.23	.1196	53.69
5.000	4.413	2.167	17.93	.1245	55.90
5.037	4.446	2.183	18.00	.1250	56.10
5.612	4.954	2.432	19.00	.1319	59.22
5.665	5.000	2.455	19.09	.1326	59.50
6.000	5.296	2.600	19.65	.1364	61.23
6.219	5.489	2.695	20.00	.1389	62.34
6.798	6.000	2.946	20.91	.1452	65.17
6.856	6.051	2.971	21.00	.1458	65.45
6.923	6.111	3.000	21.10	.1465	65.75
7.000	6.178	3.033	21.22	.1474	66.15
7.525	6.641	3.270	22.00	.1528	68.57
7.931	7.000	3.437	22.59	.1568	70.40
8.000	7.061	3.467	22.69	.1575	70.70
8.224	7.259	3.564	23.00	.1597	71.69
8.955	7.904	3.880	24.00	.1667	74.81
9.000	7.944	3.900	24.06	.1671	75.00
9.064	8.000	3.928	24.15	.1677	75.26
9.231	8.148	4.000	24.37	.1692	75.95

H	H_g	p	V	Q	G
9.717	8.576	4.211	**25.00**	.1736	77.92
10.00	8.826	4.333	25.36	.1761	79.05
10.20	**9.000**	4.419	25.61	.1779	79.82
11.00	9.709	4.767	26.60	.1847	82.93
11.33	**10.00**	4.909	27.00	.1875	84.14
11.54	10.18	**5.000**	27.24	.1892	84.90
12.00	10.59	5.200	27.78	.1929	86.60
12.46	**11.00**	5.400	28.31	.1966	88.25
13.00	11.47	5.633	28.92	.2008	90.15
13.60	**12.00**	5.891	29.57	.2054	92.17
13.85	12.22	**6.000**	29.84	.2072	92.99
13.99	12.35	6.063	**30.00**	.2083	93.51
14.00	12.36	6.067	30.01	.2084	93.54
14.73	**13.00**	6.382	30.78	.2138	95.93
15.00	13.24	6.500	31.06	.2157	96.83
15.86	**14.00**	6.873	31.94	.2218	99.55
16.00	14.12	6.933	32.08	.2228	**100.0**
16.15	14.26	**7.000**	32.24	.2239	100.5
17.00	**15.00**	7.367	33.07	.2296	103.1
18.00	15.89	7.800	34.03	.2363	106.1
18.13	**16.00**	7.855	34.15	.2371	106.4
18.46	16.30	**8.000**	34.46	.2393	107.4
19.00	16.77	8.233	34.96	.2428	109.0
19.26	**17.00**	8.346	35.20	.2444	109.7
20.00	17.65	8.667	35.87	.2491	111.8
20.39	**18.00**	8.837	36.22	.2515	112.9
20.77	18.33	**9.000**	36.55	.2538	113.9
21.00	18.54	9.100	36.75	.2552	114.6
21.53	**19.00**	9.328	37.21	.2584	116.0
22.00	19.42	9.533	37.62	.2612	117.3
22.66	**20.00**	9.819	38.18	.2651	119.0
23.00	20.30	9.967	38.46	.2671	119.9
23.08	20.37	**10.00**	38.53	.2676	120.1
23.79	**21.00**	10.31	39.12	.2717	121.9
24.00	21.18	10.40	39.29	.2729	122.5
24.88	21.96	10.78	**40.00**	.2778	124.7
24.93	**22.00**	10.80	40.04	.2780	124.8
25.00	22.07	10.83	40.10	.2785	**125.0**
25.38	22.41	**11.00**	40.41	.2806	126.0
26.00	22.95	11.27	40.89	.2840	127.5
26.06	**23.00**	11.29	40.94	.2843	127.6
27.00	23.83	11.70	41.67	.2894	129.9
27.19	**24.00**	11.78	41.82	.2904	130.4
27.69	24.44	**12.00**	42.20	.2930	131.5
28.00	24.71	12.13	42.44	.2947	132.3
28.33	**25.00**	12.27	42.68	.2964	133.0
29.00	25.60	12.57	43.19	.2999	134.6
29.46	**26.00**	12.76	43.53	.3023	135.7
30.00	26.48	**13.00**	43.93	.3051	136.9
30.59	**27.00**	13.26	44.36	.3080	138.3
31.00	27.36	13.43	44.65	.3101	139.2
31.72	**28.00**	13.75	45.17	.3137	140.8
32.00	28.24	13.87	45.37	.3151	141.4
32.31	28.52	**14.00**	45.59	.3166	142.1
32.86	**29.00**	14.24	45.97	.3192	143.3
33.00	29.13	14.30	46.07	.3200	143.6
33.99	**30.00**	14.73	46.76	.3247	145.7
34.00	30.01	14.73	46.77	.3248	145.8
34.62	30.55	**15.00**	47.19	.3277	147.1
35.00	30.89	15.17	47.45	.3295	147.9
36.00	31.78	15.60	48.12	.3341	**150.0**
36.92	32.59	**16.00**	48.73	.3384	151.9
37.00	32.66	16.03	48.78	.3388	152.1
38.00	33.54	16.47	49.44	.3434	154.1
38.87	34.31	16.84	**50.00**	.3472	155.8
39.00	34.42	16.90	50.09	.3478	156.1

H	*H_g*	*p*	*V*	*Q*	*G*
39.23	34.63	**17.00**	50.23	.3488	156.6
40.00	35.31	17.33	50.72	.3522	158.1
41.00	36.19	17.77	51.35	.3566	160.1
41.54	36.66	**18.00**	51.69	.3590	161.1
42.00	37.07	18.20	51.98	.3610	162.0
43.00	37.95	18.63	52.59	.3652	163.9
43.85	38.70	**19.00**	53.11	.3688	165.5
44.00	38.84	19.07	53.20	.3694	165.8
45.00	39.72	19.50	53.80	.3736	167.7
45.32	**40.00**	19.64	53.99	.3749	168.3
46.00	40.60	19.93	54.39	.3777	169.6
46.15	40.74	**20.00**	54.48	.3784	169.8
47.00	41.48	20.37	54.98	.3818	171.4
48.00	42.37	20.80	55.56	.3858	173.2
48.46	42.77	**21.00**	55.83	.3877	174.0
49.00	43.25	21.23	56.14	.3898	**175.0**
50.00	44.13	21.67	56.71	.3938	176.8
50.77	44.81	**22.00**	57.14	.3968	178.1
51.00	45.01	22.10	57.27	.3977	178.5
52.00	45.90	22.53	57.83	.4016	180.3
53.00	46.78	22.97	58.39	.4055	182.0
53.08	46.85	**23.00**	58.43	.4058	182.1
54.00	47.66	23.40	58.93	.4092	183.7
55.00	48.55	23.83	59.48	.4130	185.4
55.38	48.89	**24.00**	59.69	.4145	186.1
55.97	49.40	24.25	**60.00**	.4167	187.0
56.00	49.43	24.27	60.02	.4168	187.1
56.65	**50.00**	24.55	60.36	.4192	188.1
57.00	50.31	24.70	60.55	.4205	188.8
57.69	50.92	**25.00**	60.92	.4231	189.9
58.00	51.19	25.13	61.08	.4242	190.4
59.00	52.08	25.57	61.60	.4278	192.0
60.00	52.96	**26.00**	62.12	.4314	193.6
61.00	53.84	26.43	62.64	.4350	195.2
62.00	54.72	26.87	63.15	.4385	196.9
62.31	55.00	**27.00**	63.31	.4396	197.3
63.00	55.61	27.30	63.66	.4421	198.4
64.00	56.49	27.73	64.16	.4455	**200.0**
64.62	57.03	**28.00**	64.47	.4477	200.9
65.00	57.37	28.17	64.66	.4490	201.6
66.00	58.25	28.60	65.16	.4525	203.1
66.92	59.07	**29.00**	65.61	.4556	204.5
67.00	59.14	29.03	65.65	.4559	204.6
67.98	**60.00**	29.46	66.12	.4592	206.1
68.00	60.02	29.47	66.14	.4593	206.2
69.00	60.90	29.90	66.62	.4626	207.7
69.23	61.11	**30.00**	66.73	.4634	208.0
70.00	61.78	30.33	67.10	.4659	209.2
71.00	62.67	30.77	67.58	.4693	210.7
71.54	63.14	**31.00**	67.83	.4711	211.4
72.00	63.55	31.20	68.05	.4726	212.1
73.00	64.43	31.63	68.52	.4759	213.6
73.85	65.18	**32.00**	68.92	.4786	214.8
74.00	65.32	32.07	68.99	.4791	215.1
75.00	66.20	32.50	69.46	.4823	216.5
76.00	67.08	32.93	69.92	.4855	218.0
76.15	67.22	**33.00**	69.99	.4861	218.2
76.18	67.24	33.01	**70.00**	.4861	218.2
77.00	67.96	33.37	70.38	.4887	219.4
78.00	68.85	33.80	70.84	.4919	220.8
78.46	69.25	**34.00**	71.04	.4933	221.4
79.00	69.73	34.23	71.29	.4950	222.2
79.31	**70.00**	34.37	71.42	.4960	222.6
80.00	70.61	34.67	71.73	.4981	223.6
80.77	71.29	**35.00**	72.08	.5005	224.7
81.00	71.49	35.10	72.18	.5012	225.0

H	H_s	p	V	Q	G
82.00	72.38	35.53	72.63	.5043	226.4
83.00	73.26	35.97	73.07	.5074	227.7
83.08	73.33	**36.00**	73.10	.5076	227.8
84.00	74.14	36.40	73.51	.5104	229.1
85.00	75.02	36.83	73.94	.5135	230.5
85.28	75.37	**37.00**	74.11	.5147	231.0
86.00	75.91	37.27	74.38	.5165	231.9
87.00	76.79	37.70	74.81	.5195	233.2
87.69	77.40	**38.00**	75.10	.5215	234.1
88.00	77.67	38.13	75.23	.5224	234.5
89.00	78.56	38.57	75.66	.5254	235.9
90.00	79.44	**39.00**	76.08	.5284	237.2
90.64	**80.00**	39.28	76.35	.5302	238.0
91.00	80.32	39.43	76.51	.5314	238.5
92.00	81.20	39.87	76.93	.5342	239.8
92.31	81.48	**40.00**	77.05	.5352	240.2
93.00	82.09	40.30	77.34	.5371	241.1
94.00	82.97	40.73	77.76	.5400	242.4
94.62	83.51	**41.00**	78.01	.5417	243.1
95.00	83.85	41.17	78.17	.5429	243.7
96.00	84.73	41.60	78.58	.5457	245.0
96.92	85.55	**42.00**	78.96	.5483	246.1
97.00	85.62	42.03	78.99	.5485	246.2
98.00	86.50	42.47	79.39	.5513	247.5
99.00	87.38	42.90	79.80	.5542	248.8
99.23	87.59	**43.00**	79.89	.5548	249.0
99.50	87.82	43.11	**80.00**	.5556	249.4
100.0	88.26	43.33	80.20	.5569	**250.0**
101.5	89.62	**44.00**	80.81	.5613	251.9
102.0	**90.00**	44.19	80.98	.5624	252.4
103.8	91.66	**45.00**	81.73	.5676	254.7
106.2	93.70	**46.00**	82.63	.5738	257.5
108.5	95.73	**47.00**	83.52	.5800	260.3
110.0	97.09	47.67	84.12	.5842	262.2
110.8	97.77	**48.00**	84.41	.5862	263.1
113.1	99.81	**49.00**	85.28	.5922	265.8
113.3	**100.0**	49.09	85.37	.5928	266.1
115.4	101.8	**50.00**	86.15	.5983	268.5
120.0	105.9	52.00	87.85	.6101	273.9
124.6	**110.0**	54.00	89.53	.6218	279.1
125.9	111.1	54.57	**90.00**	.6250	280.5
130.0	114.7	56.33	91.44	.6350	285.0
136.0	**120.0**	58.91	93.51	.6494	291.5
138.5	122.2	**60.00**	94.37	.6554	294.1
140.0	123.6	60.67	94.89	.6590	295.8
147.3	**130.0**	63.82	97.33	.6759	303.4
150.0	132.4	65.00	98.22	.6821	306.2
155.5	137.2	67.37	**100.0**	.6944	311.7
158.6	**140.0**	68.73	101.0	.7014	314.8
160.0	141.2	69.33	101.5	.7045	316.2
161.5	142.6	**70.00**	101.9	.7080	317.7
170.0	**150.0**	73.67	104.6	.7261	326.0
180.0	158.9	78.00	107.6	.7472	335.4
181.3	**160.0**	78.55	108.0	.7499	336.6
184.6	163.0	**80.00**	109.0	.7567	339.6
190.0	167.7	82.33	110.6	.7677	344.6
192.6	**170.0**	83.46	111.3	.7730	346.9
200.0	176.5	86.67	113.4	.7875	353.6
203.9	**180.0**	88.37	114.5	.7954	357.0
207.7	183.3	**90.00**	115.6	.8027	360.2
215.3	**190.0**	93.28	117.7	.8171	366.8
226.6	**200.0**	98.19	120.7	.8384	376.3
230.8	203.7	**100.0**	121.8	.8461	379.7

TABLE LXII

$\frac{V^2}{2g}$

HEADS DUE TO VELOCITIES FROM 0 TO 10 FEET PER SECOND*

V	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001
.1	.0002	.0002	.0002	.0003	.0003	.0003	.0004	.0004	.0005	.0006
.2	.0006	.0007	.0008	.0008	.0009	.0010	.0011	.0011	.0012	.0013
.3	.0014	.0015	.0016	.0017	.0018	.0019	.0020	.0021	.0022	.0024
.4	.0025	.0026	.0027	.0029	.0030	.0031	.0033	.0034	.0036	.0037
0.5	.0039	.0040	.0042	.0044	.0045	.0047	.0049	.0051	.0052	.0054
.6	.0056	.0058	.0060	.0062	.0064	.0066	.0068	.0070	.0072	.0074
.7	.0076	.0078	.0081	.0083	.0085	.0087	.0090	.0092	.0095	.0097
.8	.0099	.0102	.0105	.0107	.0110	.0112	.0115	.0118	.0120	.0123
.9	.0126	.0129	.0132	.0134	.0137	.0140	.0143	.0146	.0149	.0152
1.0	.0155	.0159	.0162	.0165	.0168	.0171	.0175	.0178	.0181	.0185
.1	.0188	.0192	.0195	.0199	.0202	0206	.0209	.0213	.0216	.0220
.2	.0224	.0228	.0231	.0235	.0239	.0243	.0247	.0251	.0255	.0259
.3	.0263	.0267	.0271	.0275	.0279	.0283	.0288	.0292	.0296	.0300
.4	.0305	.0309	.0313	.0318	.0322	.0327	.0331	.0336	.0341	.0345
1.5	.0350	.0354	.0359	.0364	.0369	.0374	.0378	.0383	.0388	.0393
.6	.0398	.0403	.0408	.0413	.0418	.0423	.0428	.0434	.0439	.0444
.7	.0449	.0455	.0460	.0465	.0471	.0476	.0482	.0487	.0493	.0[illegible]98
.8	.0504	.0509	.0515	.0521	.0526	.0532	.0538	.0544	.0549	.0555
.9	.0561	.0567	.0573	.0579	.0585	.0591	.0597	.0603	.0609	.0616
2.0	.0622	.0628	.0634	.0641	.0647	.0653	.0660	.0666	.0673	.0679
.1	.0686	.0692	.0699	.0705	.0712	.0719	.0725	.0732	.0739	.0746
.2	.0752	.0759	.0766	.0773	.0780	.0787	.0794	.0801	.0808	.0815
.3	.0822	.0830	.0837	.0844	.0851	.0859	.0866	.0873	.0881	.0888
.4	.0895	.0903	.0910	.0918	.0926	.0933	.0941	.0948	.0956	.0964
2.5	.0972	.0979	.0987	.0995	.1003	.1011	.1019	.1027	.1035	.1043
.6	.1051	.1059	.1067	.1075	.1084	.1092	.1100	.1108	.1117	.1125
.7	.1133	.1142	.1150	.1159	.1167	.1176	.1184	.1193	.1201	.1210
.8	.1219	.1228	.1236	.1245	.1254	.1263	.1272	.1281	.1289	.1298
.9	.1307	.1316	.1326	.1335	.1344	.1353	.1362	.1371	.1381	.1390
3.0	.1399	.1409	.1418	.1427	.1437	.1446	.1456	.1465	.1475	.1484
.1	.1494	.1504	.1513	.1523	.1533	.1543	.1552	.1562	.1572	.1582
.2	.1592	.1602	.1612	.1622	.1632	.1642	.1652	.1662	.1673	.1683
.3	.1693	.1703	.1714	.1724	.1734	.1745	.1755	.1766	.1776	.1787
.4	.1797	.1808	.1818	.1829	.1840	.1850	.1861	.1872	.1883	.1894
3.5	.1904	.1915	.1926	.1937	.1948	.1959	.1970	.1981	.1992	.2004
.6	.2015	.2026	.2037	.2049	.2060	.2071	.2083	.2094	.2105	.2117
.7	.2128	.2140	.2151	.2163	.2175	.2186	.2198	.2210	.2221	.2233
.8	.2245	.2257	.2269	.2280	.2292	.2304	.2316	.2328	.2340	.2352
.9	.2365	.2377	.2389	.2401	.2413	.2426	.2438	.2450	.2463	.2475
4.0	.2487	.2500	.2512	.2525	.2537	.2550	.2563	.2575	.2588	.2601
.1	.2613	.2626	.2639	.2652	.2665	.2677	.2690	.2703	.2716	.2729
.2	.2742	.2755	.2769	.2782	.2795	.2808	.2821	.2835	.2848	.2861
.3	.2875	.2888	.2901	.2915	.2928	.2942	.2955	.2969	.2982	.2996
.4	.3010	.3023	.3037	.3051	.3065	.3079	.3092	.3106	.3120	.3134
4.5	.3148	.3162	.3176	.3190	.3204	.3218	.3233	.3247	.3261	.3275
.6	.3290	.3304	.3318	.3333	.3347	.3362	.3376	.3390	.3405	.3420
.7	.3434	.3449	.3463	.3478	.3493	.3508	.3522	.3537	.3552	.3567
.8	.3582	.3597	.3612	.3627	.3642	.3657	.3672	.3687	.3702	.3717
.9	.3733	.3748	.3763	.3779	.3794	.3809	.3825	.3840	.3856	.3871

* *Lowell Hydraulic Experiments*, p. 253, for heads from 0 to 4.99.

$\frac{V^2}{2g}$

TABLE LXII—*Continued*

V	0	1	2	3	4	5	6	7	8
5.0	.3887	.3902	.3917	.3934	.3949	.3965	.3981	.3996	.4012
.1	.4044	.4060	.4076	.4092	.4108	.4123	.4140	.4156	.417
.2	.4204	.4220	.4236	.4253	.4269	.4285	.4302	.4318	.433
.3	.4367	.4384	.4400	.4417	.4433	.4450	.4467	.4483	.450
.4	.4534	.4551	.4567	.4584	.4601	.4618	.4635	.4652	.466
5.5	.4703	.4720	.4737	.4755	.4772	.4789	.4806	.4824	.48
.6	.4876	.4893	.4910	.4928	.4946	.4963	.4981	.4998	.50
.7	.5051	.5069	.5087	.5105	.5123	.5141	.5158	.5176	.51
.8	.5231	.5248	.5266	.5284	.5303	.5321	.5339	.5357	.53
.9	.5412	.5431	.5449	.5467	.5486	.5504	.5523	.5541	.55
6.0	.5598	.5616	.5634	.5653	.5672	.5691	.5710	.5728	.57
.1	.5785	.5804	.5823	.5842	.5861	.5880	.5900	.5919	.59
.2	.5977	.5996	.6015	.6034	.6054	.6073	.6093	.6112	.61
.3	.6171	.6191	.6210	.6230	.6249	.6269	.6289	.6309	.63
.4	.6368	.6388	.6408	.6428	.6448	.6468	.6489	.6508	.65
6.5	.6569	.6589	.6609	.6630	.6650	.6670	.6691	.6711	.673
.6	.6773	.6793	.6814	.6834	.6855	.6876	.6896	.6917	.693
.7	.6979	.7000	.7021	.7042	.7063	.7084	.7105	.7126	.714
.8	.7189	.7210	.7232	.7253	.7274	.7295	.7317	.7338	.736
.9	.7402	.7424	.7445	.7467	.7488	.7510	.7532	.7553	.7575
7.0	.7618	.7640	.7662	.7684	.7706	.7727	.7749	.7771	.7794
.1	.7838	.7860	.7882	.7904	.7926	.7948	.7971	.7993	.8015
.2	.8060	.8082	.8105	.8127	.8150	.8172	.8195	.8217	.8240
.3	.8285	.8308	.8331	.8354	.8376	.8399	.8422	.8445	.8468
.4	.8514	.8537	.8560	.8583	.8606	.8629	.8653	.8676	.8699
7.5	.8746	.8769	.8792	.8816	.8839	.8863	.8886	.8910	.8933
.6	.8980	.9004	.9028	.9051	.9075	.9099	.9123	.9147	.9171
.7	.9218	.9242	.9266	.9290	.9314	.9338	.9362	.9387	.9411
.8	.9459	.9484	.9508	.9532	.9557	.9581	.9605	.9630	.9654
.9	.9703	.9728	.9753	.9777	.9802	.9827	.9851	.9876	.9901
8.0	.9951	.9975	1.000	1.002	1.005	1.007	1.010	1.012	1.015
.1	1.020	1.023	1.025	1.028	1.030	1.033	1.035	1.038	1.040
.2	1.045	1.048	1.051	1.053	1.056	1.058	1.061	1.063	1.066
.3	1.071	1.074	1.076	1.079	1.082	1.084	1.087	1.089	1.092
.4	1.097	1.099	1.102	1.105	1.108	1.110	1.113	1.115	1.118
8.5	1.123	1.126	1.129	1.131	1.134	1.136	1.139	1.142	1.145
.6	1.150	1.152	1.155	1.158	1.161	1.163	1.166	1.169	1.172
.7	1.177	1.180	1.183	1.185	1.188	1.190	1.193	1.196	1.199
.8	1.204	1.207	1.210	1.212	1.215	1.218	1.221	1.223	1.226
.9	1.232	1.234	1.237	1.240	1.243	1.245	1.248	1.251	1.254
9.0	1.260	1.262	1.265	1.268	1.270	1.273	1.276	1.279	1.281
.1	1.288	1.290	1.293	1.296	1.299	1.302	1.304	1.307	1.310
.2	1.316	1.319	1.322	1.324	1.327	1.330	1.333	1.336	1.339
.3	1.345	1.348	1.351	1.354	1.356	1.359	1.362	1.365	1.368
.4	1.374	1.377	1.379	1.383	1.386	1.389	1.392	1.394	1.397
9.5	1.403	1.406	1.409	1.412	1.415	1.418	1.421	1.424	1.427
.6	1.433	1.436	1.439	1.442	1.445	1.448	1.451	1.454	1.457
.7	1.463	1.466	1.469	1.472	1.475	1.478	1.481	1.484	1.487
.8	1.493	1.496	1.499	1.503	1.506	1.508	1.511	1.514	1.517
.9	1.524	1.526	1.530	1.533	1.536	1.539	1.542	1.545	1.549
10.0	1.555								

Possible error of 1 in last figure. $g = 32.16$

TABLE LXIII

Smith's Weir Coefficients

c_s = coefficient for weirs with the contraction suppressed at both ends and complete crest contraction

Effective Head in Feet, H	Length of Weir in Feet, L								
	.66	2	3	4	5	7	10	15	19
.1	.675				.659	.658	.658	.657	.657
.15	.662	.652	.649	.647	.645	.645	.644	.644	.643
.2	.656	.645	.642	.641	.638	.637	.637	.636	.635
.25	.653	.641	.638	.636	.634	.633	.632	.631	.630
.3	.651	.639	.636	.633	.631	.629	.628	.627	.626
.4	.650	.636	.633	.630	.628	.625	.623	.622	.621
.5	.650	.637	.633	.630	.627	.624	.621	.620	.619
.6	.651	.638	.634	.630	.627	.623	.620	.619	.618
.7	.653	.640	.635	.631	.628	.624	.620	.619	.618
.8	.656	.643	.637	.633	.629	.625	.621	.620	.618
.9		.645	.639	.635	.631	.627	.622	.620	.619
1.0		.648	.641	.637	.633	.628	.624	.621	.619
1.1			.644	.639	.635	.630	.625	.622	.620
1.2			.646	.641	.636	.632	.626	.623	.620
1.3			.648	.643	.638	.633	.628	.624	.621
1.4				.644	.640	.634	.629	.625	.622
1.5				.646	.641	.636	.630	.625	.622
1.6				.647	.642	.637	.631	.626	.623
1.7						.638	.632	.626	.623

c_c = coefficient for weirs with complete contraction at two ends and complete crest contraction

Effective Head in Feet, H	Length of Weir in Feet, L										
	.66	1	2	2.6	3	4	5	7	10	15	19
.1	.632	.639	.646	.650	.652	.653	.653	.654	.655	.655	.656
.15	.619	.625	.634	.637	.638	.639	.640	.640	.641	.642	.642
.2	.611	.618	.626	.629	.630	.631	.631	.632	.633	.634	.634
.25	.605	.612	.621	.623	.624	.625	.626	.627	.628	.628	.629
.3	.601	.608	.616	.618	.619	.621	.621	.623	.624	.624	.625
.4	.595	.601	.609	.612	.613	.614	.615	.617	.618	.619	.620
.5	.590	.596	.605	.607	.608	.610	.611	.613	.615	.616	.617
.6	.587	.593	.601	.604	.605	.607	.608	.611	.613	.614	.615
.7	.585	.590	.598	.601	.603	.604	.606	.609	.612	.613	.614
.8			.595	.598	.600	.602	.604	.607	.611	.612	.613
.9			.592	.596	.598	.600	.603	.606	.609	.611	.612
1.0			.590	.593	.595	.598	.601	.604	.608	.610	.611
1.1			.587	.591	.593	.596	.599	.603	.606	.609	.610
1.2			.585	.589	.591	.594	.597	.601	.605	.608	.610
1.3			.582	.586	.589	.592	.596	.599	.604	.607	.609
1.4			.580	.584	.587	.590	.594	.598	.602	.606	.609
1.5				.582	.585	.589	.592	.596	.601	.605	.608
1.6				.580	.582	.587	.591	.595	.600	.604	.607
1.7								.594	.599	.603	.607

TABLE LXIV

Weir Table

(Possible errors of 1 in the last figure)

1 ($n^{\frac{3}{2}}$) + 0.007

Tenths of the Tabular Diff

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8
	.0103	.0108	.0114	.0119	.0125	.0131	.0137	.0143	.0150	.0157	1	1	2	2	3	4	4	[illegible]
	.0164	.0171	.0178	.0185	.0193	.0201	.0209	.0217	.0225	.0233	1	2	2	3	4	5	5	[illegible]
	.0242	.0251	.0259	.0268	.0278	.0287	.0296	.0306	.0315	.0325	1	2	3	4	5	6	6	[illegible]
	.0335	.0345	.0355	.0365	.0376	.0386	.0397	.0407	.0418	.0429	1	2	3	4	5	6	7	[illegible]
	.0440	.0452	.0463	.0474	[illegible]	[illegible]	[illegible]	[illegible]	.0532	.0544	1	2	3	5	6	7	8	[illegible]
	.0557	.0569	.0581	.0593	[illegible]	[illegible]	[illegible]	[illegible]	.0656	.0669	1	3	4	5	6	8	9	[illegible]
	.0683	.0696	.0709	.0723	[illegible]	[illegible]	[illegible]	[illegible]	.0791	.0805	1	3	4	5	7	8	10	[illegible]
	.0819	.0833	.0847	.0861	[illegible]	[illegible]	[illegible]	[illegible]	.0934	.0949	1	3	4	6	7	9	10	[illegible]
	.0964	.0979	.0993	.1008	[illegible]	[illegible]	[illegible]	[illegible]	.1086	.1101	2	3	5	6	8	9	11	[illegible]
	.1117	.1133	.1148	.1164	.1180	.1196	[illegible]	[illegible]	.1245	.1261	2	3	5	6	8	10	11	[illegible]
	.1277	.1294	.1311	.1327	.1344	.1361	[illegible]	[illegible]	.1412	.1429	2	3	5	7	8	10	12	[illegible]
	.1446	.1464	.1480	.1498	.1515	.1533	[illegible]	[illegible]	.1586	.1604	2	4	5	7	9	11	12	[illegible]
	.1621	.1640	.1657	.1675	.1693	.1712	[illegible]	[illegible]	.1767	.1786	2	4	5	7	9	11	13	[illegible]
	.1804	.1822	.1841	.1860	.1879	.1897	[illegible]	[illegible]	.1955	.1973	2	4	6	8	9	11	13	[illegible]
	.1993	.2012	.2031	.2051	.2070	.2090	[illegible]	[illegible]	.2148	.2168	2	4	6	8	10	12	14	[illegible]
	.2188	.2208	.2228	.2248	.2268	.2289	[illegible]	[illegible]	.2349	.2370	2	4	6	8	10	12	14	[illegible]
	.2390	.2411	.2431	.2452	.2472	.2493	[illegible]	[illegible]	.2556	.2577	2	4	6	8	10	12	15	[illegible]
	.2598	.2619	.2639	.2661	.2682	.2704	[illegible]	[illegible]	.2768	.2790	2	4	6	9	11	13	15	[illegible]
	.2811	.2833	.2855	.2877	.2898	.2920	[illegible]	[illegible]	.2986	.3008	2	4	7	9	11	13	15	[illegible]
.20	.3030	.3053	.3075	.3098	.3120	.3142	.3165	.3187	.3210	.3233	2	4	7	9	11	13	16	[illegible]
1	.3255	.3277	.3300	.3323	.3347	.3370	.3393				2	5	7	9	11	14	16	[illegible]
1							.339	.342	.344	.346	0	0	1	1	1	1	2	[illegible]
2	.349	.351	.353	.356	.358	.360	.363	.365	.368	.370	0	0	1	1	1	1	2	[illegible]
3	.372	.375	.377	.380	.382	.384	.387	.389	.392	.394	0	0	1	1	1	1	2	[illegible]
4	.396	.399	.401	.404	.406	.409	.411	.414	.416	.419	0	0	1	1	1	1	2	[illegible]
.25	.421	.423	.425	.428	.430	.433	.435	.438	.440	.443	0	1	1	1	1	2	2	[illegible]
6	.446	.448	.451	.453	.456	.458	.461	.464	.466	.469	0	1	1	1	1	2	2	[illegible]
7	.471	.474	.477	.479	.482	.484	.487	.490	.492	.495	0	1	1	1	1	2	2	[illegible]
8	.498	.500	.503	.505	.508	.510	.513	.516	.519	.521	0	1	1	1	1	2	2	[illegible]
9	.524	.527	.529	.532	.535	.537	.540	.543	.546	.548	0	1	1	1	1	2	2	[illegible]
.30	.551	.554	.557	.559	.562	.565	.568	.570	.573	.576	0	1	1	1	1	2	2	[illegible]
1	.578	.581	.584	.586	.589	.592	.594	.597	.600	.603	0	1	1	1	1	2	2	[illegible]
2	.606	.609	.611	.614	.617	.620	.623	.626	.628	.631	0	1	1	1	1	2	2	[illegible]
3	.635	.637	.640	.643	.646	.649	.652	.654	.657	.660	0	1	1	1	1	2	2	[illegible]
4	.663	.666	.669	.672	.675	.678	.681	.684	.687	.690	0	1	1	1	1	2	2	[illegible]
.35	.693	.696	.698	.701	.704	.707	.710	.713	.716	.719	0	1	1	1	1	2	2	[illegible]
6	.722	.725	.728	.731	.734	.737	.740	.743	.746	.750	0	1	1	1	2	2	2	[illegible]
7	.752	.755	.758	.761	764	.767	.770	.773	.776	.779	0	1	1	1	2	2	2	[illegible]
8	.782	.785	.788	.791	.795	.798	.801	.804	.807	.810	0	1	1	1	2	2	2	[illegible]
9	.813	.816	.819	.823	.826	.829	.832	.835	.838	.841	0	1	1	1	2	2	2	[illegible]
.40	.844	.847	.851	.854	.857	.860	.863	.867	.870	.873	0	1	1	1	2	2	2	[illegible]
1	.876	.879	.883	.886	.889	.892	.895	.899	.902	.905	0	1	1	1	2	2	2	[illegible]
2	.908	.912	.915	.917	.920	.924	.927	.930	.933	.937	0	1	1	1	2	2	2	[illegible]
3	.940	.943	.946	.950	.953	.956	.960	.963	.966	.970	0.	1	1	1	2	2	2	[illegible]
4	.973	.976	.980	.983	.986	.990	.993	.996	1.000	1.003	0	1	1	1	2	2	2	[illegible]
.45	1.006	1.010	1.013	1.016	1.020	1.023	1.026	1.030	1.033	1.036	0	1	1	1	2	2	2	[illegible]
6	1.040	1.043	1.047	1.050	1.053	1.057	1.060	1.063	1.067	1.071	0	1	1	1	2	2	2	[illegible]
7	1.074	1.077	1.081	1.083	1.087	1.090	1.094	1.097	1.101	1.104	0	1	1	1	2	2	2	[illegible]
8	1.108	1.111	1.114	1.118	1.121	1.125	1.128	1.132	1.135	1.139	0	1	1	1	2	2	2	[illegible]
9	1.142	1.146	1.149	1.153	1.156	1.160	1.163	1.167	1.170	1.174	0	1	1	1	2	2	2	[illegible]
.50	1.177																	

TABLE LXIV — *Continued*

Weir Table

(Possible errors of 1 in the last figure)

									Tenths of the Tabular Differences								
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.181	1.184	1.188	1.191	1.195	1.198	1.202	1.206	1.209	0	1	1	1	2	2	2	3	3
.216	1.220	1.223	1.227	1.231	1.234	1.238	1.241	1.245	0	1	1	1	2	2	3	3	3
.252	1.256	1.259	1.263	1.267	1.270	1.274	1.278	1.281	0	1	1	1	2	2	3	3	3
.288	1.292	1.296	1.299	1.303	1.307	1.310	1.314	1.318	0	1	1	1	2	2	3	3	3
.325	1.329	1.332	1.336	1.340	1.343	1.347	1.351	1.355	0	1	1	1	2	2	3	3	3
362	1.366	1.369	1.373	1.377	1.381	1.384	1.388	1.392	0	1	1	1	2	2	3	3	3
399	1.403	1.407	1.411	1.414	1.418	1.422	1.426	1.429	0	1	1	2	2	2	3	3	3
437	1.441	1.444	1.448	1.452	1.456	1.460	1.463	1.467	0	1	1	2	2	2	3	3	3
475	1.479	1.482	1.486	1.490	1.494	1.498	1.501	1.505	0	1	1	2	2	2	3	3	3
513	1.517	1.520	1.524	1.528	1.532	1.536	1.540	1.544	0	1	1	2	2	2	3	3	3
551	1.555	1.559	1.563	1.567	1.571	1.575	1.579	1.583	0	1	1	2	2	2	3	3	3
590	1.594	1.598	1.602	1.606	1.610	1.614	1.618	1.622	0	1	1	2	2	2	3	3	4
630	1.634	1.637	1.641	1.645	1.649	1.653	1.657	1.661	0	1	1	2	2	2	3	3	4
669	1.673	1.677	1.681	1.685	1.689	1.693	1.697	1.701	0	1	1	2	2	2	3	3	4
709	1.713	1.717	1.721	1.725	1.729	1.733	1.737	1.741	0	1	1	2	2	2	3	3	4
749	1.753	1.757	1.761	1.765	1.769	1.773	1.778	1.782	0	1	1	2	2	2	3	3	4
790	1.794	1.798	1.802	1.806	1.810	1.814	1.818	1.822	0	1	1	2	2	2	3	3	4
830	1.834	1.838	1.842	1.847	1.851	1.855	1.859	1.863	0	1	1	2	2	2	3	3	4
871	1.875	1.880	1.884	1.888	1.892	1.896	1.900	1.904	0	1	1	2	2	2	3	3	4
913	1.917	1.921	1.925	1.929	1.933	1.938	1.942	1.946	0	1	1	2	2	2	3	3	4
954	1.959	1.963	1.967	1.971	1.975	1.980	1.984	1.988	0	1	1	2	2	3	3	3	4
996	2.001	2.005	2.009	2.013	2.018	2.022	2.026	2.030	0	1	1	2	2	3	3	3	4
039	2.043	2.047	2.051	2.056	2.060	2.064	2.069	2.073	0	1	1	2	2	3	3	3	4
081	2.086	2.090	2.094	2.098	2.103	2.107	2.111	2.166	0	1	1	2	2	3	3	3	4
124	2.129	2.133	2.137	2.141	2.146	2.150	2.154	2.159	0	1	1	2	2	3	3	3	4
167	2.171	2.176	2.180	2.184	2.189	2.193	2.197	2.202	0	1	1	2	2	3	3	3	4
211	2.215	2.219	2.224	2.228	2.232	2.237	2.241	2.246	0	1	1	2	2	3	3	3	4
254	2.259	2.263	2.267	2.272	2.276	2.281	2.285	2.290	0	1	1	2	2	3	3	4	4
298	2.303	2.307	2.312	2.316	2.320	2.325	2.329	2.334	0	1	1	2	2	3	3	4	4
343	2.347	2.352	2.356	2.360	2.365	2.369	2.374	2.378	0	1	1	2	2	3	3	4	4
387	2.392	2.396	2.401	2.405	2.410	2.414	2.419	2.423	0	1	1	2	2	3	3	4	4
432	2.437	2.441	2.446	2.450	2.455	2.459	2.464	2.468	0	1	1	2	2	3	3	4	4
477	2.482	2.486	2.491	2.495	2.500	2.504	2.509	2.513	0	1	1	2	2	3	3	4	4
522	2.527	2.532	2.536	2.541	2.545	2.550	2.554	2.559	0	1	1	2	2	3	3	4	4
568	2.573	2.577	2.582	2.587	2.591	2.596	2.600	2.605	0	1	1	2	2	3	3	4	4
614	2.619	2.623	2.628	2.633	2.637	2.642	2.647	2.651	0	1	1	2	2	3	3	4	4
660	2.665	2.670	2.674	2.679	2.684	2.688	2.693	2.698	0	1	1	2	2	3	3	4	4
707	2.712	2.716	2.721	2.726	2.730	2.735	2.740	2.744	0	1	1	2	2	3	3	4	4
754	2.758	2.763	2.768	2.773	2.777	2.782	2.787	2.791	0	1	1	2	2	3	3	4	4
801	2.806	2.810	2.815	2.820	2.824	2.829	2.834	2.838	0	1	1	2	2	3	3	4	4
848	2.853	2.857	2.862	2.867	2.872	2.876	2.881	2.886	0	1	1	2	2	3	3	4	4
895	2.900	2.905	2.910	2.914	2.919	2.924	2.929	2.934	0	1	1	2	2	3	3	4	4
943	2.948	2.953	2.958	2.962	2.967	2.972	2.977	2.982	0	1	1	2	2	3	3	4	4
991	2.996	3.001	3.006	3.011	3.016	3.020	3.025	3.030	0	1	1	2	2	3	3	4	4
040	3.045	3.049	3.054	3.059	3.064	3.069	3.074	3.079	0	1	1	2	2	3	3	4	4
088	3.093	3.098	3.103	3.108	3.113	3.118	3.123	3.127	0	1	1	2	2	3	3	4	4
137	3.142	3.147	3.152	3.157	3.162	3.166	3.171	3.176	0	1	1	2	2	3	3	4	4
.186	3.191	3.196	3.201	3.206	3.211	3.216	3.221	3.226	0	1	1	2	2	3	3	4	4
.235	3.240	3.245	3.250	3.255	3.260	3.265	3.270	3.275	0	1	1	2	2	3	3	4	4
.285	3.290	3.295	3.300	3.305	3.310	3.315	3.320	3.325	0	1	1	2	2	3	3	4	4

TABLE LXIV — *Continued*

Weir Table

(Possible errors of 1 in the last figure)

$3.33\,(n^{\frac{3}{2}})$

Tenths of the Tabular [difference]

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7
1.0	3.33	3.38	3.43	3.48	3.53	3.58	3.63	3.69	3.74	3.79	1	1	2	2	3	3	
.1	3.84	3.89	3.95	4.00	4.05	4.11	4.16	4.22	4.27	4.32	1	1	2	2	3	3	
.2	4.38	4.43	4.49	4.54	4.60	4.66	4.71	4.77	4.82	4.88	1	1	2	2	3	3	
.3	4.94	4.99	5.05	5.11	5.16	5.22	5.28	5.34	5.40	5.46	1	1	2	2	3	3	
.4	5.52	5.57	5.63	5.69	5.75	5.81	5.87	5.93	5.99	6.06	1	1	2	2	3	4	
1.5	6.12	6.18	6.24	6.30	6.36	6.43	6.49	6.55	6.61	6.68	1	1	2	2	3	4	
.6	6.74	6.80	6.87	6.92	6.99	7.06	7.12	7.19	7.25	7.32	1	1	2	3	3	4	
.7	7.38	7.45	7.51	7.58	7.64	7.71	7.78	7.84	7.91	7.98	1	1	2	3	3	4	
.8	8.04	8.11	8.18	8.25	8.31	8.38	8.45	8.51	8.58	8.65	1	1	2	3	3	4	
.9	8.72	8.79	8.86	8.93	9.00	9.07	9.14	9.21	9.28	9.35	1	1	2	3	3	4	
2.0	9.42	9.49	9.56	9.63	9.70	9.77	9.85	9.92	9.99	10.06	1	1	2	3	4	4	
.1	10.13	10.21	10.28	10.35	10.43	10.50	10.57	10.65	10.72	10.79	1	1	2	3	4	4	
.2	10.87	10.94	11.02	11.09	11.17	11.24	11.32	11.39	11.47	11.54	1	1	2	3	4	4	
.3	11.62	11.69	11.77	11.84	11.92	11.99	12.07	12.15	12.23	12.30	1	2	2	3	4	5	
.4	12.38	12.46	12.54	12.61	12.69	12.77	12.85	12.93	13.01	13.08	1	2	2	3	4	5	
2.5	13.16	13.24	13.32	13.40	13.48	13.56	13.64	13.72	13.80	13.88	1	2	2	3	4	5	
.6	13.96	14.04	14.12	14.20	14.28	14.37	14.45	14.53	14.61	14.69	1	2	2	3	4	5	
.7	14.78	14.86	14.94	15.02	15.10	15.18	15.27	15.35	15.43	15.52	1	2	2	3	4	5	
.8	15.60	15.68	15.77	15.85	15.94	16.02	16.11	16.19	16.28	16.36	1	2	3	3	4	5	
.9	16.45	16.53	16.62	16.70	16.79	16.87	16.96	17.04	17.13	17.22	1	2	3	3	4	5	
3.0	17.30	17.39	17.48	17.56	17.65	17.74	17.83	17.91	18.00	18.09	1	2	3	3	4	5	
.1	18.18	18.27	18.35	18.44	18.53	18.62	18.70	18.79	18.88	18.97	1	2	3	4	4	5	
.2	19.06	19.15	19.24	19.33	19.42	19.51	19.60	19.69	19.78	19.87	1	2	3	4	5	5	
.3	19.96	20.05	20.14	20.24	20.33	20.42	20.51	20.60	20.69	20.79	1	2	3	4	5	5	
.4	20.88	20.97	21.06	21.15	21.25	21.34	21.43	21.53	21.62	21.71	1	2	3	4	5	6	
3.5	21.80	21.90	21.99	22.08	22.18	22.27	22.37	22.46	22.56	22.65	1	2	3	4	5	6	
.6	22.75	22.84	22.94	23.03	23.13	23.22	23.32	23.41	23.51	23.60	1	2	3	4	5	6	
.7	23.70	23.80	23.89	23.99	24.09	24.18	24.28	24.38	24.47	24.57	1	2	3	4	5	6	
.8	24.67	24.77	24.86	24.96	25.06	25.15	25.25	25.35	25.45	25.55	1	2	3	4	5	6	
.9	25.65	25.75	25.84	25.94	26.04	26.14	26.24	26.34	26.44	26.54	1	2	3	4	5	6	
4.0	26.64	26.74	26.84	26.94	27.04	27.14	27.24	27.34	27.44	27.55	1	2	3	4	5	6	
.1	27.65	27.75	27.85	27.95	28.05	28.15	28.26	28.35	28.46	28.56	1	2	3	4	5	6	
.2	28.66	28.76	28.87	28.97	29.07	29.18	29.28	29.38	29.49	29.59	1	2	3	4	5	6	
.3	29.69	29.80	29.90	30.00	30.11	30.21	30.32	30.42	30.53	30.63	1	2	3	4	5	6	
.4	30.74	30.84	30.95	31.05	31.16	31.26	31.37	31.47	31.58	31.68	1	2	3	4	5	6	
4.5	31.79	31.89	32.00	32.11	32.21	32.32	32.42	32.53	32.64	32.75	1	2	3	4	5	6	
.6	32.85	32.96	33.07	33.18	33.28	33.39					1	2	3	4	5	6	
.6						33.4	33.5	33.6	33.7	33.8	0	0	0	0	1	1	
.7	33.9	34.0	34.1	34.3	34.4	34.5	34.6	34.7	34.8	34.9	0	0	0	0	1	1	
.8	35.0	35.1	35.2	35.4	35.5	35.6	35.7	35.8	35.9	36.0	0	0	0	0	1	1	
4.9	36.1	36.2	36.3	36.5	36.6	36.7	36.8	36.9	37.0	37.1	0	0	0	0	1	1	
5	37.2	38.4	39.5	40.6	41.8	43.0	44.1	45.3	46.5	47.7	1	2	4	5	6	7	
6	49.0	50.2	51.4	52.6	53.9	55.2	56.5	57.7	59.0	60.3	1	3	4	5	6	8	
7	61.7	63.0	64.3	65.7	67.0	68.4	69.8	71.2	72.5	73.9	1	3	4	5	7	8	
8	75.4	76.8	78.2	79.6	81.1	82.5	84.0	85.4	86.9	88.4	1	3	4	6	7	9	
9	89.9	91.4	92.9	94.4	96.0	97.5	99.0	100.6	102.2	103.7	2	3	5	6	8	9	
10	105.3	106.9	108.5	110.1	111.7	113.3	114.9	116.6	118.2	119.8	2	3	5	6	8	10	
1	121.5	123.1	124.8	126.5	128.2	129.9	131.6	133.3	135.0	136.7	2	3	5	7	8	10	
2	138.4	140.2	141.9	143.7	145.4	147.2	149.0	150.7	152.5	154.3	2	4	5	7	9	11	
3	156.1	157.9	159.7	161.5	163.3	165.2	167.0	168.9	170.7	172.6	2	4	6	7	9	11	
4	174.4	176.3	178.2	180.1	182.0	183.8	185.8	187.7	189.6	191.5	2	4	6	8	10	11	
15	193.4	195.4	197.3	199.3	201.2	203.2	205.2	207.2	209.1	211.1	2	4	6	8	10	12	
6	213.1	215.1	217.1	219.1	221.1	223.2	225.2	227.3	229.3	231.4	2	4	6	8	10	12	
7	233.4	235.5	237.5	239.6	241.7	243.8	245.9	248.0	250.1	252.2	2	4	6	8	10	13	
8	254.3	256.4	258.5	260.7	262.8	265.0	267.1	269.3	271.4	273.6	2	4	6	9	11	13	
9	275.8	278.0	280.2	282.4	284.5	286.7	288.9	291.2	293.4	295.6	2	4	7	9	11	13	
20	297.8																

TABLE LXV

$$d' = \frac{D}{2}\left(1 - \cos\frac{\phi}{2}\right) \qquad \cos\frac{\phi}{2} = 1 - \frac{2d'}{D}$$

ϕ = angle subtended at center, in radians

$$w.p. = \frac{D}{2}\phi$$

$$A = \frac{D^2}{8}(\phi - \sin\phi)$$

$$R = \frac{A}{w.p.} = \frac{D}{4}\left(1 - \frac{\sin\phi}{\phi}\right)$$

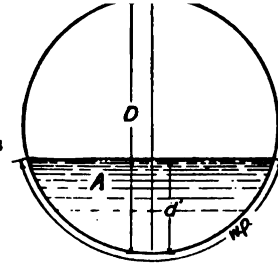


FIG. 135. — Circular Channel partly Filled.

Wet perimeter

0	1	2	3	4	5	6	7	8	9
00*	0.200*	0.284*	0.348*	0.403	0.451	0.495 +	0.536	0.574	0.609
44	0.676	0.707	0.738	0.767	0.795 +	0.823	0.850 –	0.876	0.902
27	0.952	0.976	1.000	1.024	1.047	1.070	1.093	1.115 +	1.137
59	1.181	1.203	1.224	1.245 +	1.266	1.287	1.308	1.328	1.349
69	1.390	1.410	1.430	1.450 +	1.471	1.491	1.511	1.531	1.551
71	1.591	1.611	1.631	1.651	1.671	1.691	1.711	1.731	1.752
72	1.793	1.813	1.834	1.855 –	1.875 +	1.897	1.918	1.939	1.961
82	2.004	2.026	2.049	2.071	2.094	2.118	2.141	2.165 +	2.190
14	2.240	2.265 +	2.292	2.319	2.346	2.375 –	2.404	2.434	2.465 +
98	2.532	2.568	2.606	2.647	2.691	2.739*	2.793*	2.858*	2.941*
42									

Wet area of cross section

0	1	2	3	4	5	6	7	8	9
000*	.0013*	.0037*	.0069*	.0105 +	.0147	.0192	.0242	.0294	.0350 +
09	.0470	.0534	.0600	.0668	.0739	.0811	.0885 +	.0961	.1039
18	.1199	.1281	.1365	.1449	.1535 +	.1623	.1711	.1800	.1890
32	.2074	.2167	.2260	.2355 –	.2450 –	.2545 +	.2642	.2739	.2836
34	.3032	.3130	.3229	.3328	.3428	.3527	.3627	.3727	.3827
27	.4027	.4127	.4227	.4327	.4426	.4526	.4625 –	.4724	.4822
20	.5018	.5115 +	.5212	.5308	.5404	.5499	.5594	.5687	.5780
72	.5964	.6054	.6143	.6231	.6319	.6405 –	.6489	.6573	.6655 +
36	.6815 –	.6893	.6969	.7043	.7115 +	.7186	.7254	.7320	.7384
45 +	.7504	.7560	.7612	.7662	.7707	.7749*	.7785 +*	.7816*	.7841*
54									

/ D^2

.) / D

Mean hydraulic radius

0	1	2	3	4	5	6	7	8	9
00	.0066	.0132	.0197	.0262	.0326	.0389	.0451	.0513	.0575 –
35	.0695	.0755	.0813	.0871	.0929	.0986	.1042	.1097	.1152
06	.1259	.1312	.1364	.1416	.1466	.1516	.1566	.1614	.1662
09	.1756	.1802	.1847	.1891	.1935 –	.1978	.2020	.2062	.2102
42	.2181	.2220	.2258	.2295 –	.2331	.2366	.2401	.2435 –	.2468
00	.2531	.2562	.2592	.2621	.2649	.2676	.2703	.2728	.2753
76	.2799	.2821	.2842	.2862	.2881	.2900	.2917	.2933	.2948
62	.2975	.2987	.2998	.3008	.3017	.3024	.3031	.3036	.3039
42	.3043	.3043	.3041	.3038	.3033	.3026	.3018	.3007	.2995
80	.2963	.2944	.2921	.2895 –	.2865 –	.2829*	.2787*	.2735 +*	.2666*
00									

* *Interpolation at points marked (*) may be inaccurate.*

TABLE LXVI

Circumferences of Circles by Hundredths

These two pages give the circumferences of circles for any diameter n, from $n = 1$ to $n = 10$.
To obtain the circumference of a circle whose diameter is outside the range from 1 to 10, note that moving the decim
column n is equivalent to moving it the same number of places in the body of the table.
The table serves also as a table of the multiples of π.

(n = diameter)

$n\pi$ | Tenths of the Tabular Diff

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7
.0	3.142	3.173	3.204	3.236	3.267	3.299	3.330	3.362	3.393	3.424	3	6	9	13	16	19	22
.1	3.456	3.487	3.519	3.550	3.581	3.613	3.644	3.676	3.707	3.738	3	6	9	13	16	19	22
.2	3.770	3.801	3.833	3.864	3.896	3.927	3.958	3.990	4.021	4.053	3	6	9	13	16	19	22
.3	4.084	4.115	4.147	4.178	4.210	4.241	4.273	4.304	4.335	4.367	3	6	9	13	16	19	22
.4	4.398	4.430	4.461	4.492	4.524	4.555	4.587	4.618	4.650	4.681	3	6	9	13	16	19	22
.5	4.712	4.744	4.775	4.807	4.838	4.869	4.901	4.932	4.964	4.995	3	6	9	13	16	19	22
.6	5.027	5.058	5.089	5.121	5.152	5.184	5.215	5.246	5.278	5.309	3	6	9	13	16	19	22
.7	5.341	5.372	5.404	5.435	5.466	5.498	5.529	5.561	5.592	5.623	3	6	9	13	16	19	22
.8	5.655	5.686	5.718	5.749	5.781	5.812	5.843	5.875	5.906	5.938	3	6	9	13	16	19	22
.9	5.969	6.000	6.032	6.063	6.095	6.126	6.158	6.189	6.220	6.252	3	6	9	13	16	19	22
.0	6.283	6.315	6.346	6.377	6.409	6.440	6.472	6.503	6.535	6.566	3	6	9	13	16	19	22
.1	6.597	6.629	6.660	6.692	6.723	6.754	6.786	6.817	6.849	6.880	3	6	9	13	16	19	22
.2	6.912	6.943	6.974	7.006	7.037	7.069	7.100	7.131	7.163	7.194	3	6	9	13	16	19	22
.3	7.226	7.257	7.288	7.320	7.351	7.383	7.414	7.446	7.477	7.508	3	6	9	13	16	19	22
.4	7.540	7.571	7.603	7.634	7.665	7.697	7.728	7.760	7.791	7.823	3	6	9	13	16	19	22
.5	7.854	7.885	7.917	7.948	7.980	8.011	8.042	8.074	8.105	8.137	3	6	9	13	16	19	22
.6	8.168	8.200	8.231	8.262	8.294	8.325	8.357	8.388	8.419	8.451	3	6	9	13	16	19	22
.7	8.482	8.514	8.545	8.577	8.608	8.639	8.671	8.702	8.734	8.765	3	6	9	13	16	19	22
.8	8.796	8.828	8.859	8.891	8.922	8.954	8.985	9.016	9.048	9.079	3	6	9	13	16	19	22
.9	9.111	9.142	9.173	9.205	9.236	9.268	9.299	9.331	9.362	9.393	3	6	9	13	16	19	22
.0	9.425	9.456	9.488	9.519	9.550	9.582	9.613	9.645	9.676	9.708	3	6	9	13	16	19	22
.1	9.739	9.770	9.802	9.833	9.865	9.896	9.927	9.959	9.990	10.022	3	6	9	13	16	19	22
.2	10.05	10.08	10.12	10.15	10.18	10.21	10.24	10.27	10.30	10.34	0	1	1	1	2	2	2
.3	10.37	10.40	10.43	10.46	10.49	10.52	10.56	10.59	10.62	10.65	0	1	1	1	2	2	2
.4	10.68	10.71	10.74	10.78	10.81	10.84	10.87	10.90	10.93	10.96	0	1	1	1	2	2	2
.5	11.00	11.03	11.06	11.09	11.12	11.15	11.18	11.22	11.25	11.28	0	1	1	1	2	2	2
.6	11.31	11.34	11.37	11.40	11.44	11.47	11.50	11.53	11.56	11.59	0	1	1	1	2	2	2
.7	11.62	11.66	11.69	11.72	11.75	11.78	11.81	11.84	11.88	11.91	0	1	1	1	2	2	2
.8	11.94	11.97	12.00	12.03	12.06	12.10	12.13	12.16	12.19	12.22	0	1	1	1	2	2	2
.9	12.25	12.28	12.32	12.35	12.38	12.41	12.44	12.47	12.50	12.53	0	1	1	1	2	2	2
.0	12.57	12.60	12.63	12.66	12.69	12.72	12.75	12.79	12.82	12.85	0	1	1	1	2	2	2
.1	12.88	12.91	12.94	12.97	13.01	13.04	13.07	13.10	13.13	13.16	0	1	1	1	2	2	2
.2	13.19	13.23	13.26	13.29	13.32	13.35	13.38	13.41	13.45	13.48	0	1	1	1	2	2	2
.3	13.51	13.54	13.57	13.60	13.63	13.67	13.70	13.73	13.76	13.79	0	1	1	1	2	2	2
.4	13.82	13.85	13.89	13.92	13.95	13.98	14.01	14.04	14.07	14.11	0	1	1	1	2	2	2
.5	14.14	14.17	14.20	14.23	14.26	14.29	14.33	14.36	14.39	14.42	0	1	1	1	2	2	2
.6	14.45	14.48	14.51	14.55	14.58	14.61	14.64	14.67	14.70	14.73	0	1	1	1	2	2	2
.7	14.77	14.80	14.83	14.86	14.89	14.92	14.95	14.99	15.02	15.05	0	1	1	1	2	2	2
.8	15.08	15.11	15.14	15.17	15.21	15.24	15.27	15.30	15.33	15.36	0	1	1	1	2	2	2
.9	15.39	15.43	15.46	15.49	15.52	15.55	15.58	15.61	15.65	15.68	0	1	1	1	2	2	2

TABLE LXVI — *Continued*

Circumferences of Circles by Hundredths

(n = diameter)

Tenths of the Tabular Difference

1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
15.74	15.77	15.80	15.83	15.87	15.90	15.93	15.96	15.99	0	1	1	1	2	2	2	3	3
16.05	16.08	16.12	16.15	16.18	16.21	16.24	16.27	16.30	0	1	1	1	2	2	2	3	3
16.37	16.40	16.43	16.46	16.49	16.52	16.56	16.59	16.62	0	1	1	1	2	2	2	3	3
16.68	16.71	16.74	16.78	16.81	16.84	16.87	16.90	16.93	0	1	1	1	2	2	2	3	3
7.00	17.03	17.06	17.09	17.12	17.15	17.18	17.22	17.25	0	1	1	1	2	2	2	3	3
7.31	17.34	17.37	17.40	17.44	17.47	17.50	17.53	17.56	0	1	1	1	2	2	2	3	3
7.62	17.66	17.69	17.72	17.75	17.78	17.81	17.84	17.88	0	1	1	1	2	2	2	3	3
7.94	17.97	18.00	18.03	18.06	18.10	18.13	18.16	18.19	0	1	1	1	2	2	2	3	3
8.25	18.28	18.32	18.35	18.38	18.41	18.44	18.47	18.50	0	1	1	1	2	2	2	3	3
8.57	18.60	18.63	18.66	18.69	18.72	18.76	18.79	18.82	0	1	1	1	2	2	2	3	3
8.88	18.91	18.94	18.98	19.01	19.04	19.07	19.10	19.13	0	1	1	1	2	2	2	3	3
9.20	19.23	19.26	19.29	19.32	19.35+	19.38	19.42	19.45−	0	1	1	1	2	2	2	3	3
9.51	19.54	19.57	19.60	19.63	19.67	19.70	19.73	19.76	0	1	1	1	2	2	2	3	3
9.82	19.85+	19.89	19.92	19.95−	19.98	20.01	20.04	20.07	0	1	1	1	2	2	2	3	3
0.14	20.17	20.20	20.23	20.26	20.29	20.33	20.36	20.39	0	1	1	1	2	2	2	3	3
0.45+	20.48	20.51	20.55−	20.58	20.61	20.64	20.67	20.70	0	1	1	1	2	2	2	3	3
0.77	20.80	20.83	20.86	20.89	20.92	20.95+	20.99	21.02	0	1	1	1	2	2	2	3	3
1.08	21.11	21.14	21.17	21.21	21.24	21.27	21.30	21.33	0	1	1	1	2	2	2	3	3
1.39	21.43	21.46	21.49	21.52	21.55+	21.58	21.61	21.65−	0	1	1	1	2	2	2	3	3
1.71	21.74	21.77	21.80	21.83	21.87	21.90	21.93	21.96	0	1	1	1	2	2	2	3	3
2.02	22.05+	22.09	22.12	22.15−	22.18	22.21	22.24	22.27	0	1	1	1	2	2	2	3	3
2.34	22.37	22.40	22.43	22.46	22.49	22.53	22.56	22.59	0	1	1	1	2	2	2	3	3
2.65+	22.68	22.71	22.75−	22.78	22.81	22.84	22.87	22.90	0	1	1	1	2	2	2	3	3
2.97	23.00	23.03	23.06	23.09	23.12	23.15+	23.18	23.22	0	1	1	1	2	2	2	3	3
3.28	23.31	23.34	23.37	23.40	23.44	23.47	23.50−	23.53	0	1	1	1	2	2	2	3	3
3.59	23.62	23.66	23.69	23.72	23.75+	23.78	23.81	23.84	0	1	1	1	2	2	2	3	3
3.91	23.94	23.97	24.00	24.03	24.06	24.10	24.13	24.16	0	1	1	1	2	2	2	3	3
4.22	24.25+	24.28	24.32	24.35−	24.38	24.41	24.44	24.47	0	1	1	1	2	2	2	3	3
4.54	24.57	24.60	24.63	24.66	24.69	24.72	24.76	24.79	0	1	1	1	2	2	2	3	3
4.85−	24.88	24.91	24.94	24.98	25.01	25.04	25.07	25.10	0	1	1	1	2	2	2	3	3
5.16	25.20	25.23	25.26	25.29	25.32	25.35+	25.38	25.42	0	1	1	1	2	2	2	3	3
5.48	25.51	25.54	25.57	25.60	25.64	25.67	25.70	25.73	0	1	1	1	2	2	2	3	3
5.79	25.82	25.86	25.89	25.92	25.95−	25.98	26.01	26.04	0	1	1	1	2	2	2	3	3
6.11	26.14	26.17	26.20	26.23	26.26	26.30	26.33	26.36	0	1	1	1	2	2	2	3	3
6.42	26.45+	26.48	26.52	26.55−	26.58	26.61	26.64	26.67	0	1	1	1	2	2	2	3	3
6.73	26.77	26.80	26.83	26.86	26.89	26.92	26.95+	26.99	0	1	1	1	2	2	2	3	3
7.05−	27.08	27.11	27.14	27.17	27.21	27.24	27.27	27.30	0	1	1	1	2	2	2	3	3
7.36	27.39	27.43	27.46	27.49	27.52	27.55+	27.58	27.61	0	1	1	1	2	2	2	3	3
7.68	27.71	27.74	27.77	27.80	27.83	27.87	27.90	27.93	0	1	1	1	2	2	2	3	3
7.99	28.02	28.05+	28.09	28.12	28.15−	28.18	28.21	28.24	0	1	1	1	2	2	2	3	3
8.31	28.34	28.37	28.40	28.43	28.46	28.49	28.53	28.56	0	1	1	1	2	2	2	3	3
8.62	28.65+	28.68	28.71	28.75−	28.78	28.81	28.84	28.87	0	1	1	1	2	2	2	3	3
8.93	28.97	29.00	29.03	29.06	29.09	29.12	29.15+	29.19	0	1	1	1	2	2	2	3	3
9.25−	29.28	29.31	29.34	29.37	29.41	29.44	29.47	29.50−	0	1	1	1	2	2	2	3	3
9.56	29.59	29.63	29.66	29.69	29.72	29.75+	29.78	29.81	0	1	1	1	2	2	2	3	3
29.88	29.91	29.94	29.97	30.00	30.03	30.07	30.10	30.13	0	1	1	1	2	2	2	3	3
30.19	30.22	30.25+	30.28	30.32	30.35−	30.38	30.41	30.44	0	1	1	1	2	2	2	3	3
30.50+	30.54	30.57	30.60	30.63	30.66	30.69	30.72	30.76	0	1	1	1	2	2	2	3	3
30.82	30.85+	30.88	30.91	30.94	30.98	31.01	31.04	31.07	0	1	1	1	2	2	2	3	3
31.13	31.16	31.20	31.23	31.26	31.29	31.32	31.35+	31.38	0	1	1	1	2	2	2	3	3

TABLE LXVII

Area of Circles by Hundredths

(n = diameter)

These two pages give the area of circles for any diameter n, from $n = 1$ to $n = 10$. To obtain the area of a diameter is outside the range from 1 to 10, note that moving the decimal point one place in column n is equivalent two places in the body of the table. (Compare heading to table of squares.)

$\frac{\pi}{4} n^2$

Tenths of the Tabular D

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6
1.0	0.785	0.801	0.817	0.833	0.849	0.866	0.882	0.899	0.916	0.933	2	3	5	7	8	10
1.1	0.950	0.968	0.985	1.003	1.021	1.039	1.057	1.075	1.094	1.112	2	4	5	7	9	11
1.2	1.131	1.150	1.169	1.188	1.208	1.227	1.247	1.267	1.287	1.307	2	4	6	8	10	12
1.3	1.327	1.348	1.368	1.389	1.410	1.431	1.453	1.474	1.496	1.517	2	4	6	8	11	13
1.4	1.539	1.561	1.584	1.606	1.629	1.651	1.674	1.697	1.720	1.744	2	5	7	9	11	14
1.5	1.767	1.791	1.815	1.839	1.863	1.887	1.911	1.936	1.961	1.986	2	5	7	10	12	15
1.6	2.011	2.036	2.061	2.087	2.112	2.138	2.164	2.190	2.217	2.243	3	5	8	10	13	16
1.7	2.270	2.297	2.324	2.351	2.378	2.405	2.433	2.461	2.488	2.516	3	5	8	11	14	16
1.8	2.545	2.573	2.602	2.630	2.659	2.688	2.717	2.746	2.776	2.806	3	6	9	12	15	17
1.9	2.835	2.865	2.895	2.926	2.956	2.986	3.017	3.048	3.079	3.110	3	6	9	12	15	18
2.0	3.142	3.173	3.205	3.237	3.269	3.301	3.333	3.365	3.398	3.431	3	6	10	13	16	19
2.1	3.464	3.497	3.530	3.563	3.597	3.631	3.664	3.698	3.733	3.767	3	7	10	14	17	20
2.2	3.801	3.836	3.871	3.906	3.941	3.976	4.011	4.047	4.083	4.119	4	7	11	14	18	21
2.3	4.155	4.191	4.227	4.264	4.301	4.337	4.374	4.412	4.449	4.486	4	7	11	15	18	22
2.4	4.524	4.562	4.600	4.638	4.676	4.714	4.753	4.792	4.831	4.870	4	8	12	15	19	23
2.5	4.909	4.948	4.988	5.027	5.067	5.107	5.147	5.187	5.228	5.269	4	8	12	16	20	24
2.6	5.309	5.350	5.391	5.433	5.474	5.515	5.557	5.599	5.641	5.683	4	8	12	17	21	25
2.7	5.726	5.768	5.811	5.853	5.896	5.940	5.983	6.026	6.070	6.114	4	9	13	17	22	26
2.8	6.158	6.202	6.246	6.290	6.335	6.379	6.424	6.469	6.514	6.560	4	9	13	18	22	27
2.9	6.605	6.651	6.697	6.743	6.789	6.835	6.881	6.928	6.975	7.022	5	9	14	19	23	28
3.0	7.069	7.116	7.163	7.211	7.258	7.306	7.354	7.402	7.451	7.499	5	10	14	19	24	29
3.1	7.548	7.596	7.645	7.694	7.744	7.793	7.843	7.892	7.942	7.992	5	10	15	20	25	30
3.2	8.042	8.093	8.143	8.194	8.245	8.296	8.347	8.398	8.450	8.501	5	10	15	20	26	31
3.3	8.553	8.605	8.657	8.709	8.762	8.814	8.867	8.920	8.973	9.026	5	11	16	21	26	32
3.4	9.079	9.133	9.186	9.240	9.294	9.348	9.402	9.457	9.511	9.566	5	11	16	22	27	33
3.5	9.621	9.676	9.731	9.787	9.842	9.898	9.954	10.010			6	11	17	22	28	33
								10.01	10.07	10.12	1	1	2	2	3	3
3.6	10.18	10.24	10.29	10.35	10.41	10.46	10.52	10.58	10.64	10.69	1	1	2	2	3	3
3.7	10.75	10.81	10.87	10.93	10.99	11.04	11.10	11.16	11.22	11.28	1	1	2	2	3	4
3.8	11.34	11.40	11.46	11.52	11.58	11.64	11.70	11.76	11.82	11.88	1	1	2	2	3	4
3.9	11.95	12.01	12.07	12.13	12.19	12.25	12.32	12.38	12.44	12.50	1	1	2	2	3	4
4.0	12.57	12.63	12.69	12.76	12.82	12.88	12.95	13.01	13.07	13.14	1	1	2	3	3	4
4.1	13.20	13.27	13.33	13.40	13.46	13.53	13.59	13.66	13.72	13.79	1	1	2	3	3	4
4.2	13.85	13.92	13.99	14.05	14.12	14.19	14.25	14.32	14.39	14.45	1	1	2	3	3	4
4.3	14.52	14.59	14.66	14.73	14.79	14.86	14.93	15.00	15.07	15.14	1	1	2	3	3	4
4.4	15.21	15.27	15.34	15.41	15.48	15.55	15.62	15.69	15.76	15.83	1	1	2	3	3	4
4.5	15.90	15.98	16.05	16.12	16.19	16.26	16.33	16.40	16.47	16.55	1	1	2	3	4	4
4.6	16.62	16.69	16.76	16.84	16.91	16.98	17.06	17.13	17.20	17.28	1	1	2	3	4	4
4.7	17.35	17.42	17.50	17.57	17.65	17.72	17.80	17.87	17.95	18.02	1	1	2	3	4	4
4.8	18.10	18.17	18.25	18.32	18.40	18.47	18.55	18.63	18.70	18.78	1	2	2	3	4	5
4.9	18.86	18.93	19.01	19.09	19.17	19.24	19.32	19.40	19.48	19.56	1	2	2	3	4	5

TABLE LXVII — *Continued*

Areas of Circles by Hundredths

(n = diameter)

Tenths of the Tabular Difference

0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
19.63	19.71	19.79	19.87	19.95	20.03	20.11	20.19	20.27	20.35	1	2	2	3	4	5	6	6	[illegible]
20.43	20.51	20.59	20.67	20.75	20.83	20.91	20.99	21.07	21.16	1	2	2	3	4	5	6	6	[illegible]
21.24	21.32	21.40	21.48	21.57	21.65	21.73	21.81	21.90	21.98	1	2	2	3	4	5	6	7	[illegible]
22.06	22.15	22.23	22.31	22.40	22.48	22.56	22.65	22.73	22.82	1	2	3	3	4	5	6	7	[illegible]
22.90	22.99	23.07	23.16	23.24	23.33	23.41	23.50	23.59	23.67	1	2	3	3	4	5	6	7	[illegible]
23.76	23.84	23.93	24.02	24.11	24.19	24.28	24.37	24.45	24.54	1	2	3	3	4	5	6	7	[illegible]
24.63	24.72	24.81	24.89	24.98	25.07	25.16	25.25	25.34	25.43	1	2	3	4	4	5	6	7	[illegible]
25.52	25.61	25.70	25.79	25.88	25.97	26.06	26.15	26.24	26.33	1	2	3	4	5	5	6	7	[illegible]
26.42	26.51	26.60	26.69	26.79	26.88	26.97	27.06	27.15	27.25	1	2	3	4	5	6	6	7	[illegible]
27.34	27.43	27.53	27.62	27.71	27.81	27.90	27.99	28.09	28.18	1	2	3	4	5	6	7	7	[illegible]
28.27	28.37	28.46	28.56	28.65	28.75	28.84	28.94	29.03	29.13	1	2	3	4	5	6	7	8	[illegible]
29.22	29.32	29.42	29.51	29.61	29.71	29.80	29.90	30.00	30.09	1	2	3	4	5	6	7	8	[illegible]
30.19	30.29	30.39	30.48	30.58	30.68	30.78	30.88	30.97	31.07	1	2	3	4	5	6	7	8	[illegible]
31.17	31.27	31.37	31.47	31.57	31.67	31.77	31.87	31.97	32.07	1	2	3	4	5	6	7	8	[illegible]
32.17	32.27	32.37	32.47	32.57	32.67	32.78	32.88	32.98	33.08	1	2	3	4	5	6	7	8	[illegible]
33.18	33.29	33.39	33.49	33.59	33.70	33.80	33.90	34.00	34.11	1	2	3	4	5	6	7	8	[illegible]
34.21	34.32	34.42	34.52	34.63	34.73	34.84	34.94	35.05	35.15	1	2	3	4	5	6	7	8	[illegible]
35.26	35.36	35.47	35.57	35.68	35.78	35.89	36.00	36.10	36.21	1	2	3	4	5	6	7	8	[illegible]
36.32	36.42	36.53	36.64	36.75	36.85	36.96	37.07	37.18	37.28	1	2	3	4	5	6	8	9	[illegible]
37.39	37.50	37.61	37.72	37.83	37.94	38.05	38.16	38.26	38.37	1	2	3	4	5	7	8	9	[illegible]
38.48	38.59	38.70	38.82	38.93	39.04	39.15	39.26	39.37	39.48	1	2	3	4	6	7	8	9	[illegible]
39.59	39.70	39.82	39.93	40.04	40.15	40.26	40.38	40.49	40.60	1	2	3	4	6	7	8	9	[illegible]
40.72	40.83	40.94	41.06	41.17	41.28	41.40	41.51	41.62	41.74	1	2	3	5	6	7	8	9	[illegible]
41.85	41.97	42.08	42.20	42.31	42.43	42.54	42.66	42.78	42.89	1	2	3	5	6	7	8	9	[illegible]
43.01	43.12	43.24	43.36	43.47	43.59	43.71	43.83	43.94	44.06	1	2	4	5	6	7	8	9	[illegible]
44.18	44.30	44.41	44.53	44.65	44.77	44.89	45.01	45.13	45.25	1	2	4	5	6	7	8	9	[illegible]
45.36	45.48	45.60	45.72	45.84	45.96	46.08	46.20	46.32	46.45	1	2	4	5	6	7	8	10	[illegible]
46.57	46.69	46.81	46.93	47.05	47.17	47.29	47.42	47.54	47.66	1	2	4	5	6	7	9	10	[illegible]
47.78	47.91	48.03	48.15	48.27	48.40	48.52	48.65	48.77	48.89	1	2	4	5	6	7	9	10	[illegible]
49.02	49.14	49.27	49.39	49.51	49.64	49.76	49.89	50.01	50.14	1	2	4	5	6	7	9	10	[illegible]
50.27	50.39	50.52	50.64	50.77	50.90	51.02	51.15	51.28	51.40	1	3	4	5	6	8	9	10	[illegible]
51.53	51.66	51.78	51.91	52.04	52.17	52.30	52.42	52.55	52.68	1	3	4	5	6	8	9	10	[illegible]
52.81	52.94	53.07	53.20	53.33	53.46	53.59	53.72	53.85	53.98	1	3	4	5	6	8	9	10	[illegible]
54.11	54.24	54.37	54.50	54.63	54.76	54.89	55.02	55.15	55.29	1	3	4	5	7	8	9	10	[illegible]
55.42	55.55	55.68	55.81	55.95	56.08	56.21	56.35	56.48	56.61	1	3	4	5	7	8	9	11	[illegible]
56.75	56.88	57.01	57.15	57.28	57.41	57.55	57.68	57.82	57.95	1	3	4	5	7	8	9	11	[illegible]
58.09	58.22	58.36	58.49	58.63	58.77	58.90	59.04	59.17	59.31	1	3	4	5	7	8	10	11	[illegible]
59.45	59.58	59.72	59.86	59.99	60.13	60.27	60.41	60.55	60.68	1	3	4	5	7	8	10	11	[illegible]
60.82	60.96	61.10	61.24	61.38	61.51	61.65	61.79	61.93	62.07	1	3	4	6	7	8	10	11	[illegible]
62.21	62.35	62.49	62.63	62.77	62.91	63.05	63.19	63.33	63.48	1	3	4	6	7	8	10	11	[illegible]
63.62	63.76	63.90	64.04	64.18	64.33	64.47	64.61	64.75	64.90	1	3	4	6	7	9	10	11	[illegible]
65.04	65.18	65.33	65.47	65.61	65.76	65.90	66.04	66.19	66.33	1	3	4	6	7	9	10	11	[illegible]
66.48	66.62	66.77	66.91	67.06	67.20	67.35	67.49	67.64	67.78	1	3	4	6	7	9	10	12	[illegible]
67.93	68.08	68.22	68.37	68.51	68.66	68.81	68.96	69.10	69.25	1	3	4	6	7	9	10	12	[illegible]
69.40	69.55	69.69	69.84	69.99	70.14	70.29	70.44	70.58	70.73	1	3	4	6	7	9	10	12	[illegible]
70.88	71.03	71.18	71.33	71.48	71.63	71.78	71.93	72.08	72.23	2	3	5	6	8	9	11	12	[illegible]
72.38	72.53	72.68	72.84	72.99	73.14	73.29	73.44	73.59	73.75	2	3	5	6	8	9	11	12	[illegible]
73.90	74.05	74.20	74.36	74.51	74.66	74.82	74.97	75.12	75.28	2	3	5	6	8	9	11	12	[illegible]
75.43	75.58	75.74	75.89	76.05	76.20	76.36	76.51	76.67	76.82	2	3	5	6	8	9	11	12	[illegible]
76.98	77.13	77.29	77.44	77.60	77.76	77.91	78.07	78.23	78.38	2	3	5	6	8	9	11	13	[illegible]

TABLE LXVIII

Circumferences of Circles by Eighths

Diam.	0	⅛	¼	⅜	½	⅝	¾	⅞
0	.0000	.3927	.7854	1.178	1.571	1.963	2.356	2.749
1	3.142	3.534	3.927	4.320	4.712	5.105 +	5.498	5.890
2	6.283	6.676	7.069	7.461	7.854	8.247	8.639	9.032
3	9.425 –	9.817	10.21	10.60	11.00	11.39	11.78	12.17
4	12.57	12.96	13.35	13.74	14.14	14.53	14.92	15.32
5	15.71	16.10	16.49	16.89	17.28	17.67	18.06	18.46
6	18.85	19.24	19.63	20.03	20.42	20.81	21.21	21.60
7	21.99	22.38	22.78	23.17	23.56	23.95	24.35	24.74
8	25.13	25.53	25.92	26.31	26.70	27.10	27.49	27.88
9	28.27	28.67	29.06	29.45	29.85	30.24	30.63	31.02
10	31.42	31.81	32.20	32.59	32.99	33.38	33.77	34.16
1	34.56	34.95	35.34	35.74	36.13	36.52	36.91	37.31
2	37.70	38.09	38.48	38.88	39.27	39.66	40.06	40.45
3	40.84	41.23	41.63	42.02	42.41	42.80	43.20	43.59
4	43.98	44.37	44.77	45.16	45.55	45.95	46.34	46.73
15	47.12	47.52	47.91	48.30	48.69	49.09	49.48	49.87
6	50.27	50.66	51.05	51.44	51.84	52.23	52.62	53.01
7	53.41	53.80	54.19	54.59	54.98	55.37	55.76	56.16
8	56.55	56.94	57.33	57.73	58.12	58.51	58.90	59.30
9	59.69	60.08	60.48	60.87	61.26	61.65	62.05	62.44
20	62.83	63.22	63.62	64.01	64.40	64.80	65.19	65.58
1	65.97	66.37	66.76	67.15	67.54	67.94	68.33	68.72
2	69.12	69.51	69.90	70.29	70.69	71.08	71.47	71.86
3	72.26	72.65	73.04	73.43	73.83	74.22	74.61	75.01
4	75.40	75.79	76.18	76.58	76.97	77.36	77.75	78.15
25	78.54	78.93	79.33	79.72	80.11	80.50	80.90	81.29
6	81.68	82.07	82.47	82.86	83.25	83.64	84.04	84.43
7	84.82	85.22	85.61	86.00	86.39	86.79	87.18	87.57
8	87.96	88.36	88.75	89.14	89.54	89.93	90.32	90.71
9	91.11	91.50	91.89	92.28	92.68	93.07	93.46	93.86
30	94.25	94.64	95.03	95.43	95.82	96.21	96.60	97.00
1	97.39	97.78	98.17	98.57	98.96	99.35	99.75	100.1
2	100.5	100.9	101.3	101.7	102.1	102.5	102.9	103.3
3	103.7	104.1	104.5	104.9	105.2	105.6	106.0	106.4
4	106.8	107.2	107.6	108.0	108.4	108.8	109.2	109.6
35	110.0	110.3	110.7	111.1	111.5	111.9	112.3	112.7
6	113.1	113.5	113.9	114.3	114.7	115.1	115.5	115.8
7	116.2	116.6	117.0	117.4	117.8	118.2	118.6	119.0
8	119.4	119.8	120.2	120.6	121.0	121.3	121.7	122.1
9	122.5	122.9	123.3	123.7	124.1	124.5	124.9	125.3
40	125.7	126.1	126.4	126.8	127.2	127.6	128.0	128.4
1	128.8	129.2	129.6	130.0	130.4	130.8	131.2	131.6
2	131.9	132.3	132.7	133.1	133.5	133.9	134.3	134.7
3	135.1	135.5	135.9	136.3	136.7	137.1	137.4	137.8
4	138.2	138.6	139.0	139.4	139.8	140.2	140.6	141.0
45	141.4	141.8	142.2	142.5	142.9	143.3	143.7	144.1
6	144.5	144.9	145.3	145.7	146.1	146.5	146.9	147.3
7	147.7	148.0	148.4	148.8	149.2	149.6	150.0	150.4
8	150.8	151.2	151.6	152.0	152.4	152.8	153.2	153.5
9	153.9	154.3	154.7	155.1	155.5	155.9	156.3	156.7
50	157.1							

CIRCUMFERENCES OF CIRCLES BY EIGHTHS

Diam.	0	⅛	¼	⅜	½	⅝	¾	⅞
50	157.1	157.5	157.9	158.3	158.7	159.0	159.4	159.8
1	160.2	160.6	161.0	161.4	161.8	162.2	162.6	163.0
2	163.4	163.8	164.1	164.5	164.9	165.3	165.7	166.1
3	166.5	166.9	167.3	167.7	168.1	168.5	168.9	169.3
4	169.6	170.0	170.4	170.8	171.2	171.6	172.0	172.4
55	172.8	173.2	173.6	174.0	174.4	174.8	175.1	175.5
6	175.9	176.3	176.7	177.1	177.5	177.9	178.3	178.7
7	179.1	179.5	179.9	180.2	180.6	181.0	181.4	181.8
8	182.2	182.6	183.0	183.4	183.8	184.2	184.6	185.0
9	185.4	185.7	186.1	186.5	186.9	187.3	187.7	188.1
60	188.5	188.9	189.3	189.7	190.1	190.5	190.9	191.2
1	191.6	192.0	192.4	192.8	193.2	193.6	194.0	194.4
2	194.8	195.2	195.6	196.0	196.3	196.7	197.1	197.5
3	197.9	198.3	198.7	199.1	199.5	199.9	200.3	200.7
4	201.1	201.5	201.8	202.2	202.6	203.0	203.4	203.8
65	204.2	204.6	205.0	205.4	205.8	206.2	206.6	207.0
6	207.3	207.7	208.1	208.5	208.9	209.3	209.7	210.1
7	210.5	210.9	211.3	211.7	212.1	212.5	212.8	213.2
8	213.6	214.0	214.4	214.8	215.2	215.6	216.0	216.4
9	216.8	217.2	217.6	217.9	218.3	218.7	219.1	219.5
70	219.9	220.3	220.7	221.1	221.5	221.9	222.3	222.7
1	223.1	223.4	223.8	224.2	224.6	225.0	225.4	225.8
2	226.2	226.6	227.0	227.4	227.8	228.2	228.6	228.9
3	229.3	229.7	230.1	230.5	230.9	231.3	231.7	232.1
4	232.5	232.9	233.3	233.7	234.0	234.4	234.8	235.2
75	235.6	236.0	236.4	236.8	237.2	237.6	238.0	238.4
6	238.8	239.2	239.5	239.9	240.3	240.7	241.1	241.5
7	241.9	242.3	242.7	243.1	243.5	243.9	244.3	244.7
8	245.0	245.4	245.8	246.2	246.6	247.0	247.4	247.8
9	248.2	248.6	249.0	249.4	249.8	250.1	250.5	250.9
80	251.3	251.7	252.1	252.5	252.9	253.3	253.7	254.1
1	254.5	254.9	255.3	255.6	256.0	256.4	256.8	257.2
2	257.6	258.0	258.4	258.8	259.2	259.6	260.0	260.4
3	260.8	261.1	261.5	261.9	262.3	262.7	263.1	263.5
4	263.9	264.3	264.7	265.1	265.5	265.9	266.2	266.6
85	267.0	267.4	267.8	268.2	268.6	269.0	269.4	269.8
6	270.2	270.6	271.0	271.4	271.7	272.1	272.5	272.9
7	273.3	273.7	274.1	274.5	274.9	275.3	275.7	276.1
8	276.5	276.9	277.2	277.6	278.0	278.4	278.8	279.2
9	279.6	280.0	280.4	280.8	281.2	281.6	282.0	282.4
90	282.7	283.1	283.5	283.9	284.3	284.7	285.1	285.5
1	285.9	286.3	286.7	287.1	287.5	287.8	288.2	288.6
2	289.0	289.4	289.8	290.2	290.6	291.0	291.4	291.8
3	292.2	292.6	293.0	293.3	293.7	294.1	294.5	294.9
4	295.3	295.7	296.1	296.5	296.9	297.3	297.7	298.1
95	298.5	298.8	299.2	299.6	300.0	300.4	300.8	301.2
6	301.6	302.0	302.4	302.8	303.2	303.6	303.9	304.3
7	304.7	305.1	305.5	305.9	306.3	306.7	307.1	307.5
8	307.9	308.3	308.7	309.1	309.4	309.8	310.2	310.6
9	311.0	311.4	311.8	312.2	312.6	313.0	313.4	313.8
100	314.2							

TABLE LXIX

Areas of Circles by Eighths

Diam.	0	⅛	¼	⅜	½	⅝	¾	⅞
0	.0000	.0123	.0491	.1104	.1963	.3068	.4418	.6013
1	.7854	.9940	1.227	1.495 −	1.767	2.074	2.405 +	2.761
2	3.142	3.547	3.976	4.430	4.909	5.412	5.940	6.492
3	7.069	7.670	8.296	8.946	9.621	10.32	11.04	11.79
4	12.57	13.36	14.19	15.03	15.90	16.80	17.73	18.67
5	19.63	20.63	21.65 −	22.69	23.76	24.85 +	25.97	27.11
6	28.27	29.47	30.68	31.92	33.18	34.47	35.78	37.12
7	38.48	39.87	41.28	42.72	44.18	45.66	47.17	48.71
8	50.27	51.85 −	53.46	55.09	56.75 −	58.43	60.13	61.86
9	63.62	65.40	67.20	69.03	70.88	72.76	74.66	76.59
10	78.54	80.52	82.52	84.54	86.59	88.66	90.76	92.89
1	95.03	97.21	99.40	101.6	103.9	106.1	108.4	110.8
2	113.1	115.5 −	117.9	120.3	122.7	125.2	127.7	130.2
3	132.7	135.3	137.9	140.5 +	143.1	145.8	148.5 −	151.2
4	153.9	156.7	159.5 −	162.3	165.1	168.0	170.9	173.8
15	176.7	179.7	182.7	185.7	188.7	191.7	194.8	197.9
6	201.1	204.2	207.4	210.6	213.8	217.1	220.4	223.7
7	227.0	230.3	233.7	237.1	240.5 +	244.0	247.4	250.9
8	254.5 −	258.0	261.6	265.2	268.8	272.4	276.1	279.8
9	283.5 +	287.3	291.0	294.8	298.6	302.5 −	306.4	310.2
20	314.2	318.1	322.1	326.1	330.1	334.1	338.2	342.2
1	346.4	350.5 −	354.7	358.8	363.1	367.3	371.5 +	375.8
2	380.1	384.5 −	388.8	393.2	397.6	402.0	406.5 −	411.0
3	415.5 −	420.0	424.6	429.1	433.7	438.4	443.0	447.7
4	452.4	457.1	461.9	466.6	471.4	476.3	481.1	486.0
25	490.9	495.8	500.7	505.7	510.7	515.7	520.8	525.8
6	530.9	536.0	541.2	546.4	551.5 +	556.8	562.0	567.3
7	572.6	577.9	583.2	588.6	594.0	599.4	604.8	610.3
8	615.8	621.3	626.8	632.4	637.9	643.5 +	649.2	654.8
9	660.5 +	666.2	672.0	677.7	683.5 −	689.3	695.1	701.0
30	706.9	712.8	718.7	724.6	730.6	736.6	742.6	748.7
1	754.8	760.9	767.0	773.1	779.3	785.5 +	791.7	798.0
2	804.2	810.5 +	816.9	823.2	829.6	836.0	842.4	848.8
3	855.3	861.8	868.3	874.8	881.4	888.0	894.6	901.3
4	907.9	914.6	921.3	928.1	934.8	941.6	948.4	955.3
35	962.1	969.0	975.9	982.8	989.8	996.8	1004	1011
6	1018	1025	1032	1039	1046	1054	1061	1068
7	1075 +	1082	1090	1097	1104	1112	1119	1127
8	1134	1142	1149	1157	1164	1172	1179	1187
9	1195 −	1202	1211	1218	1225 +	1233	1241	1249
40	1257	1265 −	1272	1280	1288	1296	1304	1312
1	1320	1328	1336	1345 −	1353	1361	1369	1377
2	1385 +	1394	1402	1410	1419	1427	1435 +	1444
3	1452	1461	1469	1478	1486	1495 −	1503	1512
4	1521	1529	1538	1547	1555 +	1564	1573	1582
45	1590	1599	1608	1617	1626	1635 −	1644	1653
6	1662	1671	1680	1689	1698	1707	1717	1726
7	1735 −	1744	1753	1763	1772	1781	1791	1800
8	1810	1819	1828	1838	1847	1857	1867	1876
9	1886	[illegible]	1905 +	1915 −	1924	1934	1944	1954
50	*1963*							

AREAS OF CIRCLES BY EIGHTHS

DIAM.	0	⅛	¼	⅜	½	⅝	¾	⅞
50	1963	1973	1983	1993	2003	2013	2023	2033
1	2043	2053	2063	2073	2083	2093	2103	2114
2	2124	2134	2144	2154	2165 −	2175 +	2185 +	2196
3	2206	2217	2227	2238	2248	2259	2269	2280
4	2290	2301	2311	2322	2333	2344	2354	2365 +
55	2376	2387	2397	2408	2419	2430	2441	2452
6	2463	2474	2485 +	2496	2507	2518	2529	2541
7	2552	2563 -	2574	2585 +	2597	2608	2619	2631
8	2642	2653	2665 −	2676	2688	2699	2711	2722
9	2734	2746	2757	2769	2781	2792	2804	2816
60	2827	2839	2851	2863	2875 −	2887	2899	2911
1	2922	2934	2946	2959	2971	2983	2995 −	3007
2	3019	3031	3043	3056	3068	3080	3093	3105 −
3	3117	3130	3142	3154	3167	3179	3192	3204
4	3217	3230	3242	3255 −	3267	3280	3293	3306
65	3318	3331	3344	3357	3370	3382	3395 +	3408
6	3421	3434	3447	3460	3473	3486	3499	3513
7	3526	3539	3552	3565 +	3578	3592	3605 +	3618
8	3632	3645 +	3658	3672	3685 +	3699	3712	3726
9	3739	3753	3766	3780	3794	3807	3821	3835 −
70	3848	3862	3876	3890	3904	3917	3931	3945 +
1	3959	3973	3987	4001	4015 +	4029	4043	4057
2	4072	4086	4100	4114	4128	4142	4157	4171
3	4185 +	4200	4214	4228	4243	4257	4272	4286
4	4301	4315 +	4330	4345 −	4359	4374	4388	4403
75	4418	4433	4447	4462	4477	4492	4507	4522
6	4536	4551	4566	4581	4596	4611	4626	4642
7	4657	4672	4687	4702	4717	4733	4748	4763
8	4778	4794	4809	4824	4840	4855 +	4871	4886
9	4902	4917	4933	4948	4964	4980	4995 +	5011
80	5027	5042	5058	5074	5090	5105 +	5121	5137
1	5153	5169	5185 −	5201	5217	5233	5249	5265 −
2	5281	5297	5313	5329	5346	5362	5378	5394
3	5411	5427	5443	5460	5476	5492	5509	5525 +
4	5542	5558	5575 −	5591	5608	5625 −	5641	5658
85	5675 −	5691	5708	5725 −	5741	5758	5775 +	5792
6	5809	5826	5843	5860	5877	5894	5911	5928
7	5945 −	5962	5979	5996	6013	6030	6048	6065 −
8	6082	6099	6117	6134	6151	6169	6186	6204
9	6221	6239	6256	6274	6291	6309	6326	6344
90	6362	6379	6397	6415 −	6433	6450 +	6468	6486
1	6504	6522	6540	6558	6576	6594	6612	6630
2	6648	6666	6684	6702	6720	6738	6756	6775 −
3	6793	6811	6829	6848	6866	6885 −	6903	6921
4	6940	6958	6977	6995 +	7014	7032	7051	7070
95	7088	7107	7126	7144	7163	7182	7201	7219
6	7238	7257	7276	7295 −	7314	7333	7352	7371
7	7390	7409	7428	7447	7466	7485 +	7505 −	7524
8	7543	7562	7581	7601	7620	7639	7659	7678
9	7698	7717	7737	7756	7776	7795 +	7815 −	7834
100	7854							

:he **squares** of all numbers between 1 and 10, correct to four significant
: 39.48. (Values obtained by interpolation may be in error by 1 in the last
f any number outside the range from 1 to 10, write the number as the pro
en 1 and 10, and (b) some (positive or negative) power of 10; then proce

$$ {}^2 = (6.283 \times 10^4)^2 = (6.283)^2 \times (10^4)^2 = 39.48 \times 10^8 = 3948000000. $$
$$ {}^2 = (6.283 \times 10^{-2})^2 = (6.283)^2 \times (10^{-2})^2 = 39.48 \times 10^{-4} = 0.003948. $$

lecimal point ONE *place in column* ***n*** *is equivalent to moving it* TWO *place*

ely gives the **square roots** of all numbers between 1 and 10^2. For examp

$$ \sqrt{3.142} = 1.7725; \qquad \sqrt{31.42} = 5.606. $$

root of any number outside the range from 1 to 10^2, write the number as the
etween 1 and 10^2, and (b) some (positive or negative) power of 10^2; thus:

$$ \overline{2000} = \sqrt{3.142 \times 10^6} = \sqrt{3.142} \times \sqrt{10^6} = 1.7725 \times 10^3 = 1772.5 $$
$$ \overline{42} = \sqrt{31.42 \times 10^{-2}} = \sqrt{31.42} \times \sqrt{10^{-2}} = 5.606 \times 10^{-1} = 0.5606 $$

s in the body of the table facilitate its use as an inverse table.

								TENTHS OF TI TABULAR DIFFER					
2	3	4	5	6	7	8	9	1	2	3	4	5	6
040	1.061	1.082	1.103	1.124	1.145 −	1.166	1.188						
254	1.277	1.300	1.323	1.346	1.369	1.392	1.416						
488	1.513	1.538	1.563	1.588	1.613	1.638	1.664						
742	1.769	1.796	1.823	1.850 −	1.877	1.904	1.932						
016	2.045 −	2.074	2.103	2.132	2.161	2.190	2.220						
310	2.341	2.372	2.403	2.434	2.465 −	2.496	2.528						
624	2.657	2.690	2.723	2.756	2.789	2.822	2.856						
958	2.993	3.028	3.063	3.098	3.133	3.168	3.204						
312	3.349	3.386	3.423	3.460	3.497	3.534	3.572						
686	3.725 −	3.764	3.803	3.842	3.881	3.920	3.960						
080	4.121	4.162	4.203	4.244	4.285 −	4.326	4.368						
494	4.537	4.580	4.623	4.666	4.709	4.752	4.796						
928	4.973	5.018	5.063	5.108	5.153	5.198	5.244						
382	5.429	5.476	5.523	5.570	5.617	5.664	5.712						
856	5.905 −	5.954	6.003	6.052	6.101	6.150	6.200						
.350 +	6.401	6.452	6.503	6.554	6.605 −	6.656	6.708						
.864	6.917	6.970	7.023	7.076	7.129	7.182	7.236						
.398	7.453	7.508	7.563	7.618	7.673	7.728	7.784						
.952	8.009	8.066	8.123	8.180	8.237	8.294	8.352						
.526	8.585 −	8.644	8.703	8.762	8.821	8.880	8.940						
.120	9.181	9.242	9.303	9.364	9.425 −	9.486	9.548						
.734	9.797	9.860	9.923	9.986	10.049								
					10.05 −	10.11	10.18	1	1	2	3	3	4
0.37	10.43	10.50 −	10.56	10.63	10.69	10.76	10.82	1	1	2	3	3	4
1.02	11.09	11.16	11.22	11.29	11.36	11.42	11.49	1	1	2	3	3	4
1.70	11.76	11.83	11.90	11.97	12.04	12.11	12.18	1	1	2	3	3	4
2.39	12.46	12.53	12.60	12.67	12.74	12.82	12.89	1	1	2	3	4	4
3.10	13.18	13.25 −	13.32	13.40	13.47	13.54	13.62	1	1	2	3	4	4
3.84	13.91	13.99	14.06	14.14	14.21	14.29	14.36	1	1	2	3	4	4
4.59	14.67	14.75 −	14.82	14.90	14.98	15.05 +	15.13	1	2	2	3	4	5
5.37	15.44	15.52	15.60	15.68	15.76	15.84	15.92	1	2	2	3	4	5
6.16	16.24	16.32	16.40	16.48	16.56	16.65 −	16.73	1	2	2	3	4	5
6.97	17.06	17.14	17.22	17.31	17.39	17.47	17.56	1	2	2	3	4	5
17.81	17.89	17.98	18.06	18.15 −	18.23	18.32	18.40	1	2	3	3	4	5
18.66	18.75 −	18.84	18.92	19.01	19.10	19.18	19.27	1	2	3	3	4	5
19.54	19.62	19.71	19.80	19.89	19.98	20.07	20.16	1	2	3	4	4	5
20.43	20.52	20.61	20.70	20.79	20.88	20.98	21.07	1	2	3	4	5	5
21.34	21.44	21.53	21.62	21.72	21.81	21.90	22.00	1	2	3	4	5	6
22.28	22.37	22.47	22.56	22.66	22.75 +	22.85 −	22.94	1	2	3	4	5	6
23.23	23.33	23.43	23.52	23.62	23.72	23.81	23.91	1	2	3	4	5	6
24.21	24.30	24.40	24.50 +	24.60	24.70	24.80	24.90	1	2	3	4	5	6

To avoid interpola
these twenty-two li
special table on pag
469, and 470.

TABLE LXX — *Continued*

1	2	3	4	5	6	7	8	9	Tenths of the Tabular Difference 1	2	3	4	5	6	7	8	9
10	25.20	25.30	25.40	25.50+	25.60	25.70	25.81	25.91	1	2	3	4	5	6	7	8	9
11	26.21	26.32	26.42	26.52	26.63	26.73	26.83	26.94	1	2	3	4	5	6	7	8	9
14	27.25−	27.35+	27.46	27.56	27.67	27.77	27.88	27.98	1	2	3	4	5	6	7	8	9
20	28.30	28.41	28.52	28.62	28.73	28.84	28.94	29.05+	1	2	3	4	5	6	7	9	10
27	29.38	29.48	29.59	29.70	29.81	29.92	**30**.03	30.14	1	2	3	4	5	7	8	9	10
36	30.47	30.58	30.69	30.80	30.91	31.02	31.14	31.25−	1	2	3	4	6	7	8	9	10
47	31.58	32.70	31.81	31.92	32.04	32.15−	32.26	32.38	1	2	3	5	6	7	8	9	10
60	32.72	32.83	32.95−	33.06	33.18	33.29	33.41	33.52	1	2	3	5	6	7	8	9	10
76	33.87	33.99	34.11	34.22	34.34	34.46	34.57	34.69	1	2	4	5	6	7	8	9	11
93	35.05−	35.16	35.28	35.40	35.52	35.64	35.76	35.88	1	2	4	5	6	7	8	10	11
12	36.24	36.36	36.48	36.60	36.72	36.84	36.97	37.09	1	2	4	5	6	7	8	10	11
33	37.45+	37.58	37.70	37.82	37.95−	38.07	38.19	38.32	1	2	4	5	6	7	9	10	11
56	38.69	38.81	38.94	39.06	39.19	39.31	39.44	39.56	1	2	4	5	6	7	9	10	11
82	39.94	**40**.07	40.20	40.32	40.45−	40.58	40.70	40.83	1	3	4	5	6	8	9	10	11
09	41.22	41.34	41.47	41.60	41.73	41.86	41.99	42.12	1	3	4	5	6	8	9	10	12
38	42.51	42.64	42.77	42.90	43.03	43.16	43.30	43.43	1	3	4	5	7	8	9	10	12
69	43.82	43.96	44.09	44.22	44.36	44.49	44.62	44.76	1	3	4	5	7	8	9	11	12
02	45.16	45.29	45.43	45.56	45.70	45.83	45.97	46.10	1	3	4	5	7	8	9	11	12
38	46.51	46.65−	46.79	46.92	47.06	47.20	47.33	47.47	1	3	4	5	7	8	10	11	12
75−	47.89	48.02	48.16	48.30	48.44	48.58	48.72	48.86	1	3	4	6	7	8	10	11	13
14	49.28	49.42	49.56	49.70	49.84	49.98	**50**.13	50.27	1	3	4	6	7	8	10	11	13
55+	50.69	50.84	50.98	51.12	51.27	51.41	51.55+	51.70	1	3	4	6	7	9	10	11	13
98	52.13	52.27	52.42	52.56	52.71	52.85+	53.00+	53.14	1	3	4	6	7	9	10	12	13
44	53.58	53.73	53.88	54.02	54.17	54.32	54.46	54.61	1	3	4	6	7	9	10	12	13
91	55.06	55.20	55.35+	55.50+	55.65+	55.80	55.95+	56.10	1	3	4	6	7	9	10	12	13
40	56.55+	56.70	56.85+	57.00	57.15+	57.30	57.46	57.61	2	3	5	6	8	9	11	12	14
91	58.06	58.22	58.37	58.52	58.68	58.83	58.98	59.14	2	3	5	6	8	9	11	12	14
44	59.60	59.75+	59.91	**60**.06	60.22	60.37	60.53	60.68	2	3	5	6	8	9	11	12	14
00	61.15+	61.31	61.47	61.62	61.78	61.94	62.09	62.25+	2	3	5	6	8	9	11	13	14
57	62.73	62.88	63.04	63.20	63.36	63.52	63.68	63.84	2	3	5	6	8	10	11	13	14
16	64.32	64.48	64.64	64.80	64.96	65.12	65.29	65.45−	2	3	5	6	8	10	11	13	14
77	65.93	66.10	66.26	66.42	66.59	66.75−	66.91	67.08	2	3	5	7	8	10	11	13	15
40	67.57	67.73	67.90	68.06	68.23	68.39	68.56	68.72	2	3	5	7	8	10	12	13	15
06	69.22	69.39	69.56	69.72	69.89	**70**.06	70.22	70.39	2	3	5	7	8	10	12	13	15
73	70.90	71.06	71.23	71.40	71.57	71.74	71.91	72.08	2	3	5	7	8	10	12	14	15
42	72.59	72.76	72.93	73.10	73.27	73.44	73.62	73.79	2	3	5	7	9	10	12	14	15
13	74.30	74.48	74.65−	74.82	75.00−	75.17	75.34	75.52	2	3	5	7	9	10	12	14	16
86	76.04	76.21	76.39	76.56	76.74	76.91	77.09	77.26	2	3	5	7	9	10	12	14	16
62	77.79	77.97	78.15−	78.32	78.50−	78.68	78.85+	79.03	2	4	5	7	9	11	12	14	16
39	79.57	79.74	79.92	**80**.10	80.28	80.46	80.64	80.82	2	4	5	7	9	11	13	14	16
18	81.36	81.54	81.72	81.90	82.08	82.26	82.45−	82.63	2	4	5	7	9	11	13	14	16
99	83.17	83.36	83.54	83.72	83.91	84.09	84.27	84.46	2	4	5	7	9	11	13	15	16
82	85.01	85.19	85.38	85.56	85.75−	85.93	86.12	86.30	2	4	6	7	9	11	13	15	17
68	86.86	87.05−	87.24	87.42	87.61	87.80	87.98	88.17	2	4	6	7	9	11	13	15	17
55−	88.74	88.92	89.11	89.30	89.49	89.68	89.87	**90**.06	2	4	6	8	9	11	13	15	17
44	90.63	90.82	91.01	91.20	91.39	91.58	91.78	91.97	2	4	6	8	10	11	13	15	17
35+	92.54	92.74	92.93	93.12	93.32	93.51	93.70	93.90	2	4	6	8	10	12	14	15	17
28	94.48	94.67	94.87	95.06	95.26	95.45+	95.65−	95.84	2	4	6	8	10	12	14	16	18
24	96.43	96.63	96.83	97.02	97.22	97.42	97.61	97.81	2	4	6	8	10	12	14	16	18
21	98.41	98.60	98.80	99.00	99.20	99.40	99.60	99.80	2	4	6	8	10	12	14	16	18

SPECIAL TABLE

See main table, pages 466, 467

n^2 SQUARES

n	0	1	2	3	4	5	6	7	8	9	Tenths of Tabular Difference 1	2	3	4	5
1.00	1.000	1.002	1.004	1.006	1.008	1.010	1.012	1.014	1.016	1.018	0	0	1	1	1
1	1.020	1.022	1.024	1.026	1.028	1.030	1.032	1.034	1.036	1.038	0	0	1	1	1
2	1.040	1.042	1.044	1.047	1.049	1.051	1.053	1.055−	1.057	1.059	0	0	1	1	1
3	1.061	1.063	1.065+	1.067	1.069	1.071	1.073	1.075+	1.077	1.080	0	0	1	1	1
4	1.082	1.084	1.086	1.088	1.090	1.092	1.094	1.096	1.098	1.100	0	0	1	1	1
1.05	1.102	1.105−	1.107	1.109	1.111	1.113	1.115+	1.117	1.119	1.121	0	0	1	1	1
6	1.124	1.126	1.128	1.130	1.132	1.134	1.136	1.138	1.141	1.143	0	0	1	1	1
7	1.145−	1.147	1.149	1.151	1.153	1.156	1.158	1.160	1.162	1.164	0	0	1	1	1
8	1.166	1.169	1.171	1.173	1.175+	1.177	1.179	1.182	1.184	1.186	0	0	1	1	1
9	1.188	1.190	1.192	1.195−	1.197	1.199	1.201	1.203	1.206	1.208	0	0	1	1	1
1.10	1.210	1.212	1.214	1.217	1.219	1.221	1.223	1.225+	1.228	1.230	0	0	1	1	1
1	1.232	1.234	1.237	1.239	1.241	1.243	1.245+	1.248	1.250	1.252	0	0	1	1	1
2	1.254	1.257	1.259	1.261	1.263	1.266	1.268	1.270	1.272	1.275−	0	0	1	1	1
3	1.277	1.279	1.281	1.284	1.286	1.288	1.290	1.293	1.295+	1.297	0	0	1	1	1
4	1.300	1.302	1.304	1.306	1.309	1.311	1.313	1.316	1.318	1.320	0	0	1	1	1
1.15	1.322	1.325−	1.327	1.329	1.332	1.334	1.336	1.339	1.341	1.343	0	0	1	1	1
6	1.346	1.348	1.350+	1.353	1.355−	1.357	1.360	1.362	1.364	1.367	0	0	1	1	1
7	1.369	1.371	1.374	1.376	1.378	1.381	1.383	1.385+	1.388	1.390	0	0	1	1	1
8	1.392	1.395−	1.397	1.399	1.402	1.404	1.407	1.409	1.411	1.414	0	0	1	1	1
9	1.416	1.418	1.421	1.423	1.426	1.428	1.430	1.433	1.435+	1.438	0	0	1	1	1
1.20	1.440	1.442	1.445−	1.447	1.450−	1.452	1.454	1.457	1.459	1.462	0	0	1	1	1
1	1.464	1.467	1.469	1.471	1.474	1.476	1.479	1.481	1.484	1.486	0	0	1	1	1
2	1.488	1.491	1.493	1.496	1.498	1.501	1.503	1.506	1.508	1.510	0	0	1	1	1
3	1.513	1.515+	1.518	1.520	1.523	1.525+	1.528	1.530	1.533	1.535+	0	0	1	1	1
4	1.538	1.540	1.543	1.545+	1.548	1.550+	1.553	1.555+	1.558	1.560	0	0	1	1	1
1.25	1.562	1.565+	1.568	1.570	1.573	1.575+	1.578	1.580	1.583	1.585+	0	1	1	1	1
6	1.588	1.590	1.593	1.595+	1.598	1.600	1.603	1.605+	1.608	1.610	0	1	1	1	1
7	1.613	1.615+	1.618	1.621	1.623	1.626	1.628	1.631	1.633	1.636	0	1	1	1	1
8	1.638	1.641	1.644	1.646	1.649	1.651	1.654	1.656	1.659	1.662	0	1	1	1	1
9	1.664	1.667	1.669	1.672	1.674	1.677	1.680	1.682	1.685−	1.687	0	1	1	1	1
1.30	1.690	1.693	1.695+	1.698	1.700	1.703	1.706	1.708	1.711	1.713	0	1	1	1	1
1	1.716	1.719	1.721	1.724	1.727	1.729	1.732	1.734	1.737	1.740	0	1	1	1	1
2	1.742	1.745+	1.748	1.750+	1.753	1.756	1.758	1.761	1.764	1.766	0	1	1	1	1
3	1.769	1.772	1.774	1.777	1.780	1.782	1.785−	1.788	1.790	1.793	0	1	1	1	1
4	1.796	1.798	1.801	1.804	1.806	1.809	1.812	1.814	1.817	1.820	0	1	1	1	1
1.35	1.822	1.825+	1.828	1.831	1.833	1.836	1.839	1.841	1.844	1.847	0	1	1	1	1
6	1.850−	1.852	1.855+	1.858	1.860	1.863	1.866	1.869	1.871	1.874	0	1	1	1	1
7	1.877	1.880	1.882	1.885+	1.888	1.891	1.893	1.896	1.899	1.902	0	1	1	1	1
8	1.904	1.907	1.910	1.913	1.915+	1.918	1.921	1.924	1.927	1.929	0	1	1	1	1
9	1.932	1.935−	1.938	1.940	1.943	1.946	1.949	1.952	1.954	1.957	0	1	1	1	1
1.40	1.960	1.963	1.966	1.968	1.971	1.974	1.977	1.980	1.982	1.985+	0	1	1	1	1
1	1.988	1.991	1.994	1.997	1.999	2.002	2.005+	2.008	2.011	2.014	0	1	1	1	1
2	2.016	2.019	2.022	2.025−	2.028	2.031	2.033	2.036	2.039	2.042	0	1	1	1	1
3	2.045−	2.048	2.051	2.053	2.056	2.059	2.062	2.065−	2.068	2.071	0	1	1	1	1
4	2.074	2.076	2.079	2.082	2.085+	2.088	2.091	2.094	2.097	2.100	0	1	1	1	1
1.45	2.102	2.105+	2.108	2.111	2.114	2.117	2.120	2.123	2.126	2.129	0	1	1	1	1
6	2.132	2.135−	2.137	2.140	2.143	2.146	2.149	2.152	2.155+	2.158	0	1	1	1	1
7	2.161	2.164	2.167	2.170	2.173	2.176	2.179	2.182	2.184	2.187	0	1	1	1	1
8	2.190	2.193	2.196	2.199	2.202	2.205+	2.208	2.211	2.214	2.217	0	1	1	1	1
9	2.220	2.223	2.226	2.229	2.232	2.235+	2.238	2.241	2.244	2.247	0	1	1	1	1
1.50	2.250	2.253	2.256	2.259	2.262	2.265+	2.268	2.271	2.274	2.277	0	1	1	1	2
1	2.280	2.283	2.286	2.289	2.292	2.295+	2.298	2.301	2.304	2.307	0	1	1	1	2
2	2.310	2.313	2.316	2.320	2.323	2.326	2.329	2.332	2.335−	2.338	0	1	1	1	2
3	2.341	2.344	2.347	2.350+	2.353	2.356	2.359	2.362	2.365+	2.369	0	1	1	1	2
4	2.372	2.375−	2.378	2.381	2.384	2.387	2.390	2.393	2.396	2.399	0	1	1	1	2
1.55	2.402	2.406	2.409	2.412	2.415−	2.418	2.421	2.424	2.427	2.430	0	1	1	1	2
6	2.434	2.437	2.440	2.443	2.446	2.449	2.452	2.455+	2.459	2.462	0	1	1	1	2
7	2.465−	2.468	2.471	2.474	2.477	2.481	2.484	2.487	2.490	2.493	0	1	1	1	2
8	2.496	2.500	2.503	2.506	2.509	2.512	2.515+	2.519	2.522	2.525−	0	1	1	1	2
9	2.528	2.531	2.534	2.538	2.541	2.544	2.547	2.550+	2.554	2.557	0	1	1	1	2
1.60	2.560	2.563	2.566	2.570	2.573	2.576	2.579	2.582	2.586	2.589	0	1	1	1	2
1	2.592	2.595+	2.599	2.602	2.605−	2.608	2.611	2.615−	2.618	2.621	0	1	1	1	2
2	2.624	2.628	2.631	2.634	2.637	2.641	2.644	2.647	2.650	2.654	0	1	1	1	2
3	2.657	2.660	2.663	2.667	2.670	2.673	2.676	2.680	2.683	2.686	0	1	1	1	2
4	2.690	2.693	2.696	2.699	2.703	2.706	2.709	2.713	2.716	2.719	0	1	1	1	2
1.65	*2.722*	2.726	2.729	2.732	2.736	2.739	2.742	2.746	2.749	2.752	0	1	1	1	2
6	*2.756*	2.759	2.762	2.766	2.769	2.772	2.776	2.779	2.782	2.786	0	1	1	1	2
7	*2.789*	2.792	2.796	2.799	2.802	2.806	2.809	2.812	2.816	2.819	0	1	1	1	2
8	*2.822*	2.826	2.829	2.832	2.836	2.839	2.843	2.846	2.849	2.853	0	1	1	1	2
9	*2.856*	2.859	2.863	2.866	2.870	2.873	2.876	2.880	2.883	2.887	0	1	1	1	2

									Tenths of the Tabular Difference								
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
2.893	2.897	2.900	2.904	2.907	2.910	2.914	2.917	2.921	0	1	1	1	2	2	2	3	3
2.928	2.931	2.934	2.938	2.941	2.945−	2.948	2.952	2.955−	0	1	1	1	2	2	2	3	3
2.962	2.965+	2.969	2.972	2.976	2.979	2.983	2.986	2.989	0	1	1	1	2	2	2	3	3
2.996	3.000	3.003	3.007	3.010	3.014	3.017	3.021	3.024	0	1	1	1	2	2	2	3	3
3.031	3.035−	3.038	3.042	3.045+	3.049	3.052	3.056	3.059	0	1	1	1	2	2	2	3	3
3.066	3.070	3.073	3.077	3.080	3.084	3.087	3.091	3.094	0	1	1	1	2	2	2	3	3
3.101	3.105−	3.108	3.112	3.115+	3.119	3.122	3.126	3.129	0	1	1	1	2	2	2	3	3
3.136	3.140	3.144	3.147	3.151	3.154	3.158	3.161	3.165−	0	1	1	1	2	2	2	3	3
3.172	3.176	3.179	3.183	3.186	3.190	3.193	3.197	3.201	0	1	1	1	2	2	2	3	3
3.208	3.211	3.215−	3.218	3.222	3.226	3.229	3.233	3.236	0	1	1	1	2	2	3	3	3
3.244	3.247	3.251	3.254	3.258	3.262	3.265+	3.269	3.272	0	1	1	1	2	2	3	3	3
3.280	3.283	3.287	3.291	3.294	3.298	3.301	3.305+	3.309	0	1	1	1	2	2	3	3	3
3.316	3.320	3.323	3.327	3.331	3.334	3.338	3.342	3.345+	0	1	1	1	2	2	3	3	3
3.353	3.356	3.360	3.364	3.367	3.371	3.375−	3.378	3.382	0	1	1	1	2	2	3	3	3
3.389	3.393	3.397	3.400	3.404	3.408	3.411	3.415+	3.419	0	1	1	1	2	2	3	3	3
3.426	3.430	3.434	3.437	3.441	3.445−	3.448	3.452	3.456	0	1	1	1	2	2	3	3	3
3.463	3.467	3.471	3.474	3.478	3.482	3.486	3.489	3.493	0	1	1	1	2	2	3	3	3
3.501	3.504	3.508	3.512	3.516	3.519	3.523	3.527	3.531	0	1	1	1	2	2	3	3	3
3.538	3.542	3.546	3.549	3.553	3.557	3.561	3.565−	3.568	0	1	1	2	2	2	3	3	3
3.576	3.580	3.583	3.587	3.591	3.595−	3.599	3.602	3.606	0	1	1	2	2	2	3	3	3
3.614	3.618	3.621	3.625+	3.629	3.633	3.637	3.640	3.644	0	1	1	2	2	2	3	3	3
3.652	3.656	3.660	3.663	3.667	3.671	3.675−	3.679	3.683	0	1	1	2	2	2	3	3	3
3.690	3.694	3.698	3.702	3.706	3.709	3.713	3.717	3.721	0	1	1	2	2	2	3	3	3
3.729	3.733	3.736	3.740	3.744	3.748	3.752	3.756	3.760	0	1	1	2	2	2	3	3	3
3.767	3.771	3.775+	3.779	3.783	3.787	3.791	3.795−	3.799	0	1	1	2	2	2	3	3	4
3.806	3.810	3.814	3.818	3.822	3.826	3.830	3.834	3.838	0	1	1	2	2	2	3	3	4
3.846	3.849	3.853	3.857	3.861	3.865+	3.869	3.873	3.877	0	1	1	2	2	2	3	3	4
3.885−	3.889	3.893	3.897	3.901	3.905−	3.909	3.912	3.916	0	1	1	2	2	2	3	3	4
3.924	3.928	3.932	3.936	3.940	3.944	3.948	3.952	3.956	0	1	1	2	2	2	3	3	4
3.964	3.968	3.972	3.976	3.980	3.984	3.988	3.992	3.996	0	1	1	2	2	2	3	3	4
4.004	4.008	4.012	4.016	4.020	4.024	4.028	4.032	4.036	0	1	1	2	2	2	3	3	4
4.044	4.048	4.052	4.056	4.060	4.064	4.068	4.072	4.076	0	1	1	2	2	2	3	3	4
4.084	4.088	4.093	4.097	4.101	4.105−	4.109	4.113	4.117	0	1	1	2	2	2	3	3	4
4.125−	4.129	4.133	4.137	4.141	4.145+	4.149	4.153	4.158	0	1	1	2	2	2	3	3	4
4.166	4.170	4.174	4.178	4.182	4.186	4.190	4.194	4.198	0	1	1	2	2	2	3	3	4
4.207	4.211	4.215−	4.219	4.223	4.227	4.231	4.235+	4.239	0	1	1	2	2	2	3	3	4
4.248	4.252	4.256	4.260	4.264	4.268	4.272	4.277	4.281	0	1	1	2	2	2	3	3	4
4.289	4.293	4.297	4.301	4.306	4.310	4.314	4.318	4.322	0	1	1	2	2	2	3	3	4
4.331	4.335−	4.339	4.343	4.347	4.351	4.356	4.360	4.364	0	1	1	2	2	3	3	3	4
4.372	4.376	4.381	4.385−	4.389	4.393	4.397	4.402	4.406	0	1	1	2	2	3	3	3	4
4.414	4.418	4.423	4.427	4.431	4.435+	4.439	4.444	4.448	0	1	1	2	2	3	3	3	4
4.456	4.461	4.465−	4.469	4.473	4.477	4.482	4.486	4.490	0	1	1	2	2	3	3	3	4
4.499	4.503	4.507	4.511	4.516	4.520	4.524	4.528	4.533	0	1	1	2	2	3	3	3	4
4.541	4.545+	4.550−	4.554	4.558	4.562	4.567	4.571	4.575+	0	1	1	2	2	3	3	3	4
4.584	4.588	4.592	4.597	4.601	4.605+	4.610	4.614	4.618	0	1	1	2	2	3	3	3	4
4.627	4.631	4.635+	4.640	4.644	4.648	4.653	4.657	4.661	0	1	1	2	2	3	3	3	4
4.670	4.674	4.679	4.683	4.687	4.692	4.696	4.700	4.705−	0	1	1	2	2	3	3	3	4
4.713	4.718	4.722	4.726	4.731	4.735−	4.739	4.744	4.748	0	1	1	2	2	3	3	3	4
4.757	4.761	4.765+	4.770	4.774	4.779	4.783	4.787	4.792	0	1	1	2	2	3	3	3	4
4.800	4.805−	4.809	4.814	4.818	4.822	4.827	4.831	4.836	0	1	1	2	2	3	3	4	4
4.844	4.849	4.853	4.858	4.862	4.866	4.871	4.875+	4.880	0	1	1	2	2	3	3	4	4
4.889	4.893	4.897	4.902	4.906	4.911	4.915+	4.920	4.924	0	1	1	2	2	3	3	4	4
4.933	4.937	4.942	4.946	4.951	4.955+	4.960	4.964	4.968	0	1	1	2	2	3	3	4	4
4.977	4.982	4.986	4.991	4.995+	5.000	5.004	5.009	5.013	0	1	1	2	2	3	3	4	4
5.022	5.027	5.031	5.036	5.040	5.045−	5.049	5.054	5.058	0	1	1	2	2	3	3	4	4
5.067	5.072	5.076	5.081	5.085+	5.090	5.094	5.099	5.103	0	1	1	2	2	3	3	4	4
5.112	5.117	5.121	5.126	5.130	5.135−	5.139	5.144	5.148	0	1	1	2	2	3	3	4	4
5.157	5.162	5.167	5.171	5.176	5.180	5.185−	5.189	5.194	0	1	1	2	2	3	3	4	4
5.203	5.208	5.212	5.217	5.221	5.226	5.230	5.235−	5.240	0	1	1	2	2	3	3	4	4
5.249	5.253	5.258	5.262	5.267	5.272	5.276	5.281	5.285+	0	1	1	2	2	3	3	4	4
5.295−	5.299	5.304	5.308	5.313	5.318	5.322	5.327	5.331	0	1	1	2	2	3	3	4	4
5.341	5.345+	5.350−	5.355−	5.359	5.364	5.368	5.373	5.378	0	1	1	2	2	3	3	4	4
5.387	5.392	5.396	5.401	5.406	5.410	5.415−	5.420	5.424	0	1	1	2	2	3	3	4	4
5.434	5.438	5.443	5.448	5.452	5.457	5.462	5.466	5.471	0	1	1	2	2	3	3	4	4
5.480	5.485−	5.490	5.494	5.499	5.504	5.508	5.513	5.518	0	1	1	2	2	3	3	4	4
5.527	5.532	5.537	5.541	5.546	5.551	5.555+	5.560	5.565−	0	1	1	2	2	3	3	4	4
5.574	5.579	5.584	5.588	5.593	5.598	5.603	5.607	5.612	0	1	1	2	2	3	3	4	4
5.622	5.626	5.631	5.636	5.641	5.645+	5.650+	5.655−	5.660	0	1	1	2	2	3	3	4	4
5.669	5.674	5.679	5.683	5.688	5.693	5.698	5.703	5.707	0	1	1	2	2	3	3	4	4
5.717	5.722	5.726	5.731	5.736	5.741	5.746	5.750+	5.755+	0	1	1	2	2	3	3	4	4
5.765−	5.770	5.774	5.779	5.784	5.789	5.794	5.798	5.803	0	1	1	2	2	3	3	4	4
5.813	5.818	5.823	5.827	5.832	5.837	5.842	5.847	5.852	0	1	1	2	2	3	3	4	4
5.861	5.866	5.871	5.876	5.881	5.885+	5.890	5.895+	5.900	0	1	1	2	2	3	3	4	4
5.910	5.915−	5.919	5.924	5.929	5.934	5.939	5.944	5.949	0	1	1	2	2	3	3	4	[illegible]
5.958	5.963	5.968	5.973	5.978	5.983	5.988	5.993	5.998	0	1	1	2	2	3	3	4	[illegible]

n^2 SQUARES

TENTHS
TABULAR D

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5
2.45	6.002	6.007	6.012	6.017	6.022	6.027	6.032	6.037	6.042	6.047	0	1	1	2	2
6	6.052	6.057	6.061	6.066	6.071	6.076	6.081	6.086	6.091	6.096	0	1	1	2	2
7	6.101	6.106	6.111	6.116	6.121	6.126	6.131	6.136	6.140	6.145+	0	1	1	2	2
8	6.150+	6.155+	6.160	6.165+	6.170	6.175+	6.180	6.185+	6.190	6.195+	0	1	1	2	2
9	6.200	6.205+	6.210	6.215+	6.220	6.225+	6.230	6.235+	6.240	6.245+	0	1	1	2	2
2.50	6.250	6.255+	6.260	6.265+	6.270	6.275+	6.280	6.285+	6.290	6.295+	1	1	2	2	3
1	6.300	6.305+	6.310	6.315+	6.320	6.325+	6.330	6.335+	6.340	6.345+	1	1	2	2	3
2	6.350+	6.355+	6.360	6.366	6.371	6.376	6.381	6.386	6.391	6.396	1	1	2	2	3
3	6.401	6.406	6.411	6.416	6.421	6.426	6.431	6.436	6.441	6.447	1	1	2	2	3
4	6.452	6.457	6.462	6.467	6.472	6.477	6.482	6.487	6.492	6.497	1	1	2	2	3
2.55	6.502	6.508	6.513	6.518	6.523	6.528	6.533	6.538	6.543	6.548	1	1	2	2	3
6	6.554	6.559	6.564	6.569	6.574	6.579	6.584	6.589	6.595−	6.600	1	1	2	2	3
7	6.605−	6.610	6.615+	6.620	6.625+	6.631	6.636	6.641	6.646	6.651	1	1	2	2	3
8	6.656	6.662	6.667	6.672	6.677	6.682	6.687	6.693	6.698	6.703	1	1	2	2	3
9	6.708	6.713	6.718	6.724	6.729	6.734	6.739	6.744	6.750−	6.755−	1	1	2	2	3
2.60	6.760	6.765+	6.770	6.776	6.781	6.786	6.791	6.796	6.802	6.807	1	1	2	2	3
1	6.812	6.817	6.823	6.828	6.833	6.838	6.843	6.849	6.854	6.859	1	1	2	2	3
2	6.864	6.870	6.875−	6.880	6.885+	6.891	6.896	6.901	6.906	6.912	1	1	2	2	3
3	6.917	6.922	6.927	6.933	6.938	6.943	6.948	6.954	6.959	6.964	1	1	2	2	3
4	6.970	6.975−	6.980	6.985+	6.991	6.996	7.001	7.007	7.012	7.017	1	1	2	2	3
2.65	7.022	7.028	7.033	7.038	7.044	7.049	7.054	7.060	7.065−	7.070	1	1	2	2	3
6	7.076	7.081	7.086	7.092	7.097	7.102	7.108	7.113	7.118	7.124	1	1	2	2	3
7	7.129	7.134	7.140	7.145−	7.150+	7.156	7.161	7.166	7.172	7.177	1	1	2	2	3
8	7.182	7.188	7.193	7.198	7.204	7.209	7.215−	7.220	7.225+	7.231	1	1	2	2	3
9	7.236	7.241	7.247	7.252	7.258	7.263	7.268	7.274	7.279	7.285−	1	1	2	2	3
2.70	7.290	7.295+	7.301	7.306	7.312	7.317	7.322	7.328	7.333	7.339	1	1	2	2	3
1	7.344	7.350−	7.355−	7.360	7.366	7.371	7.377	7.382	7.388	7.393	1	1	2	2	3
2	7.398	7.404	7.409	7.415−	7.420	7.426	7.431	7.437	7.442	7.447	1	1	2	2	3
3	7.453	7.458	7.464	7.469	7.475−	7.480	7.486	7.491	7.497	7.502	1	1	2	2	3
4	7.508	7.513	7.519	7.524	7.530	7.535+	7.541	7.546	7.552	7.557	1	1	2	2	3
2.75	7.562	7.568	7.574	7.579	7.585−	7.590	7.596	7.601	7.607	7.612	1	1	2	2	3
6	7.618	7.623	7.629	7.634	7.640	7.645+	7.651	7.656	7.662	7.667	1	1	2	2	3
7	7.673	7.678	7.684	7.690	7.695+	7.701	7.706	7.712	7.717	7.723	1	1	2	2	3
8	7.728	7.734	7.740	7.745+	7.751	7.756	7.762	7.767	7.773	7.779	1	1	2	2	3
9	7.784	7.790	7.795+	7.801	7.806	7.812	7.818	7.823	7.829	7.834	1	1	2	2	3
2.80	7.840	7.846	7.851	7.857	7.862	7.868	7.874	7.879	7.885−	7.890	1	1	2	2	3
1	7.896	7.902	7.907	7.913	7.919	7.924	7.930	7.935+	7.941	7.947	1	1	2	2	3
2	7.952	7.958	7.964	7.969	7.975−	7.981	7.986	7.992	7.998	8.003	1	1	2	2	3
3	8.009	8.015−	8.020	8.026	8.032	8.037	8.043	8.049	8.054	8.060	1	1	2	2	3
4	8.066	8.071	8.077	8.083	8.088	8.094	8.100	8.105+	8.111	8.117	1	1	2	2	3
2.85	8.122	8.128	8.134	8.140	8.145+	8.151	8.157	8.162	8.168	8.174	1	1	2	2	3
6	8.180	8.185+	8.191	8.197	8.202	8.208	8.214	8.220	8.225+	8.231	1	1	2	2	3
7	8.237	8.243	8.248	8.254	8.260	8.266	8.271	8.277	8.283	8.289	1	1	2	2	3
8	8.294	8.300	8.306	8.312	8.317	8.323	8.329	8.335−	8.341	8.346	1	1	2	2	3
9	8.352	8.358	8.364	8.369	8.375+	8.381	8.387	8.393	8.398	8.404	1	1	2	2	3
2.90	8.410	8.416	8.422	8.427	8.433	8.439	8.445−	8.451	8.456	8.462	1	1	2	2	3
1	8.468	8.474	8.480	8.486	8.491	8.497	8.503	8.509	8.515−	8.521	1	1	2	2	3
2	8.526	8.532	8.538	8.544	8.550−	8.556	8.561	8.567	8.573	8.579	1	1	2	2	3
3	8.585−	8.591	8.597	8.602	8.608	8.614	8.620	8.626	8.632	8.638	1	1	2	2	3
4	8.644	8.649	8.655+	8.661	8.667	8.673	8.679	8.685−	8.691	8.697	1	1	2	2	3
2.95	8.702	8.708	8.714	8.720	8.726	8.732	8.738	8.744	8.750−	8.756	1	1	2	2	3
6	8.762	8.768	8.773	8.779	8.785+	8.791	8.797	8.803	8.809	8.815−	1	1	2	2	3
7	8.821	8.827	8.833	8.839	8.845−	8.851	8.857	8.863	8.868	8.874	1	1	2	2	3
8	8.880	8.886	8.892	8.898	8.904	8.910	8.916	8.922	8.928	8.934	1	1	2	2	3
9	8.940	8.946	8.952	8.958	8.964	8.970	8.976	8.982	8.988	8.994	1	1	2	2	3
3.00	9.000	9.006	9.012	9.018	9.024	9.030	9.036	9.042	9.048	9.054	1	1	2	2	3
1	9.060	9.066	9.072	9.078	9.084	9.090	9.096	9.102	9.108	9.114	1	1	2	2	3
2	9.120	9.126	9.132	9.139	9.145−	9.151	9.157	9.163	9.169	9.175−	1	1	2	2	3
3	9.181	9.187	9.193	9.199	9.205+	9.211	9.217	9.223	9.229	9.236	1	1	2	2	3
4	9.242	9.248	9.254	9.260	9.266	9.272	9.278	9.284	9.290	9.296	1	1	2	2	3
3.05	9.302	9.309	9.315−	9.321	9.327	9.333	9.339	9.345+	9.351	9.357	1	1	2	2	3
6	9.364	9.370	9.376	9.382	9.388	9.394	9.400	9.406	9.413	9.419	1	1	2	2	3
7	9.425−	9.431	9.437	9.443	9.449	9.456	9.462	9.468	9.474	9.480	1	1	2	2	3
8	9.486	9.493	9.499	9.505−	9.511	9.517	9.523	9.530	9.536	9.542	1	1	2	2	3
9	9.548	9.554	9.560	9.567	9.573	9.579	9.585+	9.591	9.598	9.604	1	1	2	2	3
3.10	9.610	9.616	9.622	9.629	9.635−	9.641	9.647	9.653	9.660	9.666	1	1	2	2	3
1	9.672	9.678	9.685−	9.691	9.697	9.703	9.709	9.716	9.722	9.728	1	1	2	2	3
2	9.734	9.741	9.747	9.753	9.759	9.766	9.772	9.778	9.784	9.791	1	1	2	2	3
3	9.797	9.803	9.809	9.816	9.822	9.828	9.834	9.841	9.847	9.853	1	1	2	3	3
4	9.860	9.866	9.872	9.878	9.885−	9.891	9.897	9.904	9.910	9.916	1	1	2	3	3
3.15	9.922	9.929	9.935+	9.941	9.948	9.954	9.960	9.967	9.973	9.979	1	1	2	3	3
6	9.986	9.992	9.998	10.005−							1	1	2	3	

TABLE LXXI

ıree pages give the **square roots** of all numbers between 1 and 10^2, correct to four significant 'or example: $\sqrt{3.142} = 1.773$; $\sqrt{31.42} = 5.606$. (Values obtained by interpolation may be in in the last figure.)

in the square root of any number outside the range from 1 to 10^2, write the number as the prod-) some number between 1 and 10^2, and (b) some (positive or negative) power of 10^2; then proceed ollowing examples:

$$\sqrt{3142000} = \sqrt{3.142 \times 10^6} = \sqrt{3.142} \times \sqrt{10^6} = 1.773 \times 10^3 = 1773.$$
$$\sqrt{0.3142} = \sqrt{31.42 \times 10^{-2}} = \sqrt{31.42} \times \sqrt{10^{-2}} = 5.606 \times 10^{-1} = 0.5606.$$

, moving the decimal point TWO *places in column n is equivalent to moving it* ONE *place in the table.*

SQUARE ROOTS

										Tenths of the Tabular Difference								
	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
	1.005 –	1.010	1.015 –	1.020	1.025 –	1.030	1.034	1.039	1.044	0	1	1	2	2	3	3	4	4
	1.054	1.058	1.063	1.068	1.072	1.077	1.082	1.086	1.091	0	1	1	2	2	3	3	4	4
+	1.100	1.105 –	1.109	1.114	1.118	1.122	1.127	1.131	1.136	0	1	1	2	2	3	3	4	4
	1.145 –	1.149	1.153	1.158	1.162	1.166	1.170	1.175 –	1.179	0	1	1	2	2	3	3	3	4
	1.187	1.192	1.196	1.200	1.204	1.208	1.212	1.217	1.221	0	1	1	2	2	2	3	3	4
–	1.229	1.233	1.237	1.241	1.245 –	1.249	1.253	1.257	1.261	0	1	1	2	2	2	3	3	4
	1.269	.273	1.277	1.281	1.285 –	1.288	1.292	1.296	1.300	0	1	1	2	2	2	3	3	4
	1.308	1.311	1.315 +	1.319	1.323	1.327	1.330	1.334	1.338	0	1	1	2	2	2	3	3	3
	1.345 +	1.349	1.353	1.356	1.360	1.364	1.367	1.371	1.375 –	0	1	1	1	2	2	3	3	3
	1.382	1.386	1.389	1.393	1.396	1.400	1.404	1.407	1.411	0	1	1	1	2	2	3	3	3
	1.418	1.421	1.425 –	1.428	1.432	1.435 +	1.439	1.442	1.446	0	1	1	1	2	2	2	3	3
	1.453	1.456	1.459	1.463	1.466	1.470	1.473	1.476	1.480	0	1	1	1	2	2	2	3	3
	1.487	1.490	1.493	1.497	1.500	1.503	1.507	1.510	1.513	0	1	1	1	2	2	2	3	3
	1.520	1.523	1.526	1.530	1.533	1.536	1.539	1.543	1.546	0	1	1	1	2	2	2	3	3
	1.552	1.556	1.559	1.562	1.565 +	1.568	1.572	1.575 –	1.578	0	1	1	1	2	2	2	3	3
	1.584	1.587	1.591	1.594	1.597	1.600	1.603	1.606	1.609	0	1	1	1	2	2	2	3	3
	1.616	1.619	1.622	1.625 –	1.628	1.631	1.634	1.637	1.640	0	1	1	1	2	2	2	2	3
	1.646	1.649	1.652	1.655 +	1.658	1.661	1.664	1.667	1.670	0	1	1	1	2	2	2	2	3
	.676	1.679	1.682	1.685 +	1.688	1.691	1.694	1.697	1.700	0	1	1	1	1	2	2	2	3
	.706	1.709	1.712	1.715 –	1.718	1.720	1.723	1.726	1.729	0	1	1	1	1	2	2	2	3
	.735 –	1.738	1.741	1.744	1.746	1.749	1.752	1.755 –	1.758	0	1	1	1	1	2	2	2	3
	.764	1.766	1.769	1.772	1.775 –	1.778	1.780	1.783	1.786	0	1	1	1	1	2	2	2	3
	.792	1.794	1.797	1.800	1.803	1.806	1.808	1.811	1.814	0	1	1	1	1	2	2	2	2
	.819	1.822	1.825 –	1.828	1.830	1.833	1.836	1.838	1.841	0	1	1	1	1	2	2	2	2
	.847	1.849	1.852	1.855 –	1.857	1.860	1.863	1.865 +	1.868	0	1	1	1	1	2	2	2	2
	873	1.876	1.879	1.881	1.884	1.887	1.889	1.892	1.895 –	0	1	1	1	1	2	2	2	2
	900	1.903	1.905 +	1.908	1.910	1.913	1.916	1.918	1.921	0	1	1	1	1	2	2	2	2
	926	1.929	1.931	1.934	1.936	1.939	1.942	1.944	1.947	0	1	1	1	1	2	2	2	2
	952	1.954	1.957	1.960	1. 62	1.965 –	1.967	1.970	1.972	0	1	1	1	1	2	2	2	2
	977	1.980	1.982	1.985 –	1.987	1.990	1.902	1.995 –	1.997	0	1	1	1	1	2	2	2	2
	002	2.005 –	2.007	2.010	2.012	2.015 –	2.017	2.020	2.022	0	0	1	1	1	1	2	2	2
	027	2.030	2.032	2.035 –	2.037	2.040	2.042	2.045 –	2.047	0	0	1	1	1	1	2	2	2
	052	2.054	2.057	2.059	2.062	2.064	2.066	2.069	2.071	0	0	1	1	1	1	2	2	2
	076	2.078	2.081	2.083	2.086	2.088	2.090	2.093	2.095 +	0	0	1	1	1	1	2	2	2
	100	2.102	2.105 –	2.107	2.110	2.112	2.114	2.117	2.119	0	0	1	1	1	1	2	2	2
	124	2.126	2.128	2.131	2.133	2.135 +	2.138	2.140	2.142	0	0	1	1	1	1	2	2	2
	147	2.149	2.152	2.154	2.156	2.159	2.161	2.163	2.166	0	0	1	1	1	1	2	2	2
	.170	2.173	2.175 –	2.177	2.179	2.182	2.184	2.186	2.189	0	0	1	1	1	1	2	2	2
	.193	2.195 +	2.198	2.200	2.202	2.205 –	2.207	2.209	2.211	0	0	1	1	1	1	2	2	2
	.216	2.218	2.220	2.223	2.225 –	2.227	2.229	2.232	2.234	0	0	1	1	1	1	2	2	2
	.238	2.241	2.243	2.245 –	2.247	2.249	2.252	2.254	2.256	0	0	1	1	1	1	2	2	2
	.261	2.263	2.265 –	2.267	2.269	2.272	2.274	2.276	2.278	0	0	1	1	1	1	2	2	2
	.283	2.285 –	2.287	2.289	2.291	2.293	2.296	2.298	2.300	0	0	1	1	1	1	2	2	2
	.304	2.307	2.309	2.311	2.313	2.315 +	2.317	2.319	2.322	0	0	1	1	1	1	2	2	2
	.326	2.328	2.330	2.332	2.335 –	2.337	2.339	2.341	2.343	0	0	1	1	1	1	1	2	2
	2.347	2.349	2.352	2.354	2.356	2.358	2.360	2.362	2.364	0	0	1	1	1	1	1	2	2
	2.369	2.371	2.373	2.375 –	2.377	2.379	2.381	2.383	2.385 +	0	0	1	1	1	1	[illegible]	[illegible]	[illegible]
	2.390	2.392	2.394	2.396	2.398	2.400	2.402	2.404	2.406	0	0	1	1	1	1	[illegible]	[illegible]	[illegible]
	2.410	2.412	2.415 –	2.417	2.419	2.421	2.423	2.425 –	2.427	0	0	1	1	1	1	[illegible]	[illegible]	[illegible]
	2.431	2.433	2.435 +	2.437	2.439	2.441	2.443	2.445 +	2.447	0	0	1	1	1	[illegible]	[illegible]	[illegible]	[illegible]

$\sqrt{n}$ SQUARE ROOTS

TENTHS O[F] TABULAR DI[FFERENCES]

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5
6.0	2.449	2.452	2.454	2.456	2.458	2.460	2.462	2.464	2.466	2.468	0	0	1	1	1
.1	2.470	2.472	2.474	2.476	2.478	2.480	2.482	2.484	2.486	2.488	0	0	1	1	1
.2	2.490	2.492	2.494	2.496	2.498	2.500	2.502	2.504	2.506	2.508	0	0	1	1	1
.3	2.510	2.512	2.514	2.516	2.518	2.520	2.522	2.524	2.526	2.528	0	0	1	1	1
.4	2.530	2.532	2.534	2.536	2.538	2.540	2.542	2.544	2.546	2.548	0	0	1	1	1
6.5	2.550 −	2.551	2.553	2.555 +	2.557	2.559	2.561	2.563	2.565 +	2.567	0	0	1	1	1
.6	2.569	2.571	2.573	2.575 −	2.577	2.579	2.581	2.583	2.585 −	2.587	0	0	1	1	1
.7	2.588	2.590	2.592	2.594	2.596	2.598	2.600	2.602	2.604	2.606	0	0	1	1	1
.8	2.608	2.610	2.612	2.613	2.615 +	2.617	2.619	2.621	2.623	2.625 −	0	0	1	1	1
.9	2.627	2.629	2.631	2.632	2.634	2.636	2.638	2.640	2.642	2.644	0	0	1	1	1
7.0	2.646	2.648	2.650 −	2.651	2.653	2.655 +	2.657	2.659	2.661	2.663	0	0	1	1	1
.1	2.665 −	2.666	2.668	2.670	2.672	2.674	2.676	2.678	2.680	2.681	0	0	1	1	1
.2	2.683	2.685 +	2.687	2.689	2.691	2.693	2.694	2.696	2.698	2.700	0	0	1	1	1
.3	2.702	2.704	2.706	2.707	2.709	2.711	2.713	2.715 −	2.717	2.718	0	0	1	1	1
.4	2.720	2.722	2.724	2.726	2.728	2.729	2.731	2.733	2.735 −	2.737	0	0	1	1	1
7.5	2.739	2.740	2.742	2.744	2.746	2.748	2.750 −	2.751	2.753	2.755 −	0	0	1	1	1
.6	2.757	2.759	2.760	2.762	2.764	2.766	2.768	2.769	2.771	2.773	0	0	1	1	1
.7	2.775 −	2.777	2.778	2.780	2.782	2.784	2.786	2.787	2.789	2.791	0	0	1	1	1
.8	2.793	2.795 −	2.796	2.798	2.800	2.802	2.804	2.805 +	2.807	2.809	0	0	1	1	1
.9	2.811	2.812	2.814	2.816	2.818	2.820	2.821	2.823	2.825 −	2.827	0	0	1	1	1
8.0	2.828	2.830	2.832	2.834	2.835 +	2.837	2.839	2.841	2.843	2.844	0	0	1	1	1
.1	2.846	2.848	2.850 −	2.851	2.853	2.855 −	2.857	2.858	2.860	2.862	0	0	1	1	1
.2	2.864	2.865 +	2.867	2.869	2.871	2.872	2.874	2.876	2.877	2.879	0	0	1	1	1
.3	2.881	2.883	2.884	2.886	2.888	2.890	2.891	2.893	2.895 −	2.897	0	0	1	1	1
.4	2.898	2.900	2.902	2.903	2.905 +	2.907	2.909	2.910	2.912	2.914	0	0	1	1	1
8.5	2.915 +	2.917	2.919	2.921	2.922	2.924	2.926	2.927	2.929	2.931	0	0	1	1	1
.6	2.933	2.934	2.936	2.938	2.939	2.941	2.943	2.944	2.946	2.948	0	0	1	1	1
.7	2.950 −	2.951	2.953	2.955 −	2.956	2.958	2.960	2.961	2.963	2.965 −	0	0	1	1	1
.8	2.966	2.968	2.970	2.972	2.973	2.975 −	2.977	2.978	2.980	2.982	0	0	1	1	1
.9	2.983	2.985 −	2.987	2.988	2.990	2.992	2.993	2.995 −	2.997	2.998	0	0	1	1	1
9.0	3.000	3.002	3.003	3.005 −	3.007	3.008	3.010	3.012	3.013	3.015 −	0	0	0	1	1
.1	3.017	3.018	3.020	3.022	3.023	3.025 −	3.027	3.028	3.030	3.032	0	0	0	1	1
.2	3.033	3.035 −	3.036	3.038	3.040	3.041	3.043	3.045 −	3.046	3.048	0	0	0	1	1
.3	3.050 −	3.051	3.053	3.055 −	3.056	3.058	3.059	3.061	3.063	3.064	0	0	0	1	1
.4	3.066	3.068	3.069	3.071	3.072	3.074	3.076	3.077	3.079	3.081	0	0	0	1	1
9.5	3.082	3.084	3.085 +	3.087	3.089	3.090	3.092	3.094	3.095 +	3.097	0	0	0	1	1
.6	3.098	3.100	3.102	3.103	3.105 −	3.106	3.108	3.110	3.111	3.113	0	0	0	1	1
.7	3.114	3.116	3.118	3.119	3.121	3.122	3.124	3.126	3.127	3.129	0	0	0	1	1
.8	3.130	3.132	3.134	3.135 +	3.137	3.138	3.140	3.142	3.143	3.145 −	0	0	0	1	1
.9	3.146	3.148	3.150 −	3.151	3.153	3.154	3.156	3.158	3.159	3.161	0	0	0	1	1
10	3.162	3.178	3.194	3.209	3.225 −	3.240	3.256	3.271	3.286	3.302	2	3	5	6	8
1	3.317	3.332	3.347	3.362	3.376	3.391	3.406	3.421	3.435 +	3.450 −	1	3	4	6	7
2	3.464	3.479	3.493	3.507	3.521	3.536	3.550 −	3.564	3.578	3.592	1	3	4	6	7
3	3.606	3.619	3.633	3.647	3.661	3.674	3.688	3.701	3.715 −	3.728	1	3	4	5	7
4	3.742	3.755 −	3.768	3.782	3.795 −	3.808	3.821	3.834	3.847	3.860	1	3	4	5	7
15	3.873	3.886	3.899	3.912	3.924	3.937	3.950 −	3.962	3.975 −	3.987	1	3	4	5	6
6	4.000	4.012	4.025 −	4.037	4.050 −	4.062	4.074	4.087	4.099	4.111	1	2	4	5	6
7	4.123	4.135 +	4.147	4.159	4.171	4.183	4.195 +	4.207	4.219	4.231	1	2	4	5	6
8	4.243	4.254	4.266	4.278	4.290	4.301	4.113	4.324	4.336	4.347	1	2	3	5	6
9	4.359	4.370	4.382	4.393	4.405 −	4.416	4.427	4.438	4.450 −	4.461	1	2	3	5	6
20	4.472	4.483	4.494	4.506	4.517	4.528	4.539	4.550 −	4.561	4.572	1	2	3	4	6
1	4.583	4.593	4.604	4.615 +	4.626	4.637	4.648	4.658	4.669	4.680	1	2	3	4	5
2	4.690	4.701	4.712	4.722	4.733	4.743	4.754	4.764	4.775 −	4.785 +	1	2	3	4	5
3	4.796	4.806	4.817	4.827	4.837	4.848	4.858	4.868	4.879	4.889	1	2	3	4	5
4	4.899	4.909	4.919	4.930	4.940	4.950 −	4.960	4.970	4.980	4.990	1	2	3	4	5
25	5.000	5.010	5.020	5.030	5.040	5.050 −	5.060	5.070	5.079	5.089	1	2	3	4	5
6	5.099	5.109	5.119	5.128	5.138	5.148	5.158	5.167	5.177	5.187	1	2	3	4	5
7	5.196	5.206	5.215 +	5.225 −	5.235 −	5.244	5.254	5.263	5.273	5.282	1	2	3	4	5
8	5.292	5.301	5.310	5.320	5.329	5.339	5.348	5.357	5.367	5.376	1	2	3	4	5
9	5.385 +	5.394	5.404	5.413	5.422	5.431	5.441	5.450 −	5.459	5.468	1	2	3	4	5
30	5.477	5.486	5.495 +	5.505 −	5.514	5.523	5.532	5.541	5.550 −	5.559	1	2	3	4	5
1	5.568	5.577	5.586	5.595 −	5.604	5.612	5.621	5.630	5.639	5.648	1	2	3	4	4
2	5.657	5.666	5.675 −	5.683	5.692	5.701	5.710	5.718	5.727	5.736	1	2	3	4	4
3	5.745 −	5.753	5.762	5.771	5.779	5.788	5.797	5.805 +	5.814	5.822	1	2	3	3	4
4	5.831	5.840	5.848	5.857	5.865 +	5.874	5.882	5.891	5.899	5.908	1	2	3	3	4
35	5.916	5.925 −	5.933	5.941	5.950 −	5.958	5.967	5.975 −	5.983	5.992	1	2	3	3	4
6	6.000	6.008	6.017	6.025 −	6.033	6.042	6.050 −	6.058	6.066	6.075 −	1	2	2	3	4
7	6.083	6.091	6.099	6.107	6.116	6.124	6.132	6.140	6.148	6.156	1	2	2	3	4
8	6.164	6.173	6.181	6.189	6.197	6.205 −	6.213	6.221	6.229	6.237	1	2	2	3	4
9	6.245 −	6.253	6.261	6.269	6.277	6.285 −	6.293	6.301	6.309	6.317	1	2	2	3	4

TABLE LXXI — *Continued*

SQUARE ROOTS

TENTHS OF THE TABULAR DIFFERENCE

1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
6.332	6.340	6.348	6.356	6.364	6.372	6.380	6.387	6.395+	1	2	2	3	4	5	5	6	7
6.411	6.419	6.427	6.434	6.442	6.450–	6.458	6.465+	6.473	1	2	2	3	4	5	5	6	7
6.488	6.496	6.504	6.512	6.519	6.527	6.535–	6.542	6.550–	1	2	2	3	4	5	5	6	7
6.565+	6.573	6.580	6.588	6.595+	6.603	6.611	6.618	6.626	1	2	2	3	4	5	5	6	7
6.641	6.648	6.656	6.663	6.671	6.678	6.686	6.693	6.701	1	1	2	3	4	4	5	6	7
6.716	6.723	6.731	6.738	6.745+	6.753	6.760	6.768	6.775–	1	1	2	3	4	4	5	6	7
6.790	6.797	6.804	6.812	6.819	6.826	6.834	6.841	6.848	1	1	2	3	4	4	5	6	7
6.863	6.870	6.877	6.885–	6.892	6.899	6.907	6.914	6.921	1	1	2	3	4	4	5	6	7
6.935+	6.943	6.950–	6.957	6.964	6.971	6.979	6.986	6.993	1	1	2	3	4	4	5	6	6
7.007	7.014	7.021	7.029	7.036	7.043	7.050–	7.057	7.064	1	1	2	3	4	4	5	6	6
7.078	7.085+	7.092	7.099	7.106	7.113	7.120	7.127	7.134	1	1	2	3	4	4	5	6	6
7.148	7.155+	7.162	7.169	7.176	7.183	7.190	7.197	7.204	1	1	2	3	3	4	5	6	6
7.218	7.225–	7.232	7.239	7.246	7.253	7.259	7.266	7.273	1	1	2	3	3	4	5	6	6
7.287	7.294	7.301	7.308	7.314	7.321	7.328	7.335–	7.342	1	1	2	3	3	4	5	5	6
7.355+	7.362	7.369	7.376	7.382	7.389	7.396	7.403	7.409	1	1	2	3	3	4	5	5	6
7.423	7.430	7.436	7.443	7.450–	7.457	7.463	7.470	7.477	1	1	2	3	3	4	5	5	6
7.490	7.497	7.503	7.510	7.517	7.523	7.530	7.537	7.543	1	1	2	3	3	4	5	5	6
7.556	7.563	7.570	7.576	7.583	7.589	7.596	7.603	7.609	1	1	2	3	3	4	5	5	6
7.622	7.629	7.635+	7.642	7.649	7.655+	7.662	7.668	7.675–	1	1	2	3	3	4	5	5	6
7.688	7.694	7.701	7.707	7.714	7.720	7.727	7.733	7.740	1	1	2	3	3	4	5	5	6
7.752	7.759	7.765+	7.772	7.778	7.785–	7.791	7.797	7.804	1	1	2	3	3	4	4	5	6
7.817	7.823	7.829	7.836	7.842	7.849	7.855–	7.861	7.868	1	1	2	3	3	4	4	5	6
7.880	7.887	7.893	7.899	7.906	7.912	7.918	7.925–	7.931	1	1	2	3	3	4	4	5	6
7.944	7.950–	7.956	7.962	7.969	7.975–	7.981	7.987	7.994	1	1	2	3	3	4	4	5	6
8.006	8.012	8.019	8.025–	8.031	8.037	8.044	8.050–	8.056	1	1	2	2	3	4	4	5	6
8.068	8.075–	8.081	8.087	8.093	8.099	8.106	8.112	8.118	1	1	2	2	3	4	4	5	6
8.130	8.136	8.142	8.149	8.155–	8.161	8.167	8.173	8.179	1	1	2	2	3	4	4	5	6
8.191	8.198	8.204	8.210	8.216	8.222	8.228	8.234	8.240	1	1	2	2	3	4	4	5	5
8.252	8.258	8.264	8.270	8.276	8.283	8.289	8.295–	8.301	1	1	2	2	3	4	4	5	5
8.313	8.319	8.325–	8.331	8.337	8.343	8.349	8.355–	8.361	1	1	2	2	3	4	4	5	5
8.373	8.379	8.385–	8.390	8.396	8.402	8.408	8.414	8.420	1	1	2	2	3	4	4	5	5
8.432	8.438	8.444	8.450–	8.456	8.462	8.468	8.473	8.479	1	1	2	2	3	4	4	5	5
8.491	8.497	8.503	8.509	8.515–	8.521	8.526	8.532	8.538	1	1	2	2	3	4	4	5	5
8.550–	8.556	8.562	8.567	8.573	8.579	8.585–	8.591	8.597	1	1	2	2	3	3	4	5	5
8.608	8.614	8.620	8.626	8.631	8.637	8.643	8.649	8.654	1	1	2	2	3	3	4	5	5
8.666	8.672	8.678	8.683	8.689	8.695–	8.701	8.706	8.712	1	1	2	2	3	3	4	5	5
8.724	8.729	8.735–	8.741	8.746	8.752	8.758	8.764	8.769	1	1	2	2	3	3	4	5	5
8.781	8.786	8.792	8.798	8.803	8.809	8.815–	8.820	8.826	1	1	2	2	3	3	4	5	5
8.837	8.843	8.849	8.854	8.860	8.866	8.871	8.877	8.883	1	1	2	2	3	3	4	5	5
8.894	8.899	8.905+	8.911	8.916	8.922	8.927	8.933	8.939	1	1	2	2	3	3	4	4	5
8.950–	8.955+	8.961	8.967	8.972	8.978	8.983	8.989	8.994	1	1	2	2	3	3	4	4	5
9.006	9.011	9.017	9.022	9.028	9.033	9.039	9.044	9.050–	1	1	2	2	3	3	4	4	5
9.061	9.066	9.072	9.077	9.083	9.088	9.094	9.099	9.105–	1	1	2	2	3	3	4	4	5
9.116	9.121	9.127	9.132	9.138	9.143	9.149	9.154	9.160	1	1	2	2	3	3	4	4	5
9.171	9.176	9.182	9.187	9.192	9.198	9.203	9.209	9.214	1	1	2	2	3	3	4	4	5
9.225–	9.230	9.236	9.241	9.247	9.252	9.257	9.263	9.268	1	1	2	2	3	3	4	4	5
9.279	9.284	9.290	9.295+	9.301	9.306	9.311	9.317	9.322	1	1	2	2	3	3	4	4	5
9.333	9.338	9.343	9.349	9.354	9.359	9.365–	9.370	9.375+	1	1	2	2	3	3	4	4	5
9.386	9.391	9.397	9.402	9.407	9.413	9.418	9.423	9.429	1	1	2	2	3	3	4	4	5
9.439	9.445–	9.450–	9.455+	9.460	9.466	9.471	9.476	9.482	1	1	2	2	3	3	4	4	5
9.492	9.497	9.503	9.508	9.513	9.518	9.524	9.529	9.534	1	1	2	2	3	3	4	4	5
9.545–	9.550–	9.555+	9.560	9.566	9.571	9.576	9.581	9.586	1	1	2	2	3	3	4	4	5
9.597	9.602	9.607	9.612	9.618	9.623	9.628	9.633	9.638	1	1	2	2	3	3	4	4	5
9.649	9.654	9.659	9.664	9.670	9.675–	9.680	9.685+	9.690	1	1	2	2	3	3	4	4	5
9.701	9.706	9.711	9.716	9.721	9.726	9.731	9.737	9.742	1	1	2	2	3	3	4	4	5
9.752	9.757	9.762	9.767	9.772	9.778	9.783	9.788	9.793	1	1	2	2	3	3	4	4	5
9.803	9.808	9.813	9.818	9.823	9.829	9.834	9.839	9.844	1	1	2	2	3	3	4	4	5
9.854	9.859	9.864	9.869	9.874	9.879	9.884	9.889	9.894	1	1	2	2	3	3	4	4	5
9.905–	9.910	9.915–	9.920	9.925–	9.930	9.935–	9.940	9.945–	1	1	2	2	3	3	4	4	5
9.955–	9.960	9.965–	9.970	9.975–	9.980	9.985–	9.990	9.995–	1	1	2	2	3	3	4	4	5

TABLE LXXII

These two pages give the **cubes** of all numbers between 1 and 10, correct to four significant figures
example: $(2.718)^3 = 20.08$. (Values obtained by interpolation may be in error by 1 in the last figure
To obtain the cube of any number outside the range from 1 to 10, write the number as the produ
(*a*) some number between 1 and 10, and (*b*) some (positive or negative) power of 10; then proceed
the following examples:

$$(271.8)^3 = (2.718 \times 10^2)^3 = (2.718)^3 \times (10^2)^3 = 20.08 \times 10^6 = 20\,080\,000;$$
$$(0.2718)^3 = (2.718 \times 10^{-1})^3 = (2.718)^3 \times (10^{-1})^3 = 20.08 \times 10^{-3} = 0.020\,08.$$

In short, moving the decimal point ONE *place in column* **n** *is equivalent to moving it* THREE pla
the body of the table.

The table used **inversely** gives the **cube roots** of all numbers between 1 and 10^3. For example:

$$\sqrt[3]{3.142} = 1.4646; \qquad \sqrt[3]{31.42} = 3.155; \qquad \sqrt[3]{314.2} = 6.799$$

To obtain the cube root of any number outside the range from 1 to 10^3, write the number as the p
of: (*a*) some number between 1 and 10^3, and (*b*) some (positive or negative) power of 10^3; thus:

$$\sqrt[3]{3142.} = \sqrt[3]{3.142 \times 10^3} = \sqrt[3]{3.142} \times \sqrt[3]{10^3} = 1.4646 \times 10 = 14.646;$$
$$\sqrt[3]{31420000.} = \sqrt[3]{31.42 \times 10^6} = \sqrt[3]{31.42} \times \sqrt[3]{10^6} = 3.155 \times 10^2 = 315.5;$$
$$\sqrt[3]{0.0000003142} = \sqrt[3]{314.2 \times 10^{-9}} = \sqrt[3]{314.2} \times \sqrt[3]{10^{-9}} = 6.799 \times 10^{-3} = 0.006799.$$

The boldface figures in the body of the table facilitate its use as an inverse table.

n^3 CUBES

TENTHS OF THE TABULAR DIFFERENCE

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7
1.0	1.000	1.030	1.061	1.093	1.125 −	1.158	1.191	1.225 +	1.260	1.295 +							
.1	1.331	1.368	1.405 −	1.443	1.482	1.521	1.561	1.602	1.643	1.685 +							
.2	1.728	1.772	1.816	1.861	1.907	1.953	**2**.000	2.048	2.097	2.147							
.3	2.197	2.248	2.300	2.353	2.406	2.460	2.515 +	2.571	2.628	2.686							
.4	2.744	2.803	2.863	2.924	2.986	**3**.049	3.112	3.177	3.242	3.308							
1.5	3.375	3.443	3.512	3.582	3.652	3.724	3.796	3.870	3.944	**4**.020							
.6	4.096	4.173	4.252	4.331	4.411	4.492	4.574	4.657	4.742	4.827							
.7	4.913	**5**.000 +	5.088	5.178	5.268	5.359	5.452	5.545 +	5.640	5.735 +							
.8	5.832	5.930	**6**.029	6.128	6.230	6.332	6.435 −	6.539	6.645 −	6.751							
.9	6.859	6.968	**7**.078	7.189	7.301	7.415 −	7.530	7.645 +	7.762	7.881							
2.0	**8**.000	8.121	8.242	8.365 +	8.490	8.615 +	8.742	8.870	8.999	**9**.129							
.1	9.261	9.394	9.528	9.664	9.800	9.938	10.078										
.1							**10**.08	10.22	10.36	10.50 +							
.2	10.65 −	10.79	10.94	11.09	11.24	11.39	11.54	11.70	11.85 +	12.01							
.3	12.17	12.33	12.49	12.65 −	12.81	12.98	13.14	13.31	13.48	13.65 +							
.4	13.82	14.00	14.17	14.35 −	14.53	14.71	14.89	15.07	15.25 +	15.44							
2.5	15.62	15.81	16.00	16.19	16.39	16.58	16.78	16.97	17.17	17.37							
.6	17.58	17.78	17.98	18.19	18.40	18.61	18.82	19.03	19.25 −	19.47							
.7	19.68	19.90	**20**.12	20.35 −	20.57	20.80	21.02	21.25 +	21.48	21.72							
.8	21.95 +	22.19	22.43	22.67	22.91	23.15 −	23.39	23.64	23.89	24.14							
.9	24.39	24.64	24.90	25.15 +	25.41	25.67	25.93	26.20	26.46	26.73							
3.0	27.00	27.27	27.54	27.82	28.09	28.37	28.65 +	28.93	29.22	29.50 +							
.1	29.79	**30**.08	30.37	30.66	30.96	31.26	31.55 +	31.86	32.16	32.46							
.2	32.77	33.08	33.39	33.70	34.01	34.33	34.65 −	34.97	35.29	35.61							
.3	35.94	36.26	36.59	36.93	37.26	37.60	37.93	38.27	38.61	38.96							
.4	39.30	39.65 +	**40**.00	40.35 +	40.71	41.06	41.42	41.78	42.14	42.51							
3.5	42.87	43.24	43.61	43.99	44.36	44.74	45.12	45.50 −	45.88	46.27							
.6	46.66	47.05 −	47.44	47.83	48.23	48.63	49.03	49.43	49.84	**50**.24							
.7	50.65 +	51.06	51.48	51.90	52.31	52.73	53.16	53.58	54.01	54.44							
.8	54.87	55.31	55.74	56.18	56.62	57.07	57.51	57.96	58.41	58.86							
.9	59.32	59.78	**60**.24	60.70	61.16	61.63	62.10	62.57	63.04	63.52							
4.0	64.00	64.48	64.96	65.45 +	65.94	66.43	66.92	67.42	67.92	68.42							
.1	68.92	69.43	69.93	**70**.44	70.96	71.47	71.99	72.51	73.03	73.56							
.2	74.09	74.62	75.15 +	75.69	76.23	76.77	77.31	77.85 +	78.40	78.95 +							
.3	79.51	**80**.06	80.62	81.18	81.75 −	82.31	82.88	83.45 +	84.03	84.60							
.4	85.18	85.77	86.35 +	86.94	87.53	88.12	88.72	89.31	89.92	**90**.52							
4.5	91.12	91.73	92.35 −	92.96	93.58	94.20	94.82	95.44	96.07	96.70							
.6	97.34	97.97	98.61	99.25 +	99.90	100.54											
.6						**100**.5 +	101.2	101.8	102.5 +	103.2	1	1	2	3	3	4	5
.7	103.8	104.5 −	105.2	105.8	106.5 −	107.2	107.9	108.5 +	109.2	109.9	1	1	2	3	3	4	5
.8	110.6	111.3	112.0	112.7	113.4	114.1	114.8	115.5 +	116.2	116.9	1	1	2	3	4	4	5
.9	*117.6*	*118.4*	119.1	119.8	120.6	121.3	122.0	122.8	123.5 +	124.3	1	1	2	3	4	4	5

To avoid interpolation the first thirty-eight lines, special table on page 480.

BES

1	2	3	4	5	6	7	8	9	Tenths of Tabular Diff. 1	2	3	4	5	6
125.8	126.5+	127.3	128.0	128.8	129.6	130.3	131.1	131.9	1	2	2	3	4	5
133.4	134.2	135.0+	135.8	136.6	137.4	138.2	139.0	139.8	1	2	2	3	4	5
141.4	142.2	143.1	143.9	144.7	145.5+	146.4	147.2	148.0	1	2	2	3	4	5
149.7	150.6	151.4	152.3	153.1	154.0	154.9	155.7	156.6	1	2	3	3	4	5
158.3	159.2	160.1	161.0	161.9	162.8	163.7	164.6	165.5−	1	2	3	4	4	5
167.3	168.2	169.1	170.0	171.0	171.9	172.8	173.7	174.7	1	2	3	4	5	6
176.6	177.5+	178.5−	179.4	180.4	181.3	182.3	183.3	184.2	1	2	3	4	5	6
186.2	187.1	188.1	189.1	190.1	191.1	192.1	193.1	194.1	1	2	3	4	5	6
196.1	197.1	198.2	199.2	**200**.2	201.2	202.3	203.3	204.3	1	2	3	4	5	6
206.4	207.5−	208.5+	209.6	210.6	211.7	212.8	213.8	214.9	1	2	3	4	5	6
217.1	218.2	219.3	220.3	221.4	222.5+	223.6	224.8	225.9	1	2	3	4	5	7
228.1	229.2	230.3	231.5−	232.6	233.7	234.9	236.0	237.2	1	2	3	5	6	7
239.5 —	240.6	241.8	243.0	244.1	245.3	246.5−	247.7	248.9	1	2	4	5	6	7
251.2	252.4	253.6	254.8	256.0	257.3	258.5−	259.7	260.9	1	2	4	5	6	7
263.4	264.6	265.8	267.1	268.3	269.6	270.8	272.1	273.4	1	2	4	5	6	7
275.9	277.2	278.4	279.7	281.0	282.3	283.6	284.9	286.2	1	3	4	5	6	8
288.8	290.1	291.4	292.8	294.1	295.4	296.7	298.1	299.4	1	3	4	5	7	8
302.1	303.5−	304.8	306.2	307.5+	308.9	310.3	311.7	313.0	1	3	4	5	7	8
315.8	317.2	318.6	320.0	321.4	322.8	324.2	325.7	327.1	1	3	4	6	7	8
329.9	331.4	332.8	334.3	335.7	337.2	338.6	340.1	341.5+	1	3	4	6	7	9
344.5 —	345.9	347.4	348.9	350.4	351.9	353.4	354.9	356.4	1	3	4	6	7	9
359.4	360.9	362.5−	364.0	365.5+	367.1	368.6	370.1	371.7	2	3	5	6	8	9
374.8	376.4	377.9	379.5+	381.1	382.7	384.2	385.8	387.4	2	3	5	6	8	9
390.6	392.2	393.8	395.4	397.1	398.7	**400**.3	401.9	403.6	2	3	5	6	8	10
406.9	408.5+	410.2	411.8	413.5−	415.2	416.8	418.5+	420.2	2	3	5	7	8	10
423.6	425.3	427.0	428.7	430.4	432.1	433.8	435.5+	437.2	2	3	5	7	9	10
440.7	442.5−	444.2	445.9	447.7	449.5−	451.2	453.0	454.8	2	4	5	7	9	11
458.3	460.1	461.9	463.7	465.5−	467.3	469.1	470.9	472.7	2	4	5	7	9	11
476.4	478.2	480.0	481.9	483.7	485.6	487.4	489.3	491.2	2	4	6	7	9	11
494.9	496.8	498.7	**500**.6	502.5−	504.4	506.3	508.2	510.1	2	4	6	8	9	11
513.9	515.8	517.8	519.7	521.7	523.6	525.6	527.5+	529.5−	2	4	6	8	10	12
533.4	535.4	537.4	539.4	541.3	543.3	545.3	547.3	549.4	2	4	6	8	10	12
553.4	555.4	557.4	559.5−	561.5+	563.6	565.6	567.7	569.7	2	4	6	8	10	12
573.9	575.9	578.0	580.1	582.2	584.3	586.4	588.5−	590.6	2	4	6	8	10	13
594.8	596.9	599.1	**601**.2	603.4	605.5−	607.6	609.8	612.0	2	4	6	9	11	13
616.3	618.5−	620.7	622.8	625.0+	627.2	629.4	631.6	633.8	2	4	7	9	11	13
638.3	640.5+	642.7	645.0−	647.2	649.5−	651.7	654.0	656.2	2	4	7	9	11	13
660.8	663.1	665.3	667.6	669.9	672.2	674.5+	676.8	679.2	2	5	7	9	11	14
683.8	686.1	688.5−	690.8	693.2	695.5+	697.9	**700**.2	702.6	2	5	7	9	12	14
707.3	709.7	712.1	714.5+	716.9	719.3	721.7	724.2	726.6	2	5	7	10	12	14
731.4	733.9	736.3	738.8	741.2	743.7	746.1	748.6	751.1	2	5	7	10	12	15
756.1	758.6	761.0	763.6	766.1	768.6	771.1	773.6	776.2	3	5	8	10	13	15
781.2	783.8	786.3	788.9	791.5−	794.0	796.6	799.2	**801**.8	3	5	8	10	13	15
807.0	809.6	812.2	814.8	817.4	820.0	822.7	825.3	827.9	3	5	8	10	13	16
833.2	835.9	838.6	841.2	843.9	846.6	849.3	852.0	854.7	3	5	8	11	13	16
860.1	862.8	865.5+	868.3	871.0	873.7	876.5−	879.2	882.0	3	5	8	11	14	16
887.5+	890.3	893.1	895.8	898.6	**901**.4	904.2	907.0	909.9	3	6	8	11	14	17
915.5−	918.3	921.2	924.0	926.9	929.7	932.6	935.4	938.3	3	6	9	11	14	17
944.1	947.0	949.9	952.8	955.7	958.6	961.5+	964.4	967.4	3	6	9	12	15	17
973.2	976.2	979.1	982.1	985.1	988.0	991.0	994.0	997.0	3	6	9	12	15	18

SPECIAL TABLE

See main table, pages **474–475**

n^3 **CUBES**

TENTHS OF TABULAR DI[…]

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5
1.00	1.000	1.003	1.006	1.009	1.012	1.015	1.018	1.021	1.024	1.027	0	1	1	1	2
1	1.030	1.033	1.036	1.040	1.043	1.046	1.049	1.052	1.055−	1.058	0	1	1	1	2
2	1.061	1.064	1.067	1.071	1.074	1.077	1.080	1.083	1.086	1.090	0	1	1	1	2
3	1.093	1.096	1.099	1.102	1.106	1.109	1.112	1.115+	1.118	1.122	0	1	1	1	2
4	1.125−	1.128	1.131	1.135−	1.138	1.141	1.144	1.148	1.151	1.154	0	1	1	1	2
1.05	1.158	1.161	1.164	1.168	1.171	1.174	1.178	1.181	1.184	1.188	0	1	1	1	2
6	1.191	1.194	1.198	1.201	1.205−	1.208	1.211	1.215−	1.218	1.222	0	1	1	1	2
7	1.225+	1.228	1.232	1.235+	1.239	1.242	1.246	1.249	1.253	1.256	0	1	1	1	2
8	1.260	1.263	1.267	1.270	1.274	1.277	1.281	1.284	1.288	1.291	0	1	1	1	2
9	1.295+	1.299	1.302	1.306	1.309	1.313	1.317	1.320	1.324	1.327	0	1	1	1	2
1.10	1.331	1.335−	1.338	1.342	1.346	1.349	1.353	1.357	1.360	1.364	0	1	1	1	2
1	1.368	1.371	1.375+	1.379	1.382	1.386	1.390	1.394	1.397	1.401	0	1	1	1	2
2	1.405−	1.409	1.412	1.416	1.420	1.424	1.428	1.431	1.435+	1.439	0	1	1	2	2
3	1.443	1.447	1.451	1.454	1.458	1.462	1.466	1.470	1.474	1.478	0	1	1	2	2
4	1.482	1.485+	1.489	1.493	1.497	1.501	1.505+	1.509	1.513	1.517	0	1	1	2	2
1.15	1.521	1.525−	1.529	1.533	1.537	1.541	1.545−	1.549	1.553	1.557	0	1	1	2	2
6	1.561	1.565−	1.569	1.573	1.577	1.581	1.585+	1.589	1.593	1.598	0	1	1	2	2
7	1.602	1.606	1.610	1.614	1.618	1.622	1.626	1.631	1.635−	1.639	0	1	1	2	2
8	1.643	1.647	1.651	1.656	1.660	1.664	1.668	1.672	1.677	1.681	0	1	1	2	2
9	1.685+	1.689	1.694	1.698	1.702	1.706	1.711	1.715+	1.719	1.724	0	1	1	2	2
1.20	1.728	1.732	1.737	1.741	1.745+	1.750−	1.754	1.758	1.763	1.767	0	1	1	2	2
1	1.772	1.776	1.780	1.785−	1.789	1.794	1.798	1.802	1.807	1.811	0	1	1	2	2
2	1.816	1.820	1.825−	1.829	1.834	1.838	1.843	1.847	1.852	1.856	0	1	1	2	2
3	1.861	1.865+	1.870	1.875−	1.879	1.884	1.888	1.893	1.897	1.902	0	1	1	2	2
4	1.907	1.911	1.916	1.920	1.925+	1.930	1.934	1.939	1.944	1.948	0	1	1	2	2
1.25	1.953	1.958	1.963	1.967	1.972	1.977	1.981	1.986	1.991	1.996	0	1	1	2	2
6	2.000	2.005+	2.010	2.015−	2.019	2.024	2.029	2.034	2.039	2.044	0	1	1	2	2
7	2.048	2.053	2.058	2.063	2.068	2.073	2.078	2.082	2.087	2.092	0	1	1	2	2
8	2.097	2.102	2.107	2.112	2.117	2.122	2.127	2.132	2.137	2.142	0	1	1	2	2
9	2.147	2.152	2.157	2.162	2.167	2.172	2.177	2.182	2.187	2.192	1	1	2	2	3
1.30	2.197	2.202	2.207	2.212	2.217	2.222	2.228	2.233	2.238	2.243	1	1	2	2	3
1	2.248	2.253	2.258	2.264	2.269	2.274	2.279	2.284	2.290	2.295−	1	1	2	2	3
2	2.300	2.305+	2.310	2.316	2.321	2.326	2.331	2.337	2.342	2.347	1	1	2	2	3
3	2.353	2.358	2.363	2.369	2.374	2.379	2.385−	2.390	2.395+	2.401	1	1	2	2	3
4	2.406	2.411	2.417	2.422	2.428	2.433	2.439	2.444	2.449	2.455−	1	1	2	2	3
1.35	2.460	2.466	2.471	2.477	2.482	2.488	2.493	2.499	2.504	2.510	1	1	2	2	3
6	2.515+	2.521	2.527	2.532	2.538	2.543	2.549	2.554	2.560	2.566	1	1	2	2	3
7	2.571	2.577	2.583	2.588	2.594	2.600	2.605+	2.611	2.617	2.622	1	1	2	2	3
8	2.628	3.634	2.640	2.645+	2.651	2.657	2.663	2.668	2.674	2.680	1	1	2	2	3
9	2.686	2.691	2.697	2.703	2.709	2.715−	2.721	2.726	2.732	2.738	1	1	2	2	3
1.40	2.744	2.750−	2.756	2.762	2.768	2.774	2.779	2.785+	2.791	2.797	1	1	2	2	3
1	2.803	2.809	2.815+	2.821	2.827	2.833	2.839	2.845+	2.851	2.857	1	1	2	2	3
2	2.863	2.869	2.875+	2.881	2.888	2.894	2.900	2.906	2.912	2.918	1	1	2	2	3
3	2.924	2.930	2.936	2.943	2.949	2.955−	2.961	2.967	2.974	2.980	1	1	2	2	3
4	2.986	2.992	2.998	3.005−	3.011	3.017	3.023	3.030	3.036	3.042	1	1	2	3	3
1.45	3.049	3.055−	3.061	3.068	3.074	3.080	3.087	3.093	3.099	3.106	1	1	2	3	3
6	3.112	3.119	3.125−	3.131	3.138	3.144	3.151	3.157	3.164	3.170	1	1	2	3	3
7	3.177	3.183	3.190	3.196	3.203	3.209	3.216	3.222	3.229	3.235+	1	1	2	3	3
8	3.242	3.248	3.255−	3.262	3.268	3.275−	3.281	3.288	3.295−	3.301	1	1	2	3	3
9	3.308	3.315−	3.321	3.328	3.335−	3.341	3.348	3.355−	3.362	3.368	1	1	2	3	3
1.50	3.375	3.382	3.389	3.395+	3.402	3.409	3.416	3.422	3.429	3.436	1	1	2	3	3
1	3.443	3.450−	3.457	3.464	3.470	3.477	3.484	3.491	3.498	3.505−	1	1	2	3	3
2	3.512	3.519	3.526	3.533	3.540	3.547	3.554	3.561	3.567	3.575−	1	1	2	3	4
3	3.582	3.589	3.596	3.603	3.610	3.617	3.624	3.631	3.638	3.645+	1	1	2	3	4
4	3.652	3.659	3.667	3.674	3.681	3.688	3.695+	3.702	3.709	3.717	1	1	2	3	4
1.55	3.724	3.731	3.738	3.746	3.753	3.760	3.767	3.775−	3.782	3.789	1	1	2	3	4
6	3.796	3.804	3.811	3.818	3.826	3.833	3.840	3.848	3.855+	3.863	1	1	2	3	4
7	3.870	3.877	3.885−	3.892	3.900	3.907	3.914	3.922	3.929	3.937	1	1	2	3	4
8	3.944	3.952	3.959	3.967	3.974	3.982	3.989	3.997	4.005−	4.012	1	2	2	3	4
9	4.020	4.027	4.035−	4.042	4.050+	4.058	4.065+	4.073	4.081	4.088	1	2	2	3	4
1.60	4.096	4.104	4.111	4.119	4.127	4.135−	4.142	4.150−	4.158	4.166	1	2	2	3	4
1	4.173	4.181	4.189	4.197	4.204	4.212	4.220	4.228	4.236	4.244	1	2	2	3	4
2	4.252	4.259	4.267	4.275+	4.283	4.291	4.299	4.307	4.315−	4.323	1	2	2	3	4
3	4.331	4.339	4.347	4.355−	4.363	4.371	4.379	4.387	4.395−	4.403	1	2	2	3	4
4	4.411	4.419	4.427	4.435+	4.443	4.451	4.460	4.468	4.476	4.484	1	2	2	3	4
1.65	4.492	4.500+	4.508	4.517	4.525−	4.533	4.541	4.550−	4.558	4.566	1	2	2	3	4
6	4.574	4.583	4.591	4.599	4.607	4.616	4.624	4.632	4.641	4.649	1	2	2	3	4
7	4.657	4.666	4.674	4.683	4.691	4.699	4.708	4.716	4.725−	4.733	1	2	3	3	4
8	4.742	4.750+	4.759	4.767	4.776	4.784	4.793	4.801	4.810	4.818	1	2	3	3	4
9	4.827	4.835+	4.844	4.853	4.861	4.870	4.878	4.887	4.896	4.904	1	2	3	3	4

)	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
3	4.922	4.930	4.939	4.948	4.956	4.965+	4.974	4.983	4.991	1	2	3	3	4	5	6	7	8
0+	5.009	5.018	5.027	5.035+	5.044	5.053	5.062	5.071	5.080	1	2	3	4	4	5	6	7	8
8	5.097	5.106	5.115+	5.124	5.133	5.142	5.151	5.160	5.169	1	2	3	4	4	5	6	7	8
8	5.187	5.196	5.205−	5.214	5.223	5.232	5.241	5.250−	5.259	1	2	3	4	5	5	6	7	8
8	5.277	5.286	5.295+	5.304	5.314	5.323	5.332	5.341	5.350+	1	2	3	4	5	5	6	7	8
9	5.369	5.378	5.387	5.396	5.405+	5.415−	5.424	5.433	5.442	1	2	3	4	5	6	6	7	8
2	5.461	5.470	5.480	5.489	5.498	5.508	5.517	5.526	5.536	1	2	3	4	5	6	7	7	8
5+	5.555−	5.564	5.573	5.583	5.592	5.602	5.611	5.621	5.630	1	2	3	4	5	6	7	8	9
)	5.649	5.659	5.668	5.678	5.687	5.697	5.707	5.716	5.726	1	2	3	4	5	6	7	8	9
5+	5.745−	5.755−	5.764	5.774	5.784	5.793	5.803	5.813	5.822	1	2	3	4	5	6	7	8	9
	5.842	5.851	5.861	5.871	5.881	5.891	5.900	5.910	5.920	1	2	3	4	5	6	7	8	9
	5.940	5.949	5.959	5.969	5.979	5.989	5.999	6.009	6.019	1	2	3	4	5	6	7	8	9
	6.039	6.048	6.058	6.068	6.078	6.088	6.098	6.108	6.118	1	2	3	4	5	6	7	8	9
	6.139	6.149	6.159	6.169	6.179	6.189	6.199	6.209	6.219	1	2	3	4	5	6	7	8	9
	6.240	6.250−	6.260	6.270	6.280	6.291	6.301	6.311	6.321	1	2	3	4	5	6	7	8	9
	6.342	6.352	6.362	6.373	6.383	6.393	6.404	6.414	6.424	1	2	3	4	5	6	7	8	9
−	6.445+	6.456	6.466	6.476	6.487	6.497	6.508	6.518	6.529	1	2	3	4	5	6	7	8	9
	6.550−	6.560	6.571	6.581	6.592	6.602	6.613	6.623	6.634	1	2	3	4	5	6	7	8	9
−	6.655+	6.666	6.677	6.687	6.698	6.708	6.719	6.730	6.741	1	2	3	4	5	6	7	9	10
	6.762	6.773	6.783	6.794	6.805−	6.816	6.827	6.837	6.848	1	2	3	4	5	6	8	9	10
	6.870	6.881	6.892	6.902	6.913	6.924	6.935+	6.946	6.957	1	2	3	4	5	7	8	9	10
	6.979	6.990	7.001	7.012	7.023	7.034	7.045−	7.056	7.067	1	2	3	4	6	7	8	9	10
	7.089	7.100	7.111	7.122	7.133	7.144	7.156	7.167	7.178	1	2	3	4	6	7	8	9	10
	7.200	7.211	7.223	7.234	7.245+	7.256	7.268	7.279	7.290	1	2	3	4	6	7	8	9	10
	7.313	7.324	7.335+	7.347	7.358	7.369	7.381	7.392	7.403	1	2	3	5	6	7	8	9	10
−	7.426	7.438	7.449	7.461	7.472	7.484	7.495+	7.507	7.518	1	2	3	5	6	7	8	9	10
	7.541	7.553	7.564	7.576	7.587	7.599	7.610	7.622	7.634	1	2	3	5	6	7	8	9	10
+	7.657	7.669	7.680	7.692	7.704	7.715+	7.727	7.739	7.751	1	2	4	5	6	7	8	9	11
	7.774	7.786	7.798	7.810	7.821	7.833	7.845+	7.857	7.869	1	2	4	5	6	7	8	9	11
	7.892	7.904	7.916	7.928	7.940	7.952	7.964	7.976	7.988	1	2	4	5	6	7	8	10	11
)	8.012	8.024	8.036	8.048	8.060	8.072	8.084	8.096	8.108	1	2	4	5	6	7	8	10	11
	8.133	8.145−	8.157	8.169	8.181	8.194	8.206	8.218	8.230	1	2	4	5	6	7	9	10	11
	8.255−	8.267	8.279	8.291	8.304	8.316	8.328	8.341	8.353	1	2	4	5	6	7	9	10	11
5+	8.378	8.390	8.403	8.415−	8.427	8.440	8.452	8.465−	8.477	1	2	4	5	6	7	9	10	11
)	8.502	8.515−	8.527	8.540	8.552	8.565−	8.577	8.590	8.603	1	3	4	5	6	8	9	10	11
5+	8.628	8.640	8.653	8.666	8.678	8.691	8.704	8.716	8.729	1	3	4	5	6	8	9	10	11
	8.755−	8.767	8.780	8.793	8.806	8.818	8.831	8.844	8.857	1	3	4	5	6	8	9	10	12
)	8.883	8.895+	8.908	8.921	8.934	8.947	8.960	8.973	8.986	1	3	4	5	6	8	9	10	12
)	9.012	9.025−	9.038	9.051	9.064	9.077	9.090	9.103	9.116	1	3	4	5	7	8	9	10	12
	9.142	9.156	9.169	9.182	9.195+	9.208	9.221	9.235−	9.248	1	3	4	5	7	8	9	11	12
	9.274	9.287	9.301	9.314	9.327	9.341	9.354	9.367	9.381	1	3	4	5	7	8	9	11	12
	9.407	9.421	9.434	9.447	9.461	9.474	9.488	9.501	9.515−	1	3	4	5	7	8	9	11	12
	9.542	9.555+	9.569	9.582	9.596	9.609	9.623	9.636	9.650−	1	3	4	5	7	8	9	11	12
	9.677	9.691	9.704	9.718	9.732	9.745+	9.759	9.773	9.787	1	3	4	5	7	8	10	11	12
)	9.814	9.828	9.842	9.855+	9.869	9.883	9.897	9.911	9.925−	1	3	4	6	7	8	10	11	12
	9.952	9.966	9.980	9.994	**10**.008					1	3	4	6	7	8	10	11	13
					10.01	10.02	10.04	10.05−	10.06	0	0	0	1	1	1	1	1	1
	10.09	10.11	10.12	10.13	10.15−	10.16	10.18	10.19	10.20	0	0	0	1	1	1	1	1	1
	10.23	10.25−	10.26	10.27	10.29	10.30	10.32	10.33	10.35−	0	0	0	1	1	1	1	1	1
	10.37	10.39	10.40	10.42	10.43	10.45−	10.46	10.47	10.49	0	0	0	1	1	1	1	1	1
)+	10.52	10.53	10.55−	10.56	10.58	10.59	10.60	10.62	10.63	0	0	0	1	1	1	1	1	1
−	10.66	10.68	10.69	10.71	10.72	10.74	10.75−	10.76	10.78	0	0	0	1	1	1	1	1	1
	10.81	10.82	10.84	10.85+	10.87	10.88	10.90	10.91	10.93	0	0	0	1	1	1	1	1	1
	10.96	10.97	10.99	11.00	11.02	11.03	11.04	11.06	11.07	0	0	0	1	1	1	1	1	1
	11.10	11.12	11.13	11.15−	11.16	11.18	11.19	11.21	11.22	0	0	0	1	1	1	1	1	1
	11.25+	11.27	11.28	11.30	11.31	11.33	11.35−	11.36	11.38	0	0	0	1	1	1	1	1	1
	11.41	11.42	11.44	11.45+	11.47	11.48	11.50−	11.51	11.53	0	0	0	1	1	1	1	1	1
	11.56	11.57	11.59	11.60	11.62	11.64	11.65+	11.67	11.68	0	0	0	1	1	1	1	1	1
)	11.71	11.73	11.74	11.76	11.77	11.79	11.81	11.82	11.84	0	0	0	1	1	1	1	1	1
5+	11.87	11.88	11.90	11.91	11.93	11.95−	11.96	11.98	11.99	0	0	0	1	1	1	1	1	1
	12.02	12.04	12.06	12.07	12.09	12.10	12.12	12.14	12.15−	0	0	0	1	1	1	1	1	1
	12.18	12.20	12.21	12.23	12.25−	12.26	12.28	12.29	12.31	0	0	0	1	1	1	1	1	1
	12.34	12.36	12.37	12.39	12.41	12.42	12.44	12.45+	12.47	0	0	0	1	1	1	1	1	1
)	12.50+	12.52	12.54	12.55+	12.57	12.58	12.60	12.62	12.63	0	0	0	1	1	1	1	1	1
5−	12.67	12.68	12.70	12.71	12.73	12.75−	12.76	12.78	12.80	0	0	0	1	1	1	1	1	1
	12.83	12.85−	12.86	12.88	12.90	12.91	12.93	12.94	12.96	0	0	0	1	1	1	1	1	1
	12.99	13.01	13.03	13.04	13.06	13.08	13.09	13.11	13.13	0	0	0	1	1	1	1	1	1
	13.16	13.18	13.19	13.21	13.23	13.24	13.26	13.28	13.30	0	0	1	1	1	1	1	1	2
	13.33	13.35−	13.36	13.38	13.40	13.41	13.43	13.45−	13.46	0	0	1	1	1	1	1	1	2
	13.50−	13.52	13.53	13.55−	13.57	13.58	13.60	13.62	13.63	0	0	1	1	1	1	1	1	2
5+	13.67	13.69	13.70	13.72	13.74	13.75+	13.77	13.79	13.81	0	0	1	1	1	1	1	1	2
2	13.84	13.86	13.88	*13.89*	*13.91*	13.93	13.95−	13.96	13.98	0	0	1	1	1	1	1	1	2
0	14.01	*14.03*	*14.05−*	*14.07*	14.08	14.10	14.12	14.14	14.15+	0	0	1	1	1	1	1	1	1
7	*14.19*	*14.21*	*14.23*	*14.24*	*14.26*	14.28	14.30	14.31	14.33	0	0	1	1	1	1	1	1	1
−	*14.37*	*14.38*	*14.40*	*14.42*	*14.44*	14.46	14.47	14.49	14.51	0	0	1	1	1	1	1	1	1
	14.54	*14.56*	*14.58*	*14.60*	*14.62*	14.63	14.65+	14.67	14.69	0	0	1	1	1	1	1	1	1

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6
2.45	14.71	14.72	14.74	14.76	14.78	14.80	14.81	14.83	14.85+	14.87	0	0	1	1	1	1
6	14.89	14.91	14.92	14.94	14.96	14.98	15.00	15.01	15.03	15.05+	0	0	1	1	1	1
7	15.07	15.09	15.11	15.12	15.14	15.16	15.18	15.20	15.22	15.23	0	0	1	1	1	1
8	15.25+	15.27	15.29	15.31	15.33	15.35−	15.36	15.38	15.40	15.42	0	0	1	1	1	1
9	15.44	15.46	15.48	15.49	15.51	15.53	15.55+	15.57	15.59	15.61	0	0	1	1	1	1
2.50	15.62	15.64	15.66	15.68	15.70	15.72	15.74	15.76	15.78	15.79	0	0	1	1	1	1
1	15.81	15.83	15.85+	15.87	15.89	15.91	15.93	15.95−	15.96	15.98	0	0	1	1	1	1
2	16.00	16.02	16.04	16.06	16.08	16.10	16.12	16.14	16.16	16.18	0	0	1	1	1	1
3	16.19	16.21	16.23	16.25+	16.27	16.29	16.31	16.33	16.35−	16.37	0	0	1	1	1	1
4	16.39	16.41	16.43	16.45−	16.46	16.48	16.50+	16.52	16.54	16.56	0	0	1	1	1	1
2.55	16.58	16.60	16.62	16.64	16.66	16.68	16.70	16.72	16.74	16.76	0	0	1	1	1	1
6	16.78	16.80	16.82	16.84	16.86	16.88	16.90	16.92	16.93	16.95+	0	0	1	1	1	1
7	16.97	16.99	17.01	17.03	17.05+	17.07	17.09	17.11	17.13	17.15+	0	0	1	1	1	1
8	17.17	17.19	17.21	17.23	17.25+	17.27	17.29	17.31	17.33	17.35+	0	0	1	1	1	1
9	17.37	17.39	17.41	17.43	17.45+	17.47	17.50−	17.52	17.54	17. 6	0	0	1	1	1	1
2.60	17.58	17.60	17.62	17.64	17.66	17.68	17.70	17.72	17.74	17.76	0	0	1	1	1	1
1	17.78	17.80	17.82	17.84	17.86	17.88	17.90	17.92	17.94	17.96	0	0	1	1	1	1
2	17.98	18.01	18.03	18.05−	18.07	18.09	18.11	18.13	18.15−	18.17	0	0	1	1	1	1
3	18.19	18.21	18.23	18.25+	18.27	18.30	18.32	18.34	18.36	18.38	0	0	1	1	1	1
4	18.40	18.42	18.44	18.46	18.48	18.50+	18.53	18.55−	18.57	18.59	0	0	1	1	1	1
2.65	18.61	18.63	18.65+	18.67	18.69	18.72	18.74	18.76	18.78	18.80	0	0	1	1	1	1
6	18.82	18.84	18.86	18.88	18.91	18.93	18.95−	18.97	18.99	19.01	0	0	1	1	1	1
7	19.03	19.06	19.08	19.10	19.12	19.14	19.16	19.18	19.21	19.23	0	0	1	1	1	1
8	19.25−	19.27	19.29	19.31	19.34	19.36	19.38	19.40	19.42	19.44	0	0	1	1	1	1
9	19.47	19.49	19.51	19.53	19.55+	19.57	19.60	19.62	19.64	19.66	0	0	1	1	1	1
2.70	19.68	19.70	19.73	19.75−	19.77	19.79	19.81	19.84	19.86	19.88	0	0	1	1	1	1
1	19.90	19.92	19.95−	19.97	19.99	**20**.01	20.03	20.06	20.08	20.10	0	0	1	1	1	1
2	20.12	20.15−	20.17	20.19	20.21	20.23	20.26	20.28	20.30	20.32	0	0	1	1	1	1
3	20.35−	20.37	20.39	20.41	20.44	20.46	20.48	20.50+	20.53	20.55−	0	0	1	1	1	1
4	20.57	20.59	20.62	20.64	20.66	20.68	20.71	20.73	20.75+	20.77	0	0	1	1	1	1
2.75	20.80	20.82	20.84	20.87	20.89	20.91	20.93	20.96	20.98	21.00	0	0	1	1	1	1
6	21.02	21.05−	21.07	21.09	21.12	21.14	21.16	21.18	21.21	21.23	0	0	1	1	1	1
7	21.25+	21.28	21.30	21.32	21.35−	21.37	21.39	21.42	21.44	21.46	0	0	1	1	1	1
8	21.48	21.51	21.53	21.55+	21.58	21.60	21.62	21.65−	21.67	21.69	0	0	1	1	1	1
9	21.72	21.74	21.76	21.79	21.81	21.83	21.86	21.88	21.90	21.93	0	0	1	1	1	1
2.80	21.95+	21.98	22.00	22.02	22.05−	22.07	22.09	22.12	22.14	22.16	0	0	1	1	1	1
1	22.19	22.21	22.24	22.26	22.28	22.31	22.33	22.35+	22.38	22.40	0	0	1	1	1	1
2	22.43	22.45−	22.47	22.50−	22.52	22.55−	22.57	22.59	22.62	22.64	0	0	1	1	1	1
3	22.67	22.69	22.71	22.74	22.76	22.79	22.81	22.83	22.86	22.88	0	0	1	1	1	1
4	22.91	22.93	22.95+	22.98	23.00	23.03	23.05+	23.08	23.10	23.12	0	0	1	1	1	1
2.85	23.15−	23.17	23.20	23.22	23.25−	23.27	23.30	23.32	23.34	23.37	0	0	1	1	1	1
6	23.39	23.42	23.44	23.47	23.49	23.52	23.54	23.57	23.59	23.62	0	0	1	1	1	1
7	23.64	23.66	23.69	23.71	23.74	23.76	23.79	23.81	23.84	23.86	0	0	1	1	1	1
8	23.89	23.91	23.94	23.96	23.99	24.01	24.04	24.06	24.09	24.11	0	0	1	1	1	1
9	24.14	24.16	24.19	24.21	24.24	24.26	24.29	24.31	24.34	24.36	0	1	1	1	1	2
2.90	24.39	24.41	24.44	24.46	24.49	24.52	24.54	24.57	24.59	24.62	0	1	1	1	1	2
1	24.64	24.67	24.69	24.72	24.74	24.77	24.79	24.82	24.85−	24.87	0	1	1	1	1	2
2	24.90	24.92	24.95−	24.97	25.00	25.03	25.05+	25.08	25.10	25.13	0	1	1	1	1	2
3	25.15+	25.18	25.21	25.23	25.26	25.28	25.31	25.33	25.36	25.39	0	1	1	1	1	2
4	25.41	25.44	25.46	25.49	25.52	25.54	25.57	25.59	25.62	25.65−	0	1	1	1	1	2
2.95	25.67	25.70	25.72	25.75+	25.78	25.80	25.83	25.86	25.88	25.91	0	1	1	1	1	2
6	25.93	25.96	25.99	26.01	26.04	26.07	26.09	26.12	26.15−	26.17	0	1	1	1	1	2
7	26.20	26.22	26.25+	26.28	26.30	26.33	26.36	26.38	26.41	26.44	0	1	1	1	1	2
8	26.46	26.49	26.52	26.54	26.57	26.60	26.62	26.65+	26.68	26.70	0	1	1	1	1	2
9	26.73	26.76	26.78	26.81	26.84	26.87	26.89	26.92	26.95−	26.97	0	1	1	1	1	2
3.00	27.00	27.03	27.05+	27.08	27.11	27.14	27.16	27.19	27.22	27.24	0	1	1	1	1	2
1	27.27	27.30	27.33	27.35+	27.38	27.41	27.43	27.46	27.49	27.52	0	1	1	1	1	2
2	27.54	27.57	27.60	27.63	27.65+	27.68	27.71	27.74	27.76	27.79	0	1	1	1	1	2
3	27.82	27.85−	27.87	27.90	27.93	27.96	27.98	28.01	28.04	28.07	0	1	1	1	1	2
4	28.09	28.12	28.15−	28.18	28.21	28.23	28.26	28.29	28.32	28.34	0	1	1	1	1	2
3.05	28.37	28.40	28.43	28.46	28.48	28.51	28.54	28.57	28.60	28.62	0	1	1	1	1	2
6	28.65+	28.68	28.71	28.74	28.77	28.79	28.82	28.85−	28.88	28.91	0	1	1	1	1	2
7	28.93	28.96	28.99	29.02	29.05−	29.08	29.10	29.13	29.16	29.19	0	1	1	1	1	2
8	29.22	29.25−	29.28	29.30	29.33	29.36	29.39	29.42	29.45−	29.47	0	1	1	1	1	2
9	29.50+	29.53	29.56	29.59	29.62	29.65−	29.68	29.70	29.73	29.76	0	1	1	1	1	2
3.10	29.79	29.82	29.85−	29.88	29.91	29.94	29.96	29.99	**30**.02	30.05+	0	1	1	1	1	2
1	30.08	30.11	30.14	30.17	30.20	30.23	30.25+	30.28	30.31	30.34	0	1	1	1	1	2
2	30.37	30.40	30.43	30.46	30.49	30.52	30.55−	30.58	30.61	30.63	0	1	1	1	1	2
3	30.66	30.69	30.72	30.75+	30.78	30.81	30.84	30.87	30.90	30.93	0	1	1	1	1	2
4	30.96	30.99	31.02	31.05−	31.08	31.11	31.14	31.17	31.20	31.23	0	1	1	1	1	2
3.15	31.26	31.29	31.32	31.35−	31.38	31.40	31.43	31.46	31.49	31.52	0	1	1	1	1	2
6	31.55+	31.58	31.61	31.64	31.67	31.70	31.73	31.76	31.79	31.82	0	1	1	1	1	2
7	31.85	31.88	31.92	31.95−	31.98	32.01	32.04	32.07	32.10	32.13	0	1	1	1	1	2
8	32.16	[illegible]	[illegible]	32.25−	32.28	32.31	32.34	32.37	32.40	32.43	0	1	1	1	1	2
9	32.46	[illegible]	[illegible]	32.55+	32.58	32.61	32.65−	32.68	32.71	32.74	0	1	1	1	1	2

TENTHS OF THE TABULAR DIFFERENCE

1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
32.80	32.83	32.86	32.89	32.92	32.95+	32.98	33.01	33.05−	0	1	1	1	2	2	2	2	3
33.11	33.14	33.17	33.20	33.23	33.26	33.29	33.32	33.36	0	1	1	1	2	2	2	2	3
33.42	33.45−	33.48	33.51	33.54	33.57	33.60	33.64	33.67	0	1	1	1	2	2	2	2	3
33.73	33.76	33.79	33.82	33.86	33.89	33.92	33.95−	33.98	0	1	1	1	2	2	2	3	3
34.04	34.08	34.11	34.14	34.17	34.20	34.23	34.26	34.30	0	1	1	1	2	2	2	3	3
34.36	34.39	34.42	34.46	34.49	34.52	34.55+	34.58	34.61	0	1	1	1	2	2	2	3	3
34.68	34.71	34.74	34.77	34.81	34.84	34.87	34.90	34.93	0	1	1	1	2	2	2	3	3
35.00	35.03	35.06	35.09	35.13	35.16	35.19	35.22	35.26	0	1	1	1	2	2	2	3	3
35.32	35.35+	35.38	35.42	35.45−	35.48	35.51	35.55−	35.58	0		1	1	2	2	2	3	3
35.64	35.68	35.71	35.74	35.77	35.81	35.84	35.87	35.90	0	1	1	1	2	2	2	3	3
35.97	36.00	36.04	36.07	36.10	36.13	36.17	36.20	36.23	0	1	1	1	2	2	2	3	3
36.30	36.33	36.36	36.40	36.43	36.46	36.50−	36.53	36.56	0	1	1	1	2	2	2	3	3
36.63	36.66	36.69	36.73	36.76	36.79	36.83	36.86	36.89	0	1	1	1	2	2	2	3	3
36.96	36.99	37.03	37.06	37.09	37.13	37.16	37.19	37.23	0	1	1	1	2	2	2	3	3
37.29	37.33	37.36	37.39	37.43	37.46	37.49	37.53	37.56	0	1	1	1	2	2	2	3	3
37.63	37.66	37.70	37.73	37.76	37.80	37.83	37.87	37.90	0	1	1	1	2	2	2	3	3
37.97	38.00	38.03	38.07	38.10	38.14	38.17	38.20	38.24	0	1	1	1	2	2	2	3	3
38.31	38.34	38.38	38.41	38.44	38.48	38.51	38.55−	38.58	0	1	1	1	2	2	2	3	3
38.65−	38.68	38.72	38.75+	38.79	38.82	38.85+	38.89	38.92	0	1	1	1	2	2	2	3	3
38.99	39.03	39.06	39.10	39.13	39.17	39.20	39.23	39.27	0	1	1	1	2	2	2	3	3
39.34	39.37	39.41	39.44	39.48	39.51	39.55−	39.58	39.62	0	1	1	1	2	2	2	3	3
39.69	39.72	39.76	39.79	39.83	39.86	39.90	39.93	39.97	0	1	1	1	2	2	2	3	3
40.04	40.07	40.11	40.14	40.18	40.21	40.25−	40.28	40.32	0	1	1	1	2	2	2	3	3
40.39	40.42	40.46	40.49	40.53	40.57	40.60	40.64	40.67	0	1	1	1	2	2	2	3	3
40.74	40.78	40.81	40.85−	40.89	40.92	40.96	40.99	41.03	0	1	1	1	2	2	2	3	3
41.10	41.14	41.17	41.21	41.24	41.28	41.31	41.35−	41.39	0	1	1	1	2	2	3	3	3
41.46	41.49	41.53	41.57	41.60	41.64	41.67	41.71	41.75−	0	1	1	1	2	2	3	3	3
41.82	41.85+	41.89	41.93	41.96	42.00	42.04	42.07	42.11	0	1	1	1	2	2	3	3	3
42.18	42.22	42.25+	42.29	42.33	42.36	42.40	42.44	42.47	0	1	1	1	2	2	3	3	3
42.55−	42.58	42.62	42.65+	42.69	42.73	42.76	42.80	42.84	0	1	1	1	2	2	3	3	3
42.91	42.95−	42.99	43.02	43.06	43.10	43.13	43.17	43.21	0	1	1	1	2	2	3	3	3
43.28	43.32	43.35+	43.39	43.43	43.47	43.50+	43.54	43.58	0	1	1	1	2	2	3	3	3
43.65+	43.69	43.73	43.76	43.80	43.84	43.87	43.91	43.95−	0	1	1	1	2	2	3	3	3
44.02	44.06	44.10	44.14	44.17	44.21	44.25−	44.29	44.32	0	1	1	1	2	2	3	3	3
44.40	44.44	44.47	44.51	44.55+	44.59	44.63	44.66	44.70	0	1	1	2	2	2	3	3	3
44.78	44.81	44.85+	44.89	44.93	44.97	45.00	45.04	45.08	0	1	1	2	2	2	3	3	3
45.16	45.19	45.23	45.27	45.31	45.35−	45.38	45.42	45.46	0	1	1	2	2	2	3	3	3
45.54	45.58	45.61	45.65+	45.69	45.73	45.77	45.81	45.84	0	1	1	2	2	2	3	3	3
45.92	45.96	46.00	46.04	46.08	46.11	46.15+	46.19	46.23	0	1	1	2	2	2	3	3	3
46.31	46.35−	46.38	46.42	46.46	46.50+	46.54	46.58	46.62	0	1	1	2	2	2	3	3	3
46.69	46.73	46.77	46.81	46.85+	46.89	46.93	46.97	47.01	0	1	1	2	2	2	3	3	4
47.08	47.12	47.16	47.20	47.24	47.28	47.32	47.36	47.40	0	1	1	2	2	2	3	3	4
47.48	47.52	47.56	47.60	47.63	47.67	47.71	47.75+	47.79	0	1	1	2	2	2	3	3	4
47.87	47.91	47.95+	47.99	48.03	48.07	48.11	48.15−	48.19	0	1	1	2	2	2	3	3	4
48.27	48.31	48.35−	48.39	48.43	48.47	48.51	48.55−	48.59	0	1	1	2	2	2	3	3	4
48.67	48.71	48.75−	48.79	48.83	48.87	48.91	48.95−	48.99	0	1	1	2	2	2	3	3	4
9.07	49.11	49.15−	49.19	49.23	49.27	49.31	49.35+	49.39	0	1	1	2	2	2	3	3	4
9.47	49.51	49.55+	49.59	49.63	49.67	49.71	49.75+	49.80	0	1	1	2	2	2	3	3	4
9.88	49.92	49.96	50.00	50.04	50.08	50.12	50.16	50.20	0	1	1	2	2	2	3	3	4
0.28	50.33	50.37	50.41	50.45−	50.49	50.53	50.57	50.61	0	1	1	2	2	2	3	3	4
0.69	50.74	50.78	50.82	50.86	50.90	50.94	50.98	51.02	0	1	1	2	2	2	3	3	4
1.11	51.15−	51.19	51.23	51.27	51.31	51.35+	51.40	51.44	0	1	1	2	2	2	3	3	4
1.52	51.56	51.60	51.65−	51.69	51.73	51.77	51.81	51.85+	0	1	1	2	2	2	3	3	4
1.94	51.98	52.02	52.06	52.10	52.15−	52.19	52.23	52.27	0	1	1	2	2	3	3	3	4
2.36	52.40	52.44	52.48	52.52	52.57	52.61	52.65+	52.69	0	1	1	2	2	3	3	3	4
2.78	52.82	52.86	52.90	52.95−	52.99	53.03	53.07	53.11	0	1	1	2	2	3	3	3	4
3.20	53.24	53.28	53.33	53.37	53.41	53.45+	53.50−	53.54	0	1	1	2	2	3	3	3	4
3.63	53.67	53.71	53.75+	53.80	53.84	53.88	53.92	53.97	0	1	1	2	2	3	3	3	4
4.05+	54.10	54.14	54.18	54.22	54.27	54.31	54.35+	54.40	0	1	1	2	2	3	3	3	4
4.48	54.53	54.57	54.61	54.66	54.70	54.74	54.79	54.83	0	1	1	2	2	3	3	3	4
4.92	54.96	55.00	55.05−	55.09	55.13	55.18	55.22	55.26	0	1	1	2	2	3	3	3	4
5.35−	55.39	55.44	55.48	55.52	55.57	55.61	55.66	55.70	0	1	1	2	2	3	3	3	4
5.79	55.83	55.87	55.92	55.96	56.01	56.05−	56.09	56.14	0	1	1	2	2	3	3	4	4
6.23	56.27	56.31	56.36	56.40	56.45−	56.49	56.53	56.58	0	1	1	2	2	3	3	4	4
6.67	56.71	56.76	56.80	56.84	56.89	56.93	56.98	57.02	0	1	1	2	2	3	3	4	4
7.11	57.16	57.20	57.24	57.29	57.33	57.38	57.42	57.47	0	1	1	2	2	3	3	4	4
7.56	57.60	57.65−	57.69	57.74	57.78	57.83	57.87	57.92	0	1	1	2	2	3	3	4	4
8.01	58.05+	58.10	58.14	58.19	58.23	58.28	58.32	58.37	0	1	1	2	2	3	3	4	4
8.46	58.50+	58.55−	58.59	58.64	58.68	58.73	58.77	58.82	0	1	1	2	2	3	3	4	4
8.91	58.95+	59.00	59.05−	59.09	59.14	59.18	59.23	59.27	0	1	1	2	2	3	3	4	4
9.36	59.41	59.46	59.50+	59.55−	59.59	59.64	59.68	50.73	0	1	1	2	2	3	3	4	4
9.82	59.87	59.91	59.96	60.01	60.05+	60.10	60.14	60.19	0	1	1	2	2	3	3	4	4
0.28	60.33	60.37	60.42	60.47	60.51	60.56	60.61	60.65+	0	1	1	2	2	3	3	4	4
0.74	60.79	60.84	60.88	60.93	60.98	61.02	61.07	61.12	0	1	1	2	2	3	3	4	4
1.21	61.26	61.30	61.35−	61.40	61.44	61.49	61.54	61.58	0	1	1	2	2	3	3	4	4

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5
3.95	61.63	61.68	61.72	61.77	61.82	61.86	61.91	61.96	62.01	62.05+	0	1	1	2	2
6	62.10	62.15−	62.19	62.24	62.29	62.33	62.38	62.43	62.48	62.52	0	1	1	2	2
7	62.57	62.62	62.67	62.71	62.76	62.81	62.85+	62.90	62.95−	63.00	0	1	1	2	2
8	63.04	63.09	63.14	63.19	63.24	63.28	63.33	63.38	63.43	63.47	0	1	1	2	2
9	63.52	63.57	63.62	63.66	63.71	63.76	63.81	63.86	63.90	63.95+	0	1	1	2	2
4.00	64.00	64.05−	64.10	64.14	64.19	64.24	64.29	64.34	64.38	64.43	0	1	1	2	2
1	64.48	64.53	64.58	64.63	64.67	64.72	64.77	64.82	64.87	64.92	0	1	1	2	2
2	64.96	65.01	65.06	65.11	65.16	65.21	65.26	65.30	65.35+	65.40	0	1	1	2	2
3	65.45+	65.50−	65.55−	65.60	65.65−	65.69	65.74	65.79	65.84	65.89	0	1	1	2	2
4	65.94	65.99	66.04	66.09	66.14	66.18	66.23	66.28	66.33	66.38	0	1	1	2	2
4.05	66.43	66.48	66.53	66.58	66.63	66.68	66.73	66.78	66.82	66.87	0	1	1	2	2
6	66.92	66.97	67.02	67.07	67.12	67.17	67.22	67.27	67.32	67.37	0	1	1	2	2
7	67.42	67.47	67.52	67.57	67.62	67.67	67.72	67.77	67.82	67.87	0	1	1	2	2
8	67.92	67.97	68.02	68.07	68.12	68.17	68.22	68.27	68.32	68.37	1	1	2	2	3
9	68.42	68.47	68.52	68.57	68.62	68.67	68.72	68.77	68.82	68.87	1	1	2	2	3
4.10	68.92	68.97	69.02	69.07	69.12	69.17	69.22	69.27	69.33	69.38	1	1	2	2	3
1	69.43	69.48	69.53	69.58	69.63	69.68	69.73	69.78	69.83	69.88	1	1	2	2	3
2	69.93	69.99	**70**.04	70.09	70.14	70.19	70.24	70.29	70.34	70.39	1	1	2	2	3
3	70.44	70.50−	70.55−	70.60	70.65−	70.70	70.75+	70.80	70.86	70.91	1	1	2	2	3
4	70.96	71.01	71.06	71.11	71.16	71.22	71.27	71.32	71.37	71.42	1	1	2	2	3
4.15	71.47	71.53	71.58	71.63	71.68	71.73	71.78	71.84	71.89	71.94	1	1	2	2	3
6	71.99	72.04	72.10	72.15−	72.20	72.25+	72.30	72.36	72.41	72.46	1	1	2	2	3
7	72.51	72.56	72.62	72.67	72.72	72.77	72.83	72.88	72.93	72.98	1	1	2	2	3
8	73.03	73.09	73.14	73.19	73.24	73.30	73.35−	73.40	73.45+	73.51	1	1	2	2	3
9	73.56	73.61	73.67	73.72	73.77	73.82	73.88	73.93	73.98	74.04	1	1	2	2	3
4.20	74.09	74.14	74.19	74.25−	74.30	74.35+	74.41	74.46	74.51	74.57	1	1	2	2	3
1	74.62	74.67	74.72	74.78	74.83	74.88	74.94	74.99	75.04	75.10	1	1	2	2	3
2	75.15+	75.20	75.26	75.31	75.37	75.42	75.47	75.53	75.58	75.63	1	1	2	2	3
3	75.69	75.74	75.79	75.85−	75.90	75.96	76.01	76.06	76.12	76.17	1	1	2	2	3
4	76.23	76.28	76.33	76.39	76.44	76.50−	76.55−	76.60	76.66	76.71	1	1	2	2	3
4.25	76.77	76.82	76.87	76.93	76.98	77.04	77.09	77.15−	77.20	77.25+	1	1	2	2	3
6	77.31	77.36	77.42	77.47	77.53	77.58	77.64	77.69	77.75−	77.80	1	1	2	2	3
7	77.85+	77.91	77.96	78.02	78.07	78.13	78.18	78.24	78.29	78.35−	1	1	2	2	3
8	78.40	78.46	78.51	78.57	78.62	78.68	78.73	78.79	78.84	78.90	1	1	2	2	3
9	78.95+	79.01	79.06	79.12	79.17	79.23	79.29	79.34	79.40	79.45+	1	1	2	2	3
4.30	79.51	79.56	79.62	79.67	79.73	79.78	79.84	79.90	79.95+	**80**.01	1	1	2	2	3
1	80.06	80.12	80.17	80.23	80.29	80.34	80.40	80.45+	80.51	80.57	1	1	2	2	3
2	80.62	80.68	80.73	80.79	80.85−	80.90	80.96	81.01	81.07	81.13	1	1	2	2	3
3	81.18	81.24	81.30	81.35+	81.41	81.46	81.52	81.58	81.63	81.69	1	1	2	2	3
4	81.75−	81.80	81.86	81.92	81.97	82.03	82.09	82.14	82.20	82.26	1	1	2	2	3
4.35	82.31	82.37	82.43	82.48	82.54	82.60	82.65+	82.71	82.77	82.82	1	1	2	2	3
6	82.88	82.94	83.00	83.05+	83.11	83.17	83.22	83.28	83.34	83.40	1	1	2	2	3
7	83.45+	83.51	83.57	83.63	83.68	83.74	83.80	83.86	83.91	83.97	1	1	2	2	3
8	84.03	84.09	84.14	84.20	84.26	84.32	84.37	84.43	84.49	84.55−	1	1	2	2	3
9	84.60	84.66	84.72	84.78	84.84	84.89	84.95+	85.01	85.07	85.13	1	1	2	2	3
4.40	85.18	85.24	85.30	85.36	85.42	85.47	85.53	85.59	85.65−	85.71	1	1	2	2	3
1	85.77	85.82	85.88	85.94	86.00	86.06	86.12	86.18	86.23	86.29	1	1	2	2	3
2	86.35+	86.41	86.47	86.53	86.59	86.64	86.70	86.76	86.82	86.88	1	1	2	2	3
3	86.94	87.00	87.06	87.12	87.17	87.23	87.29	87.35+	87.41	87.47	1	1	2	2	3
4	87.53	87.59	87.65−	87.71	87.77	87.82	87.88	87.94	88.00	88.06	1	1	2	2	3
4.45	88.12	88.18	88.24	88.30	88.36	88.42	88.48	88.54	88.60	88.66	1	1	2	2	3
6	88.72	88.78	88.84	88.90	88.96	89.02	89.08	89.13	89.19	89.25+	1	1	2	2	3
7	89.31	89.37	89.43	89.49	89.55+	89.61	89.67	89.73	89.80	89.86	1	1	2	2	3
8	89.92	89.98	**90**.04	90.10	90.16	90.22	90.28	90.34	90.40	90.46	1	1	2	2	3
9	90.52	90.58	90.64	90.70	90.76	90.82	90.88	90.94	91.00	91.06	1	1	2	2	3
4.50	91.12	91.19	91.25−	91.31	91.37	91.43	91.49	91.55+	91.61	91.67	1	1	2	2	3
1	91.73	91.79	91.86	91.92	91.98	92.04	92.10	92.16	92.22	92.28	1	1	2	2	3
2	92.35−	92.41	92.47	92.53	92.59	92.65+	92.71	92.78	92.84	92.90	1	1	2	2	3
3	92.96	93.02	93.08	93.14	93.21	93.27	93.33	93.39	93.45+	93.51	1	1	2	2	3
4	93.58	93.64	93.70	93.76	93.82	93.89	93.95−	94.01	94.07	94.13	1	1	2	2	3
4.55	94.20	94.26	94.32	94.38	94.45−	94.51	94.57	94.63	94.69	94.76	1	1	2	2	3
6	94.82	94.88	94.94	95.01	95.07	95.13	95.19	95.26	95.32	95.38	1	1	2	3	3
7	95.44	95.51	95.57	95.63	95.69	95.76	95.82	95.88	95.95−	96.01	1	1	2	3	3
8	96.07	96.13	96.20	96.26	96.32	96.39	96.45−	96.51	96.58	96.64	1	1	2	3	3
9	96.70	96.77	96.83	96.89	96.96	97.02	97.08	97.15−	97.21	97.27	1	1	2	3	3
4.60	97.34	97.40	97.46	97.53	97.59	97.65+	97.72	97.78	97.84	97.91	1	1	2	3	3
1	97.97	98.04	98.10	98.16	98.23	98.29	98.36	98.42	98.48	98.55−	1	1	2	3	3
2	98.61	98.68	98.74	98.80	98.87	98.93	99.00	99.06	99.12	99.19	1	1	2	3	3
3	99.25+	99.32	99.38	99.45−	99.51	99.57	99.64	99.70	99.77	99.83	1	1	2	3	3
4	*99.90*	*99.96*	**100**.03								1	1	2	3	3

These four pages give the cube roots of all numbers between 1 and 10^3, correct to four sig
For example: $\sqrt[3]{2.718} = 1.395$; $\sqrt[3]{27.18} = 3.007$; $\sqrt[3]{271.8} = 6.477$. (Values obtained l
may be in error by 1 in the last figure.)

To obtain the cube root of any number outside the range from 1 to 10^3, write the numbe
of: (*a*) some number between 1 and 10^3, and (*b*) some (positive or negative) power of 10
as in the following examples:

$$\sqrt[3]{2718000} = \sqrt[3]{2.718 \times 10^6} = \sqrt[3]{2.718} \times \sqrt[3]{10^6} = 1.395 \times 10^2 = 139.5;$$
$$\sqrt[3]{27180000000} = \sqrt[3]{27.18 \times 10^9} = \sqrt[3]{27.18} \times \sqrt[3]{10^9} = 3.007 \times 10^3 = 3007;$$
$$\sqrt[3]{0.0002718} = \sqrt[3]{271.8 \times 10^{-6}} = \sqrt[3]{271.8} \times \sqrt[3]{10^{-6}} = 6.477 \times 10^{-2} = 0.06477$$

In short, moving the decimal point* THREE *places in column n is equivalent to moving it
body of the table.

$\sqrt[3]{n}$ CUBE ROOTS

TENT
TABULAI

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4
1.0	1.000	1.003	1.007	1.010	1.013	1.016	1.020	1.023	1.026	1.029	0	1	1	1
.1	1.032	1.035+	1.038	1.042	1.045−	1.048	1.051	1.054	1.057	1.060	0	1	1	1
.2	1.063	1.066	1.069	1.071	1.074	1.077	1.080	1.083	1.086	1.089	0	1	1	1
.3	1.091	1.094	1.097	1.100	1.102	1.105+	1.108	1.111	1.113	1.116	0	1	1	1
.4	1.119	1.121	1.124	1.127	1.129	1.132	1.134	1.137	1.140	1.142	0	1	1	1
1.5	1.145−	1.147	1.150−	1.152	1.155−	1.157	1.160	1.162	1.165−	1.167	0	0	1	1
.6	1.170	1.172	1.174	1.177	1.179	1.182	1.184	1.186	1.189	1.191	0	0	1	1
.7	1.193	1.196	1.198	1.200	1.203	1.205+	1.207	1.210	1.212	1.214	0	0	1	1
.8	1.216	1.219	1.221	1.223	1.225+	1.228	1.230	1.232	1.234	1.236	0	0	1	1
.9	1.239	1.241	1.243	1.245+	1.247	1.249	1.251	1.254	1.256	1.258	0	0	1	1
2.0	1.260	1.262	1.264	1.266	1.268	1.270	1.272	1.274	1.277	1.279	0	0	1	1
.1	1.281	1.283	1.285−	1.287	1.289	1.291	1.293	1.295−	1.297	1.299	0	0	1	1
.2	1.301	1.303	1.305−	1.306	1.308	1.310	1.312	1.314	1.316	1.318	0	0	1	1
.3	1.320	1.322	1.324	1.326	1.328	1.330	1.331	1.333	1.335+	1.337	0	0	1	1
.4	1.339	1.341	1.343	1.344	1.346	1.348	1.350−	1.352	1.354	1.355+	0	0	1	1
2.5	1.357	1.359	1.361	1.363	1.364	1.366	1.368	1.370	1.372	1.373	0	0	1	1
.6	1.375+	1.377	1.379	1.380	1.382	1.384	1.386	1.387	1.389	1.391	0	0	1	1
.7	1.392	1.394	1.396	1.398	1.399	1.401	1.403	1.404	1.406	1.408	0	0	1	1
.8	1.409	1.411	1.413	1.414	1.416	1.418	1.419	1.421	1.423	1.424	0	0	0	1
.9	1.426	1.428	1.429	1.431	1.433	1.434	1.436	1.437	1.439	1.441	0	0	0	1
3.0	1.442	1.444	1.445+	1.447	1.449	1.450+	1.452	1.453	1.455−	1.457	0	0	0	1
.1	1.458	1.460	1.461	1.463	1.464	1.466	1.467	1.469	1.471	1.472	0	0	0	1
.2	1.474	1.475+	1.477	1.478	1.480	1.481	1.483	1.484	1.486	1.487	0	0	0	1
.3	1.489	1.490	1.492	1.493	1.495−	1.496	1.498	1.499	1.501	1.502	0	0	0	1
.4	1.504	1.505+	1.507	1.508	1.510	1.511	1.512	1.514	1.515+	1.517	0	0	0	1
3.5	1.518	1.520	1.521	1.523	1.524	1.525+	1.527	1.528	1.530	1.531	0	0	0	1
.6	1.533	1.534	1.535+	1.537	1.538	1.540	1.541	1.542	1.544	1.545+	0	0	0	1
.7	1.547	1.548	1.549	1.551	1.552	1.554	1.555−	1.556	1.558	1.559	0	0	0	1
.8	1.560	1.562	1.563	1.565−	1.566	1.567	1.569	1.570	1.571	1.573	0	0	0	1
.9	1.574	1.575+	1.577	1.578	1.579	1.581	1.582	1.583	1.585−	1.586	0	0	0	1
4.0	1.587	1.589	1.590	1.591	1.593	1.594	1.595+	1.597	1.598	1.599	0	0	0	1
.1	1.601	1.602	1.603	1.604	1.606	1.607	1.608	1.610	1.611	1.612	0	0	0	1
.2	1.613	1.615−	1.616	1.617	1.619	1.620	1.621	1.622	1.624	1.625−	0	0	0	1
.3	1.626	1.627	1.629	1.630	1.631	1.632	1.634	1.635−	1.636	1.637	0	0	0	1
.4	1.639	1.640	1.641	1.642	1.644	1.645−	1.646	1.647	1.649	1.650−	0	0	0	0
4.5	1.651	1.652	1.653	1.655−	1.656	1.657	1.658	1.659	1.661	1.662	0	0	0	0
.6	1.663	1.664	1.666	1.667	1.668	1.669	1.670	1.671	1.673	1.674	0	0	0	0
.7	1.675+	1.676	1.677	1.679	1.680	1.681	1.682	1.683	1.685−	1.686	0	0	0	0
.8	1.687	1.688	1.689	1.690	1.692	1.693	1.694	1.695+	1.696	1.697	0	0	0	0
.9	1.698	1.700	1.701	1.702	1.703	1.704	1.705+	1.707	1.708	1.709	0	0	0	0
5.0	1.710	1.711	1.712	1.713	1.715−	1.716	1.717	1.718	1.719	1.720	0	0	0	0
.1	1.721	1.722	1.724	1.725−	1.726	1.727	1.728	1.729	1.730	1.731	0	0	0	0
.2	1.732	1.734	1.735−	1.736	1.737	1.738	1.739	1.740	1.741	1.742	0	0	0	0
.3	1.744	1.745−	1.746	1.747	1.748	1.749	1.750+	1.751	1.752	1.753	0	0	0	0
.4	1.754	1.755+	1.757	1.758	1.759	1.760	1.761	1.762	1.763	1.764	0	0	0	0
5.5	1.765+	1.766	1.767	1.768	1.769	1.771	1.772	1.773	1.774	1.775−	0	0	0	0
.6	1.776	1.777	1.778	1.779	1.780	1.781	1.782	1.783	1.784	1.785+	0	0	0	
.7	1.786	1.787	1.788	1.789	1.790	1.792	1.793	1.794	1.795−	1.796	0	0		
.8	1.797	1.798	1.799	1.800	1.801	1.802	1.803	1.804	1.805−	1.806	0	0		
.9	1.807	1.808	1.809	1.810	1.811	1.812	1.813	1.814	1.815+	1.816	0	0		

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TABLE LXXIII—*Continued*

TENTHS OF THE TABULAR DIFFERENCE

n	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
3.9	1.820	1.821	1.822	1.823	1.824	1.825+	1.826	0	0	0	0	1	1	1	1	1
	1.830	1.831	1.832	1.833	1.834	1.835+	1.836	0	0	0	0	0	1	1	1	1
	1.840	1.841	1.842	1.843	1.844	1.845−	1.846	0	0	0	0	0	1	1	1	1
	1.850−	1.851	1.852	1.853	1.854	1.855−	1.856	0	0	0	0	0	1	1	1	1
	1.860	1.860	1.861	1.862	1.863	1.864	1.865+	0	0	0	0	0	1	1	1	1
4.0	1.869	1.870	1.871	1.872	1.873	1.874	1.875−	0	0	0	0	0	1	1	1	1
	1.879	1.880	1.881	1.881	1.882	1.883	1.884	0	0	0	0	0	1	1	1	1
	1.888	1.889	1.890	1.891	1.892	1.893	1.894	0	0	0	0	0	1	1	1	1
	1.897	1.898	1.899	1.900	1.901	1.902	1.903	0	0	0	0	0	1	1	1	1
	1.907	1.907	1.908	1.909	1.910	1.911	1.912	0	0	0	0	0	1	1	1	1
4.0	1.916	1.917	1.917	1.918	1.919	1.920	1.921	0	0	0	0	0	1	1	1	1
	1.925−	1.926	1.926	1.927	1.928	1.929	1.930	0	0	0	0	0	1	1	1	1
	1.934	1.935−	1.935+	1.936	1.937	1.938	1.939	0	0	0	0	0	1	1	1	1
	1.943	1.943	1.944	1.945+	1.946	1.947	1.948	0	0	0	0	0	1	1	1	1
	1.951	1.952	1.953	1.954	1.955−	1.956	1.957	0	0	0	0	0	1	1	1	1
4.1	1.960	1.961	1.962	1.963	1.964	1.964	1.965+	0	0	0	0	0	1	1	1	1
	1.969	1.970	1.970	1.971	1.972	1.973	1.974	0	0	0	0	0	1	1	1	1
	1.977	1.978	1.979	1.980	1.981	1.981	1.982	0	0	0	0	0	1	1	1	1
	1.986	1.987	1.987	1.988	1.989	1.990	1.991	0	0	0	0	0	1	1	1	1
	1.994	1.995−	1.996	1.997	1.997	1.998	1.999	0	0	0	0	0	1	1	1	1
4.1	2.002	2.003	2.004	2.005−	2.006	2.007	2.007	0	0	0	0	0	0	1	1	1
	2.011	2.012	2.012	2.013	2.014	2.015−	2.016	0	0	0	0	0	0	1	1	1
	2.019	2.020	2.021	2.021	2.022	2.023	2.024	0	0	0	0	0	0	1	1	1
	2.027	2.028	2.029	2.030	2.030	2.031	2.032	0	0	0	0	0	0	1	1	1
	2.035+	2.036	2.037	2.038	2.038	2.039	2.040	0	0	0	0	0	0	1	1	1
4.2	2.043	2.044	2.045−	2.046	2.046	2.047	2.048	0	0	0	0	0	0	1	1	1
	2.051	2.052	2.053	2.054	2.054	2.055+	2.056	0	0	0	0	0	0	1	1	1
	2.059	2.060	2.061	2.061	2.062	2.063	2.064	0	0	0	0	0	0	1	1	1
	2.067	2.068	2.068	2.069	2.070	2.071	2.072	0	0	0	0	0	0	1	1	1
	2.075−	2.075+	2.076	2.077	2.078	2.079	2.079	0	0	0	0	0	0	1	1	1
4.2	2.082	2.083	2.084	2.085−	2.085+	2.086	2.087	0	0	0	0	0	0	1	1	1
	2.090	2.091	2.092	2.092	2.093	2.094	2.095−	0	0	0	0	0	0	1	1	1
	2.098	2.098	2.099	2.100	2.101	2.101	2.102	0	0	0	0	0	0	1	1	1
	2.105+	2.106	2.107	2.107	2.108	2.109	2.110	0	0	0	0	0	0	1	1	1
	2.113	2.113	2.114	2.115−	2.116	2.116	2.117	0	0	0	0	0	0	1	1	1
4.3	2.120	2.121	2.122	2.122	2.123	2.124	2.125−	0	0	0	0	0	0	1	1	1
	2.128	2.128	2.129	2.130	2.130	2.131	2.132	0	0	0	0	0	0	1	1	1
	2.135−	2.136	2.136	2.137	2.138	2.139	2.139	0	0	0	0	0	0	1	1	1
	2.142	2.143	2.144	2.144	2.145+	2.146	2.147	0	0	0	0	0	0	1	1	1
	2.149	2.150+	2.151	2.152	2.152	2.153	2.154	0	0	0	0	0	0	1	1	1
4.3	2.176	2.183	2.190	2.197	2.204	2.210	2.217	1	1	2	3	3	4	5	6	6
	2.244	2.251	2.257	2.264	2.270	2.277	2.283	1	1	2	3	3	4	5	5	6
	2.308	2.315−	2.321	2.327	2.333	2.339	2.345+	1	1	2	2	3	4	4	5	6
	2.369	2.375+	2.381	2.387	2.393	2.399	2.404	1	1	2	2	3	4	4	5	5
	2.427	2.433	2.438	2.444	2.450−	2.455+	2.461	1	1	2	2	3	3	4	4	5
4.4	2.483	2.488	2.493	2.499	2.504	2.509	2.515−	1	1	2	2	3	3	4	4	[illegible]
	2.535+	2.541	2.546	2.551	2.556	2.561	2.566	1	1	2	2	3	3	4	4	[illegible]
	2.586	2.591	2.596	2.601	2.606	2.611	2.616	0	1	1	2	2	3	3	4	[illegible]
	2.635+	2.640	2.645−	2.650−	2.654	2.659	2.664	0	1	1	2	2	3	3	4	[illegible]
	2.682	2.687	2.692	2.696	2.701	2.705+	2.710	0	1	1	2	2	3	3	4	[illegible]
4.4	2.728	2.732	2.737	2.741	2.746	2.750+	2.755−	0	1	1	2	2	3	3	4	[illegible]
	2.772	2.776	2.781	2.785−	2.789	2.794	2.798	0	1	1	2	2	3	3	3	[illegible]
	2.815−	2.819	2.823	2.827	2.831	2.836	2.840	0	1	1	2	2	3	3	3	[illegible]
	2.856	2.860	2.864	2.868	2.872	2.876	2.880	0	1	1	2	2	2	3	3	[illegible]
	2.896	2.900	2.904	2.908	2.912	2.916	2.920	0	1	1	2	2	2	3	3	[illegible]
4.5	2.936	2.940	2.943	2.947	2.951	2.955−	2.959	0	1	1	2	2	2	3	3	[illegible]
	2.974	2.978	2.981	2.985+	2.989	2.993	2.996	0	1	1	2	2	2	3	3	[illegible]
	3.011	3.015−	3.018	3.022	3.026	3.029	3.033	0	1	1	1	2	2	3	3	[illegible]
	3.047	3.051	3.055−	3.058	3.062	3.065+	3.069	0	1	1	1	2	2	3	3	[illegible]
	3.083	3.086	3.090	3.093	3.097	3.100	3.104	0	1	1	1	2	2	2	3	[illegible]
4.5	3.118	3.121	3.124	3.128	3.131	3.135−	3.138	0	1	1	1	2	2	2	3	[illegible]
	3.151	3.155−	3.158	3.162	3.165−	3.168	3.171	0	1	1	1	2	2	2	3	[illegible]
	3.185−	3.188	3.191	3.195−	3.198	3.201	3.204	0	1	1	1	2	2	2	3	[illegible]
	3.217	3.220	3.224	3.227	3.230	3.233	3.236	0	1	1	1	2	2	2	3	[illegible]
	3.249	3.252	3.255+	3.259	3.262	3.265−	3.268	0	1	1	1	2	2	2	3	[illegible]
4.6	3.280	3.283	3.287	3.290	3.293	3.296	3.299	0	1	1	1	2	2	2	2	[illegible]
	3.311	3.314	3.317	3.320	3.323	3.326	3.329	0	1	1	1	2	2	2	2	[illegible]
	3.341	3.344	3.347	3.350+	3.353	3.356	3.359	0	1	1	1	1	2	2	2	[illegible]
	3.371	3.374	3.377	3.380	3.382	3.385+	3.388	0	1	1	1	1	2	2	2	[illegible]
	3.400	3.403	3.406	3.409	3.411	3.414	3.417	0	1	1	1	1	2	2	2	[illegible]
	3.428	3.431	3.434	3.437	3.440	3.443	3.445+	0	1	1	1	1	2	2	2	[illegible]
	3.457	3.459	3.462	3.465−	3.468	3.471	3.473	0	1	1	1	1	2	2	2	[illegible]
	3.484	3.487	3.490	3.493	3.495+	3.498	3.501	0	1	1	1	1	2	2	2	[illegible]
	3.512	3.514	3.517	3.520	3.522	3.525−	3.528	0	1	1	1	1	2	2	2	[illegible]
	3.538	3.541	3.544	3.546	3.549	3.552	3.554	0	1	1	1	1	2	2	2	[illegible]

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n	0	1	2	3	4	5	6	7	8	9	Tenths of Tabular Diff. 1	2	3	4	5
45	3.557	3.560	3.562	3.565 −	3.567	3.570	3.573	3.575 +	3.578	3.580	0	1	1	1	1
6	3.583	3.586	3.588	3.591	3.593	3.596	3.599	3.601	3.604	3.606	0	1	1	1	1
7	3.609	3.611	3.614	3.616	3.619	3.622	3.624	3.627	3.629	3.632	0	1	1	1	1
8	3.634	3.637	3.639	3.642	3.644	3.647	3.649	3.652	3.654	3.657	0	1	1	1	1
9	3.659	3.662	3.664	3.667	3.669	3.672	3.674	3.677	3.679	3.682	0	0	1	1	1
50	3.684	3.686	3.689	3.691	3.694	3.696	3.699	3.701	3.704	3.706	0	0	1	1	1
1	3.708	3.711	3.713	3.716	3.718	3.721	3.723	3.725 +	3.728	3.730	0	0	1	1	1
2	3.733	3.735 −	3.737	3.740	3.742	3.744	3.747	3.749	3.752	3.754	0	0	1	1	1
3	3.756	3.759	3.761	3.763	3.766	3.768	3.770	3.773	3.775 +	3.777	0	0	1	1	1
4	3.780	3.782	3.784	3.787	3.789	3.791	3.794	3.796	3.798	3.801	0	0	1	1	1
55	3.803	3.805 +	3.808	3.810	3.812	3.814	3.817	3.819	3.821	3.824	0	0	1	1	1
6	3.826	3.828	3.830	3.833	3.835 −	3.837	3.839	3.842	3.844	3.846	0	0	1	1	1
7	3.849	3.851	3.853	3.855 +	3.857	3.860	3.862	3.864	3.866	3.869	0	0	1	1	1
8	3.871	3.873	3.875 +	3.878	3.880	3.882	3.884	3.886	3.889	3.891	0	0	1	1	1
9	3.893	3.895 +	3.897	3.900	3.902	3.904	3.906	3.908	3.911	3.913	0	0	1	1	1
60	3.915 −	3.917	3.919	3.921	3.924	3.926	3.928	3.930	3.932	3.934	0	0	1	1	1
1	3.936	3.939	3.941	3.943	3.945 +	3.947	3.949	3.951	3.954	3.956	0	0	1	1	1
2	3.958	3.960	3.962	3.964	3.966	3.968	3.971	3.973	3.975 −	3.977	0	0	1	1	1
3	3.979	3.981	3.983	3.985 +	3.987	3.990	3.992	3.994	3.996	3.998	0	0	1	1	1
4	4.000	4.002	4.004	4.006	4.008	4.010	4.013	4.015 −	4.017	4.019	0	0	1	1	1
65	4.021	4.023	4.025 −	4.027	4.029	4.031	4.033	4.035 +	4.037	4.039	0	0	1	1	1
6	4.041	4.043	4.045 +	4.047	4.049	4.051	4.053	4.055 +	4.058	4.060	0	0	1	1	1
7	4.062	4.064	4.066	4.068	4.070	4.072	4.074	4.076	4.078	4.080	0	0	1	1	1
8	4.082	4.084	4.086	4.088	4.090	4.092	4.094	4.096	4.098	4.100	0	0	1	1	1
9	4.102	4.104	4.106	4.108	4.109	4.111	4.113	4.115 +	4.117	4.119	0	0	1	1	1
70	4.121	4.123	4.125 +	4.127	4.129	4.131	4.133	4.135 −	4.137	4.139	0	0	1	1	1
1	4.141	4.143	4.145 −	4.147	4.149	4.151	4.152	4.154	4.156	4.158	0	0	1	1	1
2	4.160	4.162	4.164	4.166	4.168	4.170	4.172	4.174	4.176	4.177	0	0	1	1	1
3	4.179	4.181	4.183	4.185 +	4.187	4.189	4.191	4.193	4.195 −	4.196	0	0	1	1	1
4	4.198	4.200	4.202	4.204	4.206	4.208	4.210	4.212	4.213	4.215 +	0	0	1	1	1
75	4.217	4.219	4.221	4.223	4.225 −	4.227	4.228	4.230	4.232	4.234	0	0	1	1	1
6	4.236	4.238	4.240	4.241	4.243	4.245 +	4.247	4.249	4.251	4.252	0	0	1	1	1
7	4.254	4.256	4.258	4.260	4.262	4.264	4.265 +	4.267	4.269	4.271	0	0	1	1	1
8	4.273	4.274	4.276	4.278	4.280	4.282	4.284	4.285 +	4.287	4.289	0	0	1	1	1
9	4.291	4.293	4.294	4.296	4.298	4.300	4.302	4.303	4.305 +	4.307	0	0	1	1	1
80	4.309	4.311	4.312	4.314	4.316	4.318	4.320	4.321	4.323	4.325 −	0	0	1	1	1
1	4.327	4.329	4.330	4.332	4.334	4.336	4.337	4.339	4.341	4.343	0	0	1	1	1
2	4.344	4.346	4.348	4.350 −	4.352	4.353	4.355 +	4.357	4.359	4.360	0	0	1	1	1
3	4.362	4.364	4.366	4.367	4.369	4.371	4.373	4.374	4.376	4.378	0	0	1	1	1
4	4.380	4.381	4.383	4.385 −	4.386	4.388	4.390	4.392	4.393	4.395 +	0	0	1	1	1
85	4.397	4.399	4.400	4.402	4.404	4.405 +	4.407	4.409	4.411	4.412	0	0	1	1	1
6	4.414	4.416	4.417	4.419	4.421	4.423	4.424	4.426	4.428	4.429	0	0	1	1	1
7	4.431	4.433	4.434	4.436	4.438	4.440	4.441	4.443	4.445 −	4.446	0	0	1	1	1
8	4.448	4.450 −	4.451	4.453	4.455 −	4.456	4.458	4.460	4.461	4.463	0	0	1	1	1
9	4.465 −	4.466	4.468	4.470	4.471	4.473	4.475 −	4.476	4.478	4.480	0	0	0	1	1
90	4.481	4.483	4.485 −	4.486	4.488	4.490	4.491	4.493	4.495 −	4.496	0	0	0	1	1
1	4.498	4.500	4.501	4.503	4.505 −	4.506	4.508	4.509	4.511	4.513	0	0	0	1	1
2	4.514	4.516	4.518	4.519	4.521	4.523	4.524	4.526	4.527	4.529	0	0	0	1	1
3	4.531	4.532	4.534	4.536	4.537	4.539	4.540	4.542	4.544	4.545 +	0	0	0	1	1
4	4.547	4.548	4.550 +	4.552	4.553	4.555 −	4.556	4.558	4.560	4.561	0	0	0	1	1
95	4.563	4.565 −	4.566	4.568	4.569	4.571	4.572	4.574	4.576	4.577	0	0	0	1	1
6	4.579	4.580	4.582	4.584	4.585 +	4.587	4.588	4.590	4.592	4.593	0	0	0	1	1
7	4.595 −	4.596	4.598	4.599	4.601	4.603	4.604	4.606	4.607	4.609	0	0	0	1	1
8	4.610	4.612	4.614	4.615 +	4.617	4.618	4.620	4.621	4.623	4.625 −	0	0	0	1	1
9	4.626	4.628	4.629	4.631	4.632	4.634	4.635 +	4.637	4.638	4.640	0	0	0	1	1
10	4.642	4.657	4.672	4.688	4.703	4.718	4.733	4.747	4.762	4.777	1	3	4	6	7
1	4.791	4.806	4.820	4.835 −	4.849	4.863	4.877	4.891	4.905 −	4.919	1	3	4	6	7
2	4.932	4.946	4.960	4.973	4.987	5.000	5.013	5.027	5.040	5.053	1	3	4	5	7
3	5.066	5.079	5.092	5.104	5.117	5.130	5.143	5.155 +	5.168	5.180	1	3	4	5	6
4	5.192	5.205 −	5.217	5.229	5.241	5.254	5.266	5.278	5.290	5.301	1	2	4	5	6
15	5.313	5.325 +	5.337	5.348	5.360	5.372	5.383	5.395 −	5.406	5.418	1	2	3	5	6
6	5.429	5.440	5.451	5.463	5.474	5.485 −	5.496	5.507	5.518	5.529	1	2	3	4	6
7	5.540	5.550 +	5.561	5.572	5.583	5.593	5.604	5.615 −	5.625 +	5.636	1	2	3	4	5
8	5.646	5.657	5.667	5.677	5.688	5.698	5.708	5.718	5.729	5.739	1	2	3	4	5
9	5.749	5.759	5.769	5.779	5.789	5.799	5.809	5.819	5.828	5.838	1	2	3	4	5
20	5.848	5.858	5.867	5.877	5.887	5.896	5.906	5.915 +	5.925 −	5.934	1	2	3	4	5
1	5.944	5.953	5.963	5.972	5.981	5.991	6.000	6.009	6.018	6.028	1	2	3	4	5
2	6.037	6.046	6.055 +	6.064	6.073	6.082	6.091	6.100	6.109	6.118	1	2	3	4	5
3	6.127	6.136	6.145 −	6.153	6.162	6.171	6.180	6.188	6.197	6.206	1	2	3	4	4
4	6.214	6.223	6.232	6.240	6.249	6.257	6.266	6.274	6.283	6.291	1	2	3	3	4
25	6.300	6.308	6.316	6.325 −	6.333	6.341	6.350 −	6.358	6.366	6.374	1	2	2	3	4
6	6.383	6.391	6.399	6.407	6.415 +	6.423	6.431	6.439	6.447	6.455 +	1	2	[illegible]	[illegible]	[illegible]
7	6.463	6.471	6.479	6.487	6.495 +	6.503	6.511	6.519	6.527	6.534	1	2	[illegible]	[illegible]	[illegible]
8	6.542	6.550 −	6.558	6.565 +	6.573	6.581	6.589	6.596	6.604	6.611	1	[illegible]	[illegible]	[illegible]	[illegible]
9	6.619	6.627	6.634	6.642	6.649	6.657	6.664	6.672	6.679	6.687	1	[illegible]	[illegible]	[illegible]	[illegible]

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	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
								TENTHS OF THE TABULAR DIFFERENCE								
	6.717	6.724	6.731	6.739	6.746	6.753	6.761	1	1	2	3	4	4	5	6	[illegible]
	6.790	6.797	6.804	6.811	6.818	6.826	6.833	1	1	2	3	4	4	5	6	[illegible]
	6.861	6.868	6.875 +	6.882	6.889	6.896	6.903	1	1	2	3	4	4	5	[illegible]	6
	6.931	6.938	6.945 +	6.952	6.959	6.966	6.973	1	1	2	3	3	4	5	6	6
	7.000	7.007	7.014	7.020	7.027	7.034	7.041	1	1	2	3	3	4	5	5	6
	7.067	7.074	7.081	7.087	7.094	7.101	7.107	1	1	2	3	3	4	5	5	6
	7.133	7.140	7.147	7.153	7.160	7.166	7.173	1	1	2	3	3	4	5	5	6
	7.198	7.205 −	7.211	7.218	7.224	7.230	7.237	1	1	2	3	3	4	4	5	6
	7.262	7.268	7.275 −	7.281	7.287	7.294	7.300	1	1	2	3	3	4	4	5	6
	7.325 −	7.331	7.337	7.343	7.350 −	7.356	7.362	1	1	2	2	3	4	4	5	6
	7.386	7.393	7.399	7.405 −	7.411	7.417	7.423	1	1	2	2	3	4	4	5	5
	7.447	7.453	7.459	7.465 +	7.471	7.477	7.483	1	1	2	2	3	4	4	5	5
	7.507	7.513	7.518	7.524	7.530	7.536	7.542	1	1	2	2	3	4	4	5	5
	7.565 +	7.571	7.577	7.583	7.589	7.594	7.600	1	1	2	2	3	3	4	5	5
	7.623	7.629	7.635 −	7.640	7.646	7.652	7.657	1	1	2	2	3	3	4	5	5
	7.680	7.686	7.691	7.697	7.703	7.708	7.714	1	1	2	2	3	3	4	5	5
	7.736	7.742	7.747	7.753	7.758	7.764	7.769	1	1	2	2	3	3	4	4	5
	7.791	7.797	7.802	7.808	7.813	7.819	7.824	1	1	2	2	3	3	4	4	5
	7.846	7.851	7.857	7.862	7.868	7.873	7.878	1	1	2	2	3	3	4	4	5
	7.900	7.905 +	7.910	7.916	7.921	7.926	7.932	1	1	2	2	3	3	4	4	5
	7.953	7.958	7.963	7.969	7.974	7.979	7.984	1	1	2	2	3	3	4	4	5
	8.005 +	8.010	8.016	8.021	8.026	8.031	8.036	1	1	2	2	3	3	4	4	5
	8.057	8.062	8.067	8.072	8.077	8.082	8.088	1	1	2	2	3	3	4	4	5
	8.108	8.113	8.118	8.123	8.128	8.133	8.138	1	1	2	2	3	3	4	4	5
	8.158	8.163	8.168	8.173	8.178	8.183	8.188	0	1	1	2	2	3	3	4	4
	8.208	8.213	8.218	8.223	8.228	8.233	8.238	0	1	1	2	2	3	3	4	4
	8.257	8.262	8.267	8.272	8.277	8.282	8.286	0	1	1	2	2	3	3	4	4
	8.306	8.311	8.316	8.320	8.325 +	8.330	8.335 −	0	1	1	2	2	3	3	4	4
	8.354	8.359	8.363	8.368	8.373	8.378	8.382	0	1	1	2	2	3	3	4	4
	8.401	8.406	8.411	8.416	8.420	8.425 −	8.430	0	1	1	2	2	3	3	4	4
	8.448	8.453	8.458	8.462	8.467	8.472	8.476	0	1	1	2	2	3	3	4	4
	8.495 −	8.499	8.504	8.509	8.513	8.518	8.522	0	1	1	2	2	3	3	4	4
	8.541	8.545 +	8.550 −	8.554	8.559	8.564	8.568	0	1	1	2	2	3	3	4	4
	8.586	8.591	8.595 +	8.600	8.604	8.609	8.613	0	1	1	2	2	3	3	4	4
	8.631	8.636	8.640	8.645 −	8.649	8.653	8.658	0	1	1	2	2	3	3	4	4
	8.676	8.680	8.685 −	8.689	8.693	8.698	8.702	0	1	1	2	2	3	3	4	4
+	8.720	8.724	8.729	8.733	8.737	8.742	8.746	0	1	1	2	2	3	3	4	4
	8.763	8.768	8.772	8.776	8.781	8.785 +	8.789	0	1	1	2	2	3	3	3	4
	8.807	8.811	8.815 +	8.819	8.824	8.828	8.832	0	1	1	2	2	3	3	3	4
+	8.849	8.854	8.858	8.862	8.866	8.871	8.875 −	0	1	1	2	2	3	3	3	4
	8.892	8.896	8.900	8.904	8.909	8.913	8.917	0	1	1	2	2	3	3	3	4
	8.934	8.938	8.942	8.946	8.950 +	8.955 −	8.959	0	1	1	2	2	3	3	3	4
	8.975 +	8.979	8.984	8.988	8.992	8.996	9.000	0	1	1	2	2	2	3	3	4
	9.016	9.021	9.025 −	9.029	9.033	9.037	9.041	0	1	1	2	2	2	3	3	4
	9.057	9.061	9.065 +	9.069	9.073	9.078	9.082	0	1	1	2	2	2	3		4
	9.098	9.102	9.106	9.110	9.114	9.118	9.122	0	1	1	2	2	2	3	3	4
	9.138	9.142	9.146	9.150 −	9.154	9.158	9.162	0	1	1	2	2	2	3	3	4
	9.178	9.182	9.185 +	9.189	9.193	9.197	9.201	0	1	1	2	2	2	3	3	4
	9.217	9.221	9.225 −	9.229	9.233	9.237	9.240	0	1	1	2	2	2	3	3	4
	9.256	9.260	9.264	9.268	9.272	9.275 +	9.279	0	1	1	2	2	2	3	3	3
	9.295 −	9.299	9.302	9.306	9.310	9.314	9.318	0	1	1	2	2	2	3	3	3
	9.333	9.337	9.341	9.345 −	9.348	9.352	9.356	0	1	1	2	2	2	3	3	3
	9.371	9.375 +	9.379	9.383	9.386	9.390	9.394	0	1	1	2	2	2	3	3	3
+	9.409	9.413	9.417	9.420	9.424	9.428	9.432	0	1	1	2	2	2	3	3	3
	9.447	9.450 +	9.454	9.458	9.462	9.465 +	9.469	0	1	1	1	2	2	3	3	3
	9.484	9.488	9.491	9.495 −	9.499	9.502	9.506	0	1	1	1	2	2	3	3	3
	9.521	9.524	9.528	9.532	9.535 +	9.539	9.543	0	1	1	1	2	2	3	3	3
	9.557	9.561	9.565 −	9.568	9.572	9.576	9.579	0	1	1	1	2	2	3	3	3
	9.594	9.597	9.601	9.605 −	9.608	9.612	9.615 +	0	1	1	1	2	2	3	3	3
	9.630	9.633	9.637	9.641	9.644	9.648	9.651	0	1	1	1	2	2	3	3	3
	9.666	9.669	9.673	9.676	9.680	9.683	9.687	0	1	1	1	2	2	2	3	3
	9.701	9.705 −	9.708	9.712	9.715 +	9.719	9.722	0	1	1	1	2	2	2	3	3
	9.736	9.740	9.743	9.747	9.750 +	9.754	9.758	0	1	1	1	2	2	2	3	3
	9.771	9.775 −	9.778	9.782	9.785 +	9.789	9.792	0	1	1	1	2	2	2	3	3
	9.806	9.810	9.813	9.817	9.820	9.824	9.827	0	1	1	1	2	2	2	3	3
	9.841	9.844	9.848	9.851	9.855 −	9.858	9.861	0	1	1	1	2	2	2	3	3
	9.875 +	9.879	9.882	9.885 +	9.889	9.892	9.896	0	1	1	1	2	2	2	3	3
	9.909	9.913	9.916	9.919	9.923	9.926	9.930	0	1	1	1	2	2	2	3	3
	9.943	9.946	9.949	9.953	9.956	9.960	9.963	0	1	1	1	2	2	2	3	3
	9.977	9.980	9.983	9.987	9.990	9.993	9.997	0	1	1	1	2	2	2	3	3

pages give the three-halves powers of all numbers betwee
example: $(2.718)^{\frac{3}{2}} = 4.482$; $(27.18)^{\frac{3}{2}} = 141.7$. (Values
the last figure.)
three-halves power of any number outside the range from 1 t
number between 1 and 10^2, and (b) some (positive or neg
ing examples:

$$(27180.)^{\frac{3}{2}} = (2.718 \times 10^4)^{\frac{3}{2}} = (2.718)^{\frac{3}{2}} \times (10^4)^{\frac{3}{2}} = 4.$$
$$(0.2718)^{\frac{3}{2}} = (27.18 \times 10^{-2})^{\frac{3}{2}} = (27.18)^{\frac{3}{2}} \times (10^{-2})^{\frac{3}{2}} = 14$$

moving the decimal point TWO *places in column n is equiva*
able.

EE–HALVES POWERS

1	2	3	4	5	6	7	8
1.154	1.315−	1.482	1.657	1.837	2.024	2.217	2.415−
3.043	3.263	3.488	3.718	3.953	4.192	4.437	4.685+
5.458	5.724	5.995−	6.269	6.548	6.831	7.117	7.408
8.302	8.607	8.917	9.230	9.546	9.866	10.190	
						10.19	10.52
11.52	11.86	12.20	12.55−	12.90	13.25+	13.61	13.97
15.07	15.44	15.81	16.19	16.57	16.96	17.34	17.73
18.92	19.32	19.72	20.13	20.54	20.95+	21.37	21.78
23.05+	23.48	23.91	24.35−	24.78	25.22	25.66	26.11
27.45+	27.90	28.36	28.82	29.28	29.74	30.21	30.68
32.10	32.58	33.06	33.54	34.02	34.51	35.00+	35.49
36.98	37.48	37.99	38.49	39.00	39.51	40.02	40.53
42.09	42.61	43.14	43.66	44.19	44.73	45.26	45.79
47.41	47.96	48.50+	49.05+	49.60	50.15+	50.71	51.26
52.95−	53.51	54.08	54.64	55.21	55.79	56.36	56.94
58.68	59.26	59.85−	60.43	61.02	61.62	62.21	62.80
64.60	65.20	65.81	66.41	67.02	67.63	68.25−	68.86
70.71	71.33	71.96	72.58	73.21	73.84	74.47	75.10
77.00	77.64	78.28	78.93	79.57	80.22	80.87	81.51
83.47	84.13	84.79	85.45−	86.11	86.77	87.44	88.10
90.11	90.79	91.46	92.14	92.82	93.50−	94.18	94.86
96.92	97.61	98.30	99.00	99.69	100.39		
					100.4	101.1	101.8
103.9	104.6	105.3	106.0	106.7	107.4	108.2	108.9
111.0	111.7	112.5−	113.2	113.9	114.6	115.4	116.1
118.3	119.0	119.8	120.5+	121.3	122.0	122.8	123.5
125.8	126.5+	127.3	128.0	128.8	129.5+	130.3	131.0
133.3	134.1	134.9	135.6	136.4	137.2	138.0	138.7
141.1	141.9	142.6	143.4	144.2	145.0−	145.8	146.6
149.0	149.8	150.5+	151.3	152.1	152.9	153.8	154.6
157.0	157.8	158.6	159.4	160.2	161.0	161.9	162.7
165.1	166.0	166.8	167.6	168.4	169.3	170.1	170.9
173.4	174.3	175.1	176.0	176.8	177.6	178.5−	179.3
181.9	182.7	183.6	184.4	185.3	186.1	187.0	187.8
190.4	191.3	192.2	193.0	193.9	194.8	195.6	196.5
199.1	200.0	200.9	201.8	202.6	203.5+	204.4	205.3
208.0	208.8	209.7	210.6	211.5+	212.4	213.3	214.2
216.9	217.8	218.7	219.6	220.5+	221.4	222.3	223.2
226.0	226.9	227.8	228.7	229.6	230.6	231.5−	232.4
235.2	236.1	237.0	238.0	238.9	239.8	240.8	241.7
244.5−	245.4	246.4	247.3	248.3	249.2	250.1	251.1
253.9	254.9	255.8	256.8	257.7	258.7	259.7	260.6
263.5−	264.5−	265.4	266.4	267.3	268.3	269.3	270.2
273.2	274.1	275.1	276.1	277.1	278.0	279.0	280.0
283.0	283.9	284.9	285.9	286.9	287.9	288.9	289.9
292.9	293.9	294.9	295.9	296.9	297.9	298.9	299.9
302.9	303.9	304.9	305.9	306.9	307.9	308.9	310.0
313.0	314.0	315.0+	316.1	317.1	318.1	319.1	320.2
323.2	324.3	325.3	326.3	327.4	328.4	329.4	330.5
333.6	334.6	335.7	336.7	337.8	338.8	339.9	340.9
344.1	345.1	346.2	347.2	348.3	349.3	350.4	351.4

Moving the decimal

$n^{\frac{3}{2}}$ **THREE–**

n	0	1
50	353.6	354.6
1	364.2	365.3
2	375.0−	376.1
3	385.8	386.9
4	396.8	397.9
55	407.9	409.0
6	419.1	420.2
7	430.3	431.5
8	441.7	442.9
9	453.2	454.3
60	464.8	465.9
1	476.4	477.6
2	488.2	489.4
3	500.0+	501.2
4	512.0	513.2
65	524.0	525.3
6	536.2	537.4
7	548.4	549.6
8	560.7	562.0
9	573.2	574.4
70	585.7	586.9
1	598.3	599.5+
2	610.9	612.2
3	623.7	625.0−
4	636.6	637.9
75	649.5+	650.8
6	662.6	663.9
7	675.7	677.0
8	688.9	690.2
9	702.2	703.5+
80	715.5+	716.9
1	729.0	730.4
2	742.5+	743.9
3	756.2	757.5+
4	769.9	771.2
85	783.7	785.0+
6	797.5+	798.9
7	811.5−	812.9
8	825.5+	826.9
9	839.6	841.0
90	853.8	855.2
1	868.1	869.5+
2	882.4	883.9
3	896.9	898.3
4	911.4	912.8
95	925.9	927.4
6	940.6	942.1
7	955.3	956.8
8	970.2	971.6
9	985.0+	986.5+
100	1000.0	

TABLE LXXIV — *Continued*

point TWO *places in column* **n** *is equivalent to moving it* THREE *places in the body of t*

HALVES POWERS

TENTHS OF THE TABULAR DIFFERENC

2	3	4	5	6	7	8	9	1	2	3	4	5	6	7
355.7	356.7	357.8	358.9	359.9	361.0	362.1	363.1	1	2	3	4	5	6	7
366.4	367.4	368.5+	369.6	370.7	371.7	372.8	373.9	1	2	3	4	5	6	8
377.1	378.2	379.3	380.4	381.5−	382.6	383.7	384.8	1	2	3	4	5	7	8
388.0	389.1	390.2	391.3	392.4	393.5+	394.6	395.7	1	2	3	4	5	7	8
399.0	400.1	401.2	402.3	403.4	404.6	405.7	406.8	1	2	3	4	6	7	8
410.1	411.2	412.3	413.5−	414.6	415.7	416.8	417.9	1	2	3	4	6	7	8
421.3	422.4	423.6	424.7	425.8	426.9	428.1	429.2	1	2	3	5	6	7	8
432.6	433.7	434.9	436.0	437.2	438.3	439.4	440.6	1	2	3	5	6	7	8
444.0	445.1	446.3	447.4	448.6	449.7	450.9	452.0	1	2	3	5	6	7	8
455.5−	456.6	457.8	459.0	460.1	461.3	462.4	463.6	1	2	3	5	6	7	8
467.1	468.2	469.4	470.6	471.7	472.9	474.1	475.3	1	2	4	5	6	7	8
478.8	479.9	481.1	482.3	483.5−	484.6	485.8	487.0	1	2	4	5	6	7	8
490.6	491.7	492.9	494.1	495.3	496.5−	497.7	498.9	1	2	4	5	6	7	8
502.4	503.6	504.8	506.0	507.2	508.4	509.6	510.8	1	2	4	5	6	7	8
514.4	515.6	516.8	518.0	519.2	520.4	521.6	522.8	1	2	4	5	6	7	8
526.5−	527.7	528.9	530.1	531.3	532.5+	533.8	535.0−	1	2	4	5	6	7	8
538.6	539.8	541.1	542.3	543.5+	544.7	546.0	547.2	1	2	4	5	6	7	9
550.9	552.1	553.3	554.6	555.8	557.0	558.3	559.5+	1	2	4	5	6	7	9
563.2	564.5−	565.7	566.9	568.2	569.4	570.7	571.9	1	2	4	5	6	7	9
575.7	576.9	578.1	579.4	580.6	581.9	583.2	584.4	1	3	4	5	6	8	9
588.2	589.4	590.7	591.9	593.2	594.5−	595.7	597.0	1	3	4	5	6	8	9
600.8	602.1	603.3	604.6	605.9	607.1	608.4	609.7	1	3	4	5	6	8	9
613.5−	614.8	616.0	617.3	618.6	619.9	621.2	622.4	1	3	4	5	6	8	9
626.3	627.6	628.8	630.1	631.4	632.7	634.0	635.3	1	3	4	5	6	8	9
639.2	640.4	641.7	643.0	644.3	645.6	646.9	648.2	1	3	4	5	6	8	9
652.1	653.4	654.7	656.0	657.3	658.6	659.9	661.2	1	3	4	5	7	8	9
665.2	666.5−	667.8	669.1	670.4	671.7	673.0	674.4	1	3	4	5	7	8	9
678.3	679.6	680.9	682.3	683.6	684.9	686.2	687.6	1	3	4	5	7	8	9
691.5+	692.9	694.2	695.5+	696.8	698.2	699.5+	700.8	1	3	4	5	7	8	9
704.8	706.2	707.5+	708.8	710.2	711.5+	712.9	714.2	1	3	4	5	7	8	9
718.2	719.6	720.9	722.3	723.6	725.0−	726.3	727.7	1	3	4	5	7	8	9
731.7	733.1	734.4	735.8	737.1	738.5−	739.8	741.2	1	3	4	5	7	8	9
745.3	746.6	748.0	749.3	750.7	752.1	753.4	754.8	1	3	4	5	7	8	10
758.9	760.3	761.6	763.0	764.4	765.8	767.1	768.5−	1	3	4	5	7	8	10
772.6	774.0	775.4	776.8	778.1	779.5+	780.9	782.3	1	3	4	6	7	8	10
786.4	787.8	789.2	790.6	792.0	793.4	794.8	796.1	1	3	4	6	7	8	10
800.3	801.7	803.1	804.5−	805.9	807.3	808.7	810.1	1	3	4	6	7	8	10
814.3	815.7	817.1	818.5−	819.9	821.3	822.7	824.1	1	3	4	6	7	8	10
828.3	829.7	831.1	832.6	834.0	835.4	836.8	838.2	1	3	4	6	7	8	10
842.5−	843.9	845.3	846.7	848.1	849.5+	851.0	852.4	1	3	4	6	7	9	10
856.7	858.1	859.5+	860.9	862.4	863.8	865.2	866.7	1	3	4	6	7	9	10
870.9	872.4	873.8	875.2	876.7	878.1	879.6	881.0	1	3	4	6	7	9	10
885.3	886.8	888.2	889.6	891.1	892.5+	894.0	895.4	1	3	4	6	7	9	10
899.8	901.2	902.7	904.1	905.6	907.0	908.5−	909.9	1	3	4	6	7	9	10
914.3	915.7	917.2	918.6	920.1	921.6	923.0	924.5−	1	3	4	6	7	9	10
928.9	930.3	931.8	933.3	934.7	936.2	937.7	939.1	1	3	4	6	7	9	10
943.5+	945.0+	946.5−	948.0	949.4	950.9	952.4	953.9	1	3	4	6	7	9	10
958.3	959.8	961.3	962.7	964.2	965.7	967.2	968.7	1	3	4	6	7	9	10
973.1	974.6	976.1	977.6	979.1	980.6	982.1	983.5+	1	3	4	6	7	9	10
988.0	989.5+	991.0	992.5+	994.0	995.5+	997.0	998.5+	1	3	4	6	7	9	10

Moving the dec

$n^{\frac{3}{2}}$ THREE

n	0
1.0	1.000
.1	1.154
.2	1.315 −
.3	1.482
.4	1.657
1.5	1.837
.6	2.024
.7	2.217
.8	2.415 −
.9	2.619
2.0	2.828
.1	3.043
.2	3.263
.3	3.488
.4	3.718
2.5	3.953
.6	4.192
.7	4.437
.8	4.685 +
.9	4.939
3.0	5.196
.1	5.458
.2	5.724
.3	5.995 −
.4	6.269
3.5	6.548
.6	6.831
.7	7.117
.8	7.408
.9	7.702
4.0	8.000
.1	8.302
.2	8.607
.3	8.917
.4	9.230
4.5	9.546
.6	9.866

These two pages give the **reciproc** example, $1/3.142 = 0.3183$. (Values reciprocal of any number outside the column n is equivalent to moving it th

$\frac{1}{n}$ RECIPROCALS

n	0	1	2	3	4
1.0		.9901	.9804	.9709	.9615
.1	.9091	.9009	.8929	.8850 −	.8772
.2	.8333	.8264	.8197	.8130	.8065
.3	.7692	.7634	.7576	.7519	.7463
.4	.7143	.7092	.7042	.6993	.6944
1.5	.6667	.6623	.6579	.6536	.6494
.6	.6250	.6211	.6173	.6135 −	.6098
.7	.5882	.5848	.5814	.5780	.5747
.8	.5556	.5525 −	.5495 −	.5464	.5435
.9	.5263	.5236	.5208	.5181	.5155
2.0	.5000	.4975 +	.4950 +	.4926	.4902
.1	.4762	.4739	.4717	.4695 −	.4673
.2	.4545 +	.4525 −	.4505 −	.4484	.4464
.3	.4348	.4329	.4310	.4292	.4274
.4	.4167	.4149	.4132	.4115 +	.4098
2.5	.4000	.3984	.3968	.3953	.3937
.6	.3846	.3831	.3817	.3802	.3788
.7	.3704	.3690	.3676	.3663	.3650
.8	.3571	.3559	.3546	.3534	.3521
.9	.3448	.3436	.3425 −	.3413	.3401
3.0	.3333	.3322	.3311	.3300	.3289
.1	.3226	.3215 +	.3205 +	.3195 −	.3185
.2	.3125	.3115 +	.3106	.3096	.3086
.3	.3030	.3021	.3012	.3003	.2994
.4	.2941	.2933	.2924	.2915 +	.2907
3.5	.2857	.2849	.2841	.2833	.2825
.6	.2778	.2770	.2762	.2755 −	.2747
.7	.2703	.2695 +	.2688	.2681	.2674
.8	.2632	.2625 −	.2618	.2611	.2604
.9	.2564	.2558	.2551	.2545 −	.2538
4.0	.2500	2494	.2488	.2481	.2475
.1	.2439	.2433	.2427	.2421	.2415
.2	.2381	.2375 +	.2370	.2364	.2358
.3	.2326	.2320	.2315 −	.2309	.2304
.4	.2273	.2268	.2262	.2257	.2252
4.5	.2222	.2217	.2212	.2208	.2203
.6	.2174	.2169	.2165 −	.2160	.2155
.7	.2128	.2123	.2119	.2114	.2110
.8	.2083	.2079	.2075 −	.2070	.2066
.9	.2041	.2037	.2033	.2028	.2024

TABLE LXXV

…ls of all numbers between 1 and 10, correct to four significant figures. For …btained by interpolation may be in error by 1 in the last figure.) To obtain the … range from 1 to 10, note that moving the decimal point in either direction in … same number of places in the *opposite* direction in the body of the table.

					Tenths of the Tabular Difference								
5	**6**	**7**	**8**	**9**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
.9524	.9434	.9346	.9259	.9174									
.8696	.8621	.8547	.8475 −	.8403									
.8000	.7937	.7874	.7813	.7752									
.7407	.7353	.7299	.7246	.7194									
.6897	.6849	.6803	.6757	.6711	To avoid interpolation in the first ten lines, use special table on pages 490 and 491.								
.6452	.6410	.6369	.6329	.6289									
.6061	.6024	.5988	.5952	.5917									
.5714	.5682	.5650 −	.5618	.5587									
.5405 +	.5376	.5348	.5319	.5291									
.5128	.5102	.5076	.5051	.5025 +									
.4878	.4854	.4831	.4808	.4785 −	−2	−5	−7	−10	−12	−14	−17	−19	−21
.4651	.4630	.4608	.4587	.4566	−2	−4	−6	−9	−11	−13	−15	−17	−19
.4444	.4425 −	.4405 +	.4386	.4367	−2	−4	−6	−8	−10	−12	−14	−16	−18
.4255 +	.4237	.4219	.4202	.4184	−2	−4	−5	−7	−9	−11	−13	−14	−16
.4082	.4065 +	.4049	.4032	.4016	−2	−3	−5	−7	−8	−10	−12	−13	−15
.3922	.3906	.3891	.3876	.3861	−2	−3	−5	−6	−8	−9	−11	−12	−14
.3774	.3759	.3745 +	.3731	.3717	−1	−3	−4	−6	−7	−9	−10	−11	−13
.3636	.3623	.3610	.3597	.3584	−1	−3	−4	−5	−7	−8	−9	−11	−12
.3509	.3497	.3484	.3472	.3460	−1	−2	−4	−5	−6	−7	−9	−10	−11
.3390	.3378	.3367	.3356	.3344	−1	−2	−3	−5	−6	−7	−8	−9	−10
.3279	.3268	.3257	.3247	.3236	−1	−2	−3	−4	−5	−6	−8	−9	−10
.3175 −	.3165 −	.3155 −	.3145 −	.3135 −	−1	−2	−3	−4	−5	−6	−7	−8	−9
.3077	.3067	.3058	.3049	.3040	−1	−2	−3	−4	−5	−6	−7	−8	−9
.2985 +	.2976	.2967	.2959	.2950 −	−1	−2	−3	−4	−4	−5	−6	−7	−8
.2899	.2890	.2882	.2874	.2865 +	−1	−2	−3	−3	−4	−5	−6	−7	−8
.2817	.2809	.2801	.2793	.2786	−1	−2	−2	−3	−4	−5	−6	−6	−7
.2740	.2732	.2725 −	.2717	.2710	−1	−2	−2	−3	−4	−5	−5	−6	−7
.2667	.2660	.2653	.2646	.2639	−1	−1	−2	−3	−4	−4	−5	−6	−6
.2597	.2591	.2584	.2577	.2571	−1	−1	−2	−3	−3	−4	−5	−5	−6
.2532	.2525 +	.2519	.2513	.2506	−1	−1	−2	−3	−3	−4	−4	−5	−6
.2469	.2463	.2457	.2451	.2445 −	−1	−1	−2	−2	−3	−4	−4	−5	−5
.2410	.2404	.2398	.2392	.2387	−1	−1	−2	−2	−3	−3	−4	−5	−5
.2353	.2347	.2342	.2336	.2331	−1	−1	−2	−2	−3	−3	−4	−4	−5
.2299	.2294	.2288	.2283	.2278	−1	−1	−2	−2	−3	−3	−4	−4	−5
.2247	.2242	.2237	.2232	.2227	−1	−1	−2	−2	−3	−3	−4	−4	−5
.2198	.2193	.2188	.2183	.2179	0	−1	−1	−2	−2	−3	−3	−4	−4
.2151	.2146	.2141	.2137	.2132	0	−1	−1	−2	−2	−3	−3	−4	−4
.2105 +	.2101	.2096	.2092	.2088	0	−1	−1	−2	−2	−3	−3	−4	−4
.2062	.2058	.2053	.2049	.2045 −	0	−1	−1	−2	−2	−3	−3	−3	−4
.2020	.2016	.2012	.2008	.2004	0	−1	−1	−2	−2	−2	−3	−3	−4

$\frac{1}{n}$ RECIPROCALS

n	0	1
5.0	.2000	.1996
.1	.1961	.1957
.2	.1923	.1919
.3	.1887	.1883
.4	.1852	.1848
5.5	.1818	.1815−
.6	.1786	.1783
.7	.1754	.1751
.8	.1724	.1721
.9	.1695−	.1692
6.0	.1667	.1664
.1	.1639	.1637
.2	.1613	.1610
.3	.1587	.1585−
.4	.1563	.1560
6.5	.1538	.1536
.6	.1515+	.1513
.7	.1493	.1490
.8	.1471	.1468
.9	.1449	.1447
7.0	.1429	.1427
.1	.1408	.1406
.2	.1389	.1387
.3	.1370	.1368
.4	.1351	.1350−
7.5	.1333	.1332
.6	.1316	.1314
.7	.1299	.1297
.8	.1282	.1280
.9	.1266	.1264
8.0	.1250	.1248
.1	.1235−	.1233
.2	.1220	.1218
.3	.1205−	.1203
.4	.1190	.1189
8.5	.1176	.1175+
.6	.1163	.1161
.7	.1149	.1148
.8	.1136	.1135+
.9	.1124	.1122
9.0	.1111	.1110
.1	.1099	.1098
.2	.1087	.1086
.3	.1075+	.1074
.4	.1064	.1063
9.5	.1053	.1052
.6	.1042	.1041
.7	.1031	.1030
.8	.1020	.1019
.9	.1010	.1009

$\frac{1}{n}$ RECIPROCALS

n	0	1	2	3
1.00		.9990	.9980	.9970
1	.9901	.9891	.9881	.9872
2	.9804	.9794	.9785−	.9775+
3	.9709	.9699	.9690	.9681
4	.9615+	.9606	.9597	.9588
1.05	.9524	.9515−	.9506	.9497
6	.9434	.9425+	.9416	.9407
7	.9346	.9337	.9328	.9320
8	.9259	.9251	.9242	.9234
9	.9174	.9166	.9158	.9149
1.10	.9091	.9083	.9074	.9066
1	.9009	.9001	.8993	.8985−
2	.8929	.8921	.8913	.8905−
3	.8850−	.8842	.8834	.8826
4	.8772	.8764	.8757	.8749
1.15	.8696	.8688	.8681	.8673
6	.8621	.8613	.8606	.8598
7	.8547	.8540	.8532	.8525+
8	.8475−	.8467	.8460	.8453
9	.8403	.8396	.8389	.8382
1.20	.8333	.8326	.8319	.8313
1	.8264	.8258	.8251	.8244
2	.8197	.8190	.8183	.8177
3	.8130	.8123	.8117	.8110
4	.8065−	.8058	.8052	.8045+
1.25	.8000	.7994	.7987	.7981
6	.7937	.7930	.7924	.7918
7	.7874	.7868	.7862	.7855+
8	.7812	.7806	.7800	.7794
9	.7752	.7746	.7740	.7734
1.30	.7692	.7686	.7680	.7675−
1	.7634	.7628	.7622	.7616
2	.7576	.7570	.7564	.7559
3	.7519	.7513	.7508	.7502
4	.7463	.7457	.7452	.7446
1.35	.7407	.7402	.7396	.7391
6	.7353	.7348	.7342	.7337
7	.7299	.7294	.7289	.7283
8	.7246	.7241	.7236	.7231
9	.7194	.7189	.7184	.7179
1.40	.7143	.7138	.7133	.7128
1	.7092	.7087	.7082	.7077
2	.7042	.7037	.7032	.7027
3	.6993	.6988	.6983	.6978
4	.6944	.6940	.6935−	.6930
1.45	.6897	.6892	.6887	.6882
6	.6849	.6845−	.6840	.6835+
7	.6803	.6798	.6793	.6789
8	.6757	.6752	.6748	.6743
9	.6711	.6707	.6702	.6698

SPECIAL TABLE

4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
						Tenths of the Tabular Difference								
960	.9050 +	.9940	.9930	.9921	.9911	−1	−2	−3	−4	−5	−6	−7	−8	−9
862	.9852	.9843	.9833	.9823	.9814	−1	−2	−3	−4	−5	−6	−7	−8	−9
766	.9756	.9747	.9737	.9728	.9718	−1	−2	−3	−4	−5	−6	−7	−8	−9
671	.9662	.9653	.9643	.9634	.9625 −	−1	−2	−3	−4	−5	−6	−7	−7	−8
579	.9569	.9560	.9551	.9542	.9533	−1	−2	−3	−4	−5	−5	−6	−7	−8
488	.9479	.9470	.9461	.9452	.9443	−1	−2	−3	−4	−4	−5	−6	−7	−8
398	.9390	.9381	.9372	.9363	.9355 −	−1	−2	−3	−4	−4	−5	−6	−7	−8
311	.9302	.9294	.9285 +	.9276	.9268	−1	−2	−3	−3	−4	−5	−6	−7	−8
225 +	.9217	.9208	.9200	.9191	.9183	−1	−2	−3	−3	−4	−5	−6	−7	−8
141	.9132	.9124	.9116	.9107	.9099	−1	−2	−3	−3	−4	−5	−6	−7	−8
058	.9050 −	.9042	.9033	.9025 +	.9017	−1	−2	−2	−3	−4	−5	−6	−7	−7
977	.8969	.8961	.8953	.8945 −	.8937	−1	−2	−2	−3	−4	−5	−6	−6	−7
897	.8889	.8881	.8873	.8865 +	.8857	−1	−2	−2	−3	−4	−5	−6	−6	−7
818	.8811	.8803	.8795 +	.8787	.8780	−1	−2	−2	−3	−4	−5	−5	−6	−7
741	.8734	.8726	.8718	.8711	.8703	−1	−2	−2	−3	−4	−5	−5	−6	−7
666	.8658	.8651	.8643	.8636	.8628	−1	−1	−2	−3	−4	−4	−5	−6	−7
591	.8584	.8576	.8569	.8562	.8554	−1	−1	−2	−3	−4	−4	−5	−6	−7
518	.8511	.8503	.8496	.8489	.8482	−1	−1	−2	−3	−4	−4	−5	−6	−7
446	.8439	.8432	.8425 −	.8418	.8410	−1	−1	−2	−3	−4	−4	−5	−6	−6
375 +	.8368	.8361	.8354	.8347	.8340	−1	−1	−2	−3	−4	−4	−5	−6	−6
306	.8299	.8292	.8285 +	.8278	.8271	−1	−1	−2	−3	−3	−4	−5	−6	−6
237	.8230	.8224	.8217	.8210	.8203	−1	−1	−2	−3	−3	−4	−5	−5	−6
170	.8163	.8157	.8150 −	.8143	.8137	−1	−1	−2	−3	−3	−4	−5	−5	−6
104	.8097	.8091	.8084	.8078	.8071	−1	−1	−2	−3	−3	−4	−5	−5	−6
039	.8032	.8026	.8019	.8013	.8006	−1	−1	−2	−3	−3	−4	−5	−5	−6
974	.7968	.7962	.7955 +	.7949	.7943	−1	−1	−2	−3	−3	−4	−4	−5	−6
911	.790 +	.7899	.7893	.7886	.7880	−1	−1	−2	−2	−3	−4	−4	−5	−6
849	.7843	.7837	.7831	.7825 −	.7819	−1	−1	−2	−2	−3	−4	−4	−5	−6
788	.7782	.7776	.7770	.7764	.7758	−1	−1	−2	−2	−3	−4	−4	−5	−5
728	.7722	.7716	.7710	.7704	.7698	−1	−1	−2	−2	−3	−4	−4	−5	−5
669	.7663	.7657	.7651	.7645 +	.7639	−1	−1	−2	−2	−3	−4	−4	−5	−5
610	.7605 −	.7599	.7593	.7587	.7582	−1	−1	−2	−2	−3	−3	−4	−5	−5
553	.7547	.7541	.7536	.7530	.7524	−1	−1	−2	−2	−3	−3	−4	−5	−5
496	.7491	.7485 +	.7479	.7474	.7468	−1	−1	−2	−2	−3	−3	−4	−4	−5
440	.7435 −	.7429	.7424	.7418	.7413	−1	−1	−2	−2	−3	−3	−4	−4	−5
386	.7380	.7375 −	.7369	.7364	.7358	−1	−1	−2	−2	−3	−3	−4	−4	−5
331	.7326	.7321	.7315 +	.7310	.7305 −	−1	−1	−2	−2	−3	−3	−4	−4	−5
278	.7273	.7267	.7262	.7257	.7252	−1	−1	−2	−2	−3	−3	−4	−4	−5
225 +	.7220	.7215 +	.7210	.7205 −	.7199	−1	−1	−2	−2	−3	−3	−4	−4	−5
174	.7168	.7163	.7158	.7153	.7148	−1	−1	−2	−2	−3	−3	−4	−4	−5
123	.7117	.7112	.7107	.7102	.7097	−1	−1	−2	−2	−3	−3	−4	−4	−5
072	.7067	.7062	.7057	.7052	.7047	0	−1	−1	−2	−2	−3	−3	−4	−4
022	.7018	.7013	.7008	.7003	.6998	0	−1	−1	−2	−2	−3	−3	−4	−4
974	.6969	.6964	.6959	.6954	.6949	0	−1	−1	−2	−2	−3	−3	−4	−4
925 +	.6920	.6916	.6911	.6906	.6901	0	−1	−1	−2	−2	−3	−3	−4	−4
878	.6873	.6868	.6863	.6859	.6854	0	−1	−1	−2	−2	−3	−3	−4	−4
831	.6826	.6821	.6817	.6812	.6807	0	−1	−1	−2	−2	−3	−3	−4	−4
784	.6780	.6775 +	.6770	.6766	.6761	0	−1	−1	−2	−2	−3	−3	−4	−4
739	.6734	.6729	.6725 −	.6720	.6716	0	−1	−1	−2	−2	−3	−3	−4	−4
693	.6689	.6684	.6680	.6676	.6671	0	−1	−1	−2	−2	−3	−3	−4	−4

$\frac{1}{n}$ RECIPROCA

n	0	1
1.50	.6667	.6662
1	.6623	.6618
2	.6579	.6575 —
3	.6536	.6532
4	.6494	.6489
1.55	.6452	.6447
6	.6410	.6406
7	.6369	.6365 +
8	.6329	.6325 +
9	.6289	.6285 +
1.60	.6250	.6246
1	.6211	.6207
2	.6173	.6169
3	.6135 —	.6131
4	.6098	.6094
1.65	.6061	.6057
6	.6024	.6020
7	.5988	.5984
8	.5952	.5949
9	.5917	.5914
1.70	.5882	.5879
1	.5848	.5845 —
2	.5814	.5811
3	.5780	.5777
4	.5747	.5744
1.75	.5714	.5711
6	.5682	.5679
7	.5650 —	.5647
8	.5618	.5615 —
9	.5587	.5583
1.80	.5556	.5552
1	.5525 —	.5522
2	.5495 —	.5491
3	.5464	.5461
4	.5435 —	.5432
1.85	.5405 +	.5402
6	.5376	.5373
7	.5348	.5345 —
8	.5319	.5316
9	.5291	.5288
1.90	.5263	.5260
1	.5236	.5233
2	.5208	.5206
3	.5181	.5179
4	.5155 —	.5152
1.95	.5128	.5126
6	.5102	.5099
8	.5076	.5074
8	.5051	.5048
9	.5025 +	.5023

°	RADIAN	sin	tan	sec
0°	0.0000	0.0000	0.0000	1.0
1	.0175	.0175	.0175	1.0
2	.0349	.0349	.0349	1.0
3	.0524	.0523	.0524	1.0
4	.0698	.0698	.0699	1.0
5°	0.0873	0.0872	0.0875	1.0
6	.1047	.1045	.1051	1.0
7	.1222	.1219	.1228	1.0
8	.1396	.1392	.1405	1.0
9	.1571	.1564	.1584	1.0
10°	0.1745	0.1736	0.1763	1.0
11	.1920	.1908	.1944	1.0
12	.2094	.2079	.2126	1.0
13	.2269	.2250	.2309	1.0
14	.2443	.2419	.2493	1.0
15°	0.262	0.259	0.268	1.0
16	.279	.276	.287	1.0
17	.297	.292	.306	1.0
18°	.314	.309	.325	1.0
19	.332	.326	.344	1.0
20°	0.349	0.342	0.364	1.0
21	.367	.358	.384	1.0
22	.384	.375	.404	1.0
23	.401	.391	.424	1.0
24	.419	.407	.445	1.0
25°	0.436	0.423	0.466	1.1
26	.454	.438	.488	1.1
27	.471	.454	.510	1.1
28	.489	.469	.532	1.1
29	.506	.485	.554	1.1
30°	0.524	0.500	0.577	1.1
31	.541	.515	.601	1.1
32	.559	.530	.625	1.1
33	.576	.545	.649	1.1
34	.593	.559	.675	1.2
35°	0.611	0.574	0.700	1.2
36	.628	.588	.727	1.2
37	.646	.602	.754	1.2
38	.663	.616	.781	1.2
39	.681	.629	.810	1.2
40°	0.698	0.643	0.839	1.3
41	.716	.656	.869	1.3
42	.733	.669	.900	1.3
43	.750	.682	.933	1.3
44	.768	.695	0.966	1.3
45°	0.785	0.707	1.000	1.4
		cos	ctn	cs

UTES
0″
3″.6
7″.2
10″.8
14″.4
18″.
21″.6
25″.2
28″.8
32″.4
36″.

* I

d Table of Trigonometric Functions

ers	
0000	**90°**
0002	89
0006	88
0014	87
0024	86
0038	85°
0055	84
0075	83
0097	82
0123	81
0152	**80°**
0184	79
0219	78
0256	77
0297	76
0341	75°
0387	74
0437	73
0489	72
0545	71
0603	**70°**
0664	69
0728	68
0795	67
0856	66
0937	65°
1012	64
1090	63
1171	62
1254	61
1340	**60°**
1428	59
1520	58
1613	57
1710	56
1808	55°
1910	54
2014	53
2120	52
2229	51
234	**50°**
245	49
257	48
269	47
281	46
293	**45°**
rs	°

°	Radian	sin	tan	sec	vers	
45°	0.785	0.707	1.000	1.414	0.293	**45°**
46	.803	.719	1.036	1.440	.305	44
47°	.820	.731	1.072	1.466	.318	43
48	.838	.743	1.111	1.494	.331	42
49	.855	.755	1.150	1.524	.344	41
50°	0.873	0.766	1.192	1.556	0.357	**40°**
51	.890	.777	1.235	1.589	.371	39
52	.908	.788	1.280	1.624	.384	38
53	.925	.799	1.327	1.662	.398	37
54	.942	.809	1.376	1.701	.412	36
55°	0.960	0.819	1.428	1.743	0.426	35
56	.977	.829	1.483	1.788	.441	34
57	0.995	.839	1.540	1.836	.455	33
58	1.012	.848	1.600	1.887	.470	32
59	1.030	.857	1.664	1.942	.485	31
60°	1.047	0.866	1.732	2.00	0.500	**30°**
61	1.065	.875	1.804	2.06	.515	29
62	1.082	.883	1.881	2.13	.531	28
63	1.100	.891	1.963	2.20	.546	27
64	1.117	.899	2.050	2.28	.562	26
65°	1.134	0.906	2.14	2.37	0.577	25°
66	1.152	.914	2.25	2.46	.593	24
67	1.169	.921	2.36	2.56	.609	23
68	1.187	.927	2.48	2.67	.625	22
69	1.204	.934	2.61	2.79	.642	21
70°	1.222	0.940	2.75	2.92	0.658	**20**
71	1.239	.946	2.90	3.07	.674	19
72	1.257	.951	3.08	3.24	.691	18
73	1.274	.956	3.27	3.42	.708	17
74	1.292	.961	3.49	3.63	.724	16
75°	1.309	0.966	3.73	3.86	0.741	15°
76	1.326	.970	4.01	4.13	.758	14
77	1.344	.974	4.33	4.45	.775	13
78	1.361	.978	4.70	4.81	.792	12
79	1.379	.982	5.14	5.24	.809	11
80°	1.396	0.985	5.67*	5.76*	0.826	**10°**
81	1.414	.988	6.31*	6.39*	.844	9
82	1.431	.990	7.12*	7.19*	.861	8
83	1.449	.993	8.14*	8.21*	.878	7
84	1.466	.995	9.51*	9.57*	.895	6
85°	1.484	0.996	11.43*	11.47*	0.913	**5°**
86	1.501	.998	14.30*	14.34*	.930	4
87	1.518	.999	19.08*	19.11*	.948	3
88	1.536	0.999	28.64*	28.65*	.965	2
89	1.553	1.000	57.29*	57.30*	0.983	1
90°	1.571	1.000	∞	∞	1.000	**0°**
		cos	ctn	csc	cvrs	°

tion at points marked * may be inaccurate.

ND SECONDS INTO
S OF A DEGREE
peats indefinitely)

30′	=0°.500
31′	.516
32′	.533
33′	.550
34′	.566
35′	.583
36′	.600
37′	.616
38′	.633
39′	.650
40′	=0°.666
41′	.683
42′	.700
43′	.716
44′	.733
45′	.750
46′	.766
47′	.783
48′	.800
49′	.816
50′	=0°.833
51′	.850
52′	.866
53′	.883
54′	.900
55′	.916
56′	.933
57′	.950
58′	.966
59′	.983
60′	=1°.000

0°.00416
0°.00833
0°.01250

°.00277
°.00555
°.00833
°.01111
°.01388

°.00166
°.00194
°.00222
°.00250

°.29578 –

329 + radians
888 +
84814 –

FROM DECIMAL PARTS OF A DEGREE INTO MINUTES AND SECONDS. — (*Exact Values*)

0°.00 =	0′	**0°.50** =	30′	**°.000** =	0″
.01	0′36″	.51	30′36″	°.001 =	3″.6
.02	1′12″	.52	31′12″	°.002 =	7″.2
.03	1′48″	.53	31′48″	°.003 =	10″.8
.04	2′24″	.54	32′24″	°.004 =	14″.4
.05	3′	.55	33′	°.005 =	18″.
.06	3′36″	.56	33′36″	°.006 =	21″.6
.07	4′12″	.57	34′12″	°.007 =	25″.2
.08	4′48″	.58	34′48″	°.008 =	28″.8
.09	5′24″	.59	35′24″	°.009	32″.4
0°.10 =	6′	**0°.60** =	36′	**°.01** =	36″.
.11	6′36″	.61	36′36″		
.12	7′12″	.62	37′12″		
.13	7′48″	.63	37′48″		
.14	8′24″	.64	38′24″		
.15	9′	.65	39′		
.16	9′36″	.66	39′36″		
.17	10′12″	.67	40′12″		
.18	10′48″	.68	40′48″		
.19	11′24″	.69	41′24″		
0°.20 =	12′	**0°.70** =	42′		
.21	12′36″	.71	42′36″		
.22	13′12″	.72	43′12″		
.23	13′48″	.73	43′48″		
.24	14′24″	.74	44′24″		
.25	15′	.75	45′		
.26	15′36″	.76	45′36″		
.27	16′12″	.77	46′12″		
.28	16′48″	.78	46′48″		
.29	17′24″	.79	47′24″		
0°.30 =	18′	**0°.80** =	48′		
.31	18′36″	.81	48′36″		
.32	19′12″	.82	49′12″		
.33	19′48″	.83	49′48″		
.34	20′24″	.84	50′24″		
.35	21′	.85	51′		
.36	21′36″	.86	51′36″		
.37	22′12″	.87	52′12″		
.38	22′48″	.88	52′48″		
.39	23′24″	.89	53′24″		
0°.40 =	24′	**0°.90** =	54′		
.41	24′36″	.91	54′36″		
.42	25′12″	.92	55′12″		
.43	25′48″	.93	55′48″		
.44	26′24″	.94	56′24″		
.45	27′	.95	57′		
.46	27′36″	.96	57′36″		
.47	28′12″	.97	58′12″		
.48	28′48″	.98	58′48″		
.49	29′24″	.99	59′24″		
0°50. =	30′	**1°.00** =	60′		

TABLE LXXVIII

These two pages give the common logarithms of numbers between 1 and 10, correct to fo
Moving the decimal point n places to the right (or left) in the number is equivalent to adding n (o
the logarithm. Thus, $\log 0.017453 = 0.2419 - 2$ $[= \bar{2}.2419]$.

For any Angle x,	When x is between $0°$ and $1°.1$	When x is between $88°.9$ and 9
$\log$ (x in radians $=$ $\bar{2}.2419 + \log(x°)$	$\log \sin x = \bar{2}.2419 + \log(x°)$ $\log \tan x = \bar{2}.2419 + \log(x°)$ $\log \cot x = 1.7581 - \log(x°)$	$\log \cos x = \bar{2}.2419 + \log(90° - x$ $\log \cot x = \bar{2}.2419 + \log(90° - x$ $\log \tan x = 1.7581 - \log(90° - x$

Log

	0	1	2	3	4	5	6	7	8	9	10	Tenth Tabular 1	2
1.0	0.0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	0414		
1.1	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	0792		
1.2	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	1139		
1.3	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	1461		
1.4	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	1761		
1.5	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	2041		
1.6	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	2304		
1.7	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2553		
1.8	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2788		
1.9	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	3010		
2.0	0.3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	3222	2	4
2.1	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	3424	2	4
2.2	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	3617	2	4
2.3	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	3802	2	4
2.4	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	3979	2	4
2.5	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	4150	2	3
2.6	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	4314	2	3
2.7	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	4472	2	3
2.8	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	4624	2	3
2.9	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	4771	1	3
3.0	0.4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	4914	1	3
3.1	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	5051	1	3
3.2	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	5185	1	3
3.3	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	5315	1	3
3.4	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	5441	1	3
3.5	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	5563	1	2
3.6	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	5682	1	2
3.7	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	5798	1	2
3.8	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	5911	1	2
3.9	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	6021	1	2
4.0	0.6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	6128	1	2
4.1	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	6232	1	2
4.2	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	6335	1	2
4.3	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	6435	1	2
4.4	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	6532	1	2
4.5	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	6628	1	2
4.6	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	6721	1	2
4.7	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	6812	1	2
4.8	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	6902	1	2
4.9	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	6990	1	2

To av
lation
ten line
special
pages 4

$= 0.4971$ $\log \pi/2 = 0.1961$ $\log \pi^2 = 0.9943$ $\log \sqrt{\pi} = 0.2486$

$.4343$ $\log(0.4343) = \bar{1}.6378$

0	1	2	3	4	5	6	7
0.6990	6998	7007	7016	7024	7033	7042	705
7076	7084	7093	7101	7110	7118	7126	713
7160	7168	7177	7185	7193	7202	7210	721
7243	7251	7259	7267	7275	7284	7292	730
7324	7332	7340	7348	7356	7364	7372	73
7404	7412	7419	7427	7435	7443	7451	74
7482	7490	7497	7505	7513	7520	7528	75
7559	7566	7574	7582	7589	7597	7604	76
7634	7642	7649	7657	7664	7672	7679	76
7709	7716	7723	7731	7738	7745	7752	77
0.7782	7789	7796	7803	7810	7818	7825	78
7853	7860	7868	7875	7882	7889	7896	79
7924	7931	7938	7945	7952	7959	7966	79
7993	8000	8007	8014	8021	8028	8035	80
8062	8069	8075	8082	8089	8096	8102	81
8129	8136	8142	8149	8156	8162	8169	81
8195	8202	8209	8215	8222	8228	8235	82
8261	8267	8274	8280	8287	8293	8299	8
8325	8331	8338	8344	8351	8357	8363	8
8388	8395	8401	8407	8414	8420	8426	8
0.8451	8457	8463	8470	8476	8482	8488	8
8513	8519	8525	8531	8537	8543	8549	8
8573	8579	8585	8591	8597	8603	8609	8
8633	8639	8645	8651	8657	8663	8669	8
8692	8698	8704	8710	8716	8722	8727	8
8751	8756	8762	8768	8774	8779	8785	8
8808	8814	8820	8825	8831	8837	8842	8
8865	8871	8876	8882	8887	8893	8899	8
8921	8927	8932	8938	8943	8949	8954	8
8976	8982	8987	8993	8998	9004	9009	9
0.9031	9036	9042	9047	9053	9058	9063	9
9085	9090	9096	9101	9106	9112	9117	9
9138	9143	9149	9154	9159	9165	9170	9
9191	9196	9201	9206	9212	9217	9222	9
9243	9248	9253	9258	9263	9269	9274	9
9294	9299	9304	9309	9315	9320	9325	
9345	9350	9355	9360	9365	9370	9375	
9395	9400	9405	9410	9415	9420	9425	
9445	9450	9455	9460	9465	9469	9474	
9494	9499	9504	9509	9513	9518	9523	
0.9542	9547	9552	9557	9562	9566	9571	
9590	9595	9600	9605	9609	9614	9619	
9638	9643	9647	9652	9657	9661	9666	
9685	9689	9694	9699	9703	9708	9713	
9731	9736	9741	9745	9750	9754	9759	
9777	9782	9786	9791	9795	9800	9805	
9823	9827	9832	9836	9841	9845	9850	
9868	9872	9877	9881	9886	9890	9894	
9912	9917	9921	9926	9930	9934	9939	
9956	9961	9965	9969	9974	9978	9983	

Log

1.00	0 00
1.01	18
1.02	47
1.03	75
1.04	03
1.05	31
1.06	59
1.07	87
1.08	14
1.09	41
1.10	0 68
1.11	95
1.12	22
1.13	48
1.14	75
1.15	01
1.16	27
1.17	53
1.18	79
1.19	04
1.20	0. 330
1.21	355
1.22	380
1.23	405
1.24	430
1.25	455
1.26	480
1.27	504
1.28	529
1.29	553
1.30	0.1 577
1.31	601
1.32	625
1.33	648
1.34	672
1.35	2695
1.36	2718
1.37	2742
1.38	2765
1.39	2788
1.40	0.1 2810
1.41	2833
1.42	2856
1.43	2878
1.44	2900
1.45	2923
1.46	2945
1.47	2967
1.48	2989
1.49	3010

0	1	2	3	4	5	6	7	8	9	10
0.1761	1764	1767	1770	1772	1775	1778	1781	1784	1787	1790
1790	1793	1796	1798	1801	1804	1807	1810	1813	1816	1818
1818	1821	1824	1827	1830	1833	1836	1838	1841	1844	1847
1847	1850	1853	1855	1858	1861	1864	1867	1870	1872	1875
1875	1878	1881	1884	1886	1889	1892	1895	1898	1901	1903
1903	1906	1909	1912	1915	1917	1920	1923	1926	1928	1931
1931	1934	1937	1940	1942	1945	1948	1951	1953	1956	1959
1959	1962	1965	1967	1970	1973	1976	1978	1981	1984	1987
1987	1989	1992	1995	1998	2000	2003	2006	2009	2011	2014
2014	2017	2019	2022	2025	2028	2030	2033	2036	2038	2041
0.2041	2044	2047	2049	2052	2055	2057	2060	2063	2066	2068
2068	2071	2074	2076	2079	2082	2084	2087	2090	2092	2095
2095	2098	2101	2103	2106	2109	2111	2114	2117	2119	2122
2122	2125	2127	2130	2133	2135	2138	2140	2143	2146	2148
2148	2151	2154	2156	2159	2162	2164	2167	2170	2172	2175
2175	2177	2180	2183	2185	2188	2191	2193	2196	2198	2201
2201	2204	2206	2209	2212	2214	2217	2219	2222	2225	2227
2227	2230	2232	2235	2238	2240	2243	2245	2248	2251	2253
2253	2256	2258	2261	2263	2266	2269	2271	2274	2276	2279
2279	2281	2284	2287	2289	2292	2294	2297	2299	2302	2304
0.2304	2307	2310	2312	2315	2317	2320	2322	2325	2327	2330
2330	2333	2335	2338	2340	2343	2345	2348	2350	2353	2355
2355	2358	2360	2363	2365	2368	2370	2373	2375	2378	2380
2380	2383	2385	2388	2390	2393	2395	2398	2400	2403	2405
2405	2408	2410	2413	2415	2418	2420	2423	2425	2428	2430
2430	2433	2435	2438	2440	2443	2445	2448	2450	2453	2455
2455	2458	2460	2463	2465	2467	2470	2472	2475	2477	2480
2480	2482	2485	2487	2490	2492	2494	2497	2499	2502	2504
2504	2507	2509	2512	2514	2516	2519	2521	2524	2526	2529
2529	2531	2533	2536	2538	2541	2543	2545	2548	2550	2553
0.2553	2555	2558	2560	2562	2565	2567	2570	2572	2574	2577
2577	2579	2582	2584	2586	2589	2591	2594	2596	2598	2601
2601	2603	2605	2608	2610	2613	2615	2617	2620	2622	2625
2625	2627	2629	2632	2634	2636	2639	2641	2643	2646	2648
2648	2651	2653	2655	2658	2660	2662	2665	2667	2669	2672
2672	2674	2676	2679	2681	2683	2686	2688	2690	2693	2695
2695	2697	2700	2702	2704	2707	2709	2711	2714	2716	2718
2718	2721	2723	2725	2728	2730	2732	2735	2737	2739	2742
2742	2744	2746	2749	2751	2753	2755	2758	2760	2762	2765
2765	2767	2769	2772	2774	2776	2778	2781	2783	2785	2788
0.2788	2790	2792	2794	2797	2799	2801	2804	2806	2808	2810
2810	2813	2815	2817	2819	2822	2824	2826	2828	2831	2833
2833	2835	2838	2840	2842	2844	2847	2849	2851	2853	2856
2856	2858	2860	2862	2865	2867	2869	2871	2874	2876	2878
2878	2880	2882	2885	2887	2889	2891	2894	2896	2898	2900
2900	2903	2905	2907	2909	2911	2914	2916	2918	2920	2923
2923	2925	2927	2929	2931	2934	2936	2938	2940	2942	2945
2945	2947	2949	2951	2953	2956	2958	2960	2962	2964	2967
2967	2969	2971	2973	2975	2978	2980	2982	2984	2986	2989
2989	2991	2993	2995	2997	2999	3002	3004	3006	3008	3010

These two pa
loving the deci
he logarithm.

For an
log (x in
$\bar{2}.2419$

Log

	0
1.0	0.0000
1.1	0414
1.2	0792
1.3	1139
1.4	1461
1.5	1761
1.6	2041
1.7	2304
1.8	2553
1.9	2788
2.0	0.3010
2.1	3222
2.2	3424
2.3	3617
2.4	3802
2.5	3979
2.6	4150
2.7	4314
2.8	4472
2.9	4624
3.0	0.4771
3.1	4914
3.2	5051
3.3	5185
3.4	5315
3.5	5441
3.6	5563
3.7	5682
3.8	5798
3.9	5911
4.0	0.6021
4.1	6128
4.2	6232
4.3	6335
4.4	6435
4.5	6532
4.6	6628
4.7	6721
4.8	6812
4.9	6902

log π = 0.4971
log e = 0.4343

0	1	2	3	4	5	6	7	8	9	10
.0000	0004	0009	0013	0017	0022	0026	0030	0035	0039	0043
0043	0048	0052	0056	0060	0065	0069	0073	0077	0082	0086
0086	0090	0095	0099	0103	0107	0111	0116	0120	0124	0128
0128	0133	0137	0141	0145	0149	0154	0158	0162	0166	0170
0170	0175	0179	0183	0187	0191	0195	0199	0204	0208	0212
0212	0216	0220	0224	0228	0233	0237	0241	0245	0249	0253
0253	0257	0261	0265	0269	0273	0278	0282	0286	0290	0294
0294	0298	0302	0306	0310	0314	0318	0322	0326	0330	0334
0334	0338	0342	0346	0350	0354	0358	0362	0366	0370	0374
0374	0378	0382	0386	0390	0394	0398	0402	0406	0410	0414
.0414	0418	0422	0426	0430	0434	0438	0441	0445	0449	0453
0453	0457	0461	0465	0469	0473	0477	0481	0484	0488	0492
0492	0496	0500	0504	0508	0512	0515	0519	0523	0527	0531
0531	0535	0538	0542	0546	0550	0554	0558	0561	0565	0569
0569	0573	0577	0580	0584	0588	0592	0596	0599	0603	0607
0607	0611	0615	0618	0622	0626	0630	0633	0637	0641	0645
0645	0648	0652	0656	0660	0663	0667	0671	0674	0678	0682
0682	0686	0689	0693	0697	0700	0704	0708	0711	0715	0719
0719	0722	0726	0730	0734	0737	0741	0745	0748	0752	0755
0755	0759	0763	0766	0770	0774	0777	0781	0785	0788	0792
0792	0795	0799	0803	0806	0810	0813	0817	0821	0824	0828
0828	0831	0835	0839	0842	0846	0849	0853	0856	0860	0864
0864	0867	0871	0874	0878	0881	0885	0888	0892	0896	0899
0899	0903	0906	0910	0913	0917	0920	0924	0927	0931	0934
0934	0938	0941	0945	0948	0952	0955	0959	0962	0966	0969
0969	0973	0976	0980	0983	0986	0990	0993	0997	1000	1004
1004	1007	1011	1014	1017	1021	1024	1028	1031	1035	1038
1038	1041	1045	1048	1052	1055	1059	1062	1065	1069	1072
1072	1075	1079	1082	1086	1089	1092	1096	1099	1103	1106
1106	1109	1113	1116	1119	1123	1126	1129	1133	1136	1139
139	1143	1146	1149	1153	1156	1159	1163	1166	1169	1173
173	1176	1179	1183	1186	1189	1193	1196	1199	1202	1206
206	1209	1212	1216	1219	1222	1225	1229	1232	1235	1239
239	1242	1245	1248	1252	1255	1258	1261	1265	1268	1271
271	1274	1278	1281	1284	1287	1290	1294	1297	1300	1303
303	1307	1310	1313	1316	1319	1323	1326	1329	1332	1335
335	1339	1342	1345	1348	1351	1355	1358	1361	1364	1367
367	1370	1374	1377	1380	1383	1386	1389	1392	1396	1399
399	1402	1405	1408	1411	1414	1418	1421	1424	1427	1430
430	1433	1436	1440	1443	1446	1449	1452	1455	1458	1461
461	1464	1467	1471	1474	1477	1480	1483	1486	1489	1492
492	1495	1498	1501	1504	1508	1511	1514	1517	1520	1523
523	1526	1529	1532	1535	1538	1541	1544	1547	1550	1553
553	1556	1559	1562	1565	1569	1572	1575	1578	1581	1584
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644	1647	1649	1652	1655	1658	1661	1664	1667	1670	1673
673	1676	1679	1682	1685	1688	1691	1694	1697	1700	1703
703	1706	1708	1711	1714	1717	1720	1723	1726	1729	1732
732	1735	1738	1741	1744	1746	1749	1752	1755	1758	1761

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	1818	1821	1824	1827	1830	1833	1836	1838	1841	1844	1847
	1847	1850	1853	1855	1858	1861	1864	1867	1870	1872	1875
	1875	1878	1881	1884	1886	1889	1892	1895	1898	1901	1903
	1903	1906	1909	1912	1915	1917	1920	1923	1926	1928	1931
	1931	1934	1937	1940	1942	1945	1948	1951	1953	1956	1959
	1959	1962	1965	1967	1970	1973	1976	1978	1981	1984	1987
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	2014	2017	2019	2022	2025	2028	2030	2033	2036	2038	2041
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	2095	2098	2101	2103	2106	2109	2111	2114	2117	2119	2122
	2122	2125	2127	2130	2133	2135	2138	2140	2143	2146	2148
	2148	2151	2154	2156	2159	2162	2164	2167	2170	2172	2175
	2175	2177	2180	2183	2185	2188	2191	2193	2196	2198	2201
	2201	2204	2206	2209	2212	2214	2217	2219	2222	2225	2227
	2227	2230	2232	2235	2238	2240	2243	2245	2248	2251	2253
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	2279	2281	2284	2287	2289	2292	2294	2297	2299	2302	2304
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	2455	2458	2460	2463	2465	2467	2470	2472	2475	2477	2480
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	2504	2507	2509	2512	2514	2516	2519	2521	2524	2526	2529
	2529	2531	2533	2536	2538	2541	2543	2545	2548	2550	2553
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	2648	2651	2653	2655	2658	2660	2662	2665	2667	2669	2672
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	2695	2697	2700	2702	2704	2707	2709	2711	2714	2716	2718
	2718	2721	2723	2725	2728	2730	2732	2735	2737	2739	2742
	2742	2744	2746	2749	2751	2753	2755	2758	2760	2762	2765
	2765	2767	2769	2772	2774	2776	2778	2781	2783	2785	2788
90	0.2788	2790	2792	2794	2797	2799	2801	2804	2806	2808	2810
.91	2810	2813	2815	2817	2819	2822	2824	2826	2828	2831	2833
.92	2833	2835	2838	2840	2842	2844	2847	2849	2851	2853	2856
.93	2856	2858	2860	2862	2865	2867	2869	2871	2874	2876	2878
.94	2878	2880	2882	2885	2887	2889	2891	2894	2896	2898	2900
.95	2900	2903	2905	2907	2909	2911	2914	2916	2918	2920	2923
.96	2923	2925	2927	2929	2931	2934	2936	2938	2940	2942	2945
.97	2945	2947	2949	2951	2953	2956	2958	2960	2962	2964	2967
.98	2967	2969	2971	2973	2975	2978	2980	2982	2984	2986	2989
.99	2989	2991	2993	2995	2997	2999	3002	3004	3006	3008	3010

INDEX

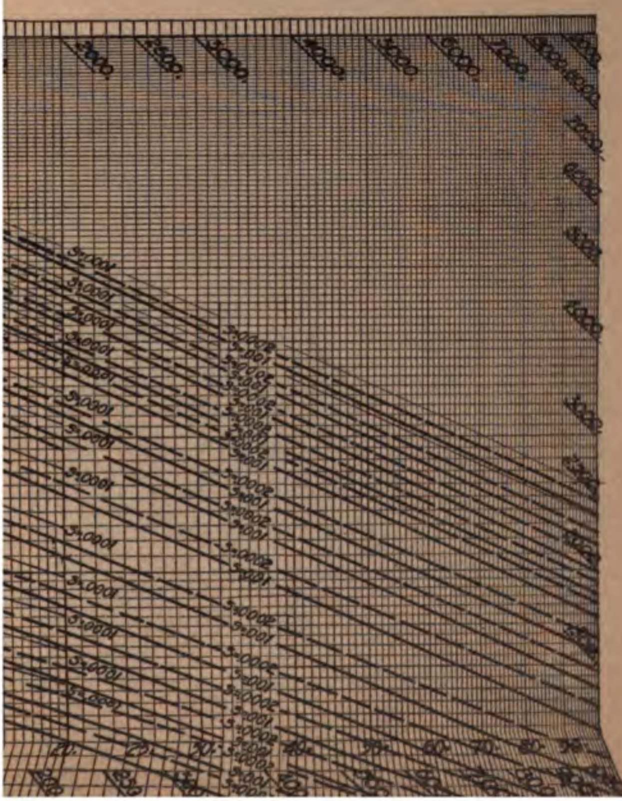
ɔG DIAGRAM FOR COMPUTING THE FLOW OF WATER IN OPEN
NNELS BY THE CHEZY, AND THE KUTTER FORMULAS.

Given the area of the cross section of a stream (A) = 42.5 square an hydraulic radius (R) = 4.4 feet; the slope (S) = .0015; and ıd the velocity (V), the discharge (Q), and the value of C.

The vertical through $R = 4.4$ cuts the curve for $S = .001$ and), the product line through D is cut at E by a horizontal from he abscissa of E is 11.7, the velocity (V), in feet per second.

Continue the perpendicular through E to its intersection at F ntal through 42.5 (A). The product line through this point reads arge (Q) in cubic feet per second.

Trace a horizontal through D to I, its intersection with a vertical ($R^{\frac{1}{2}} = 2.097$), a product line IJ through this point reads $C = 145$.



132. Log Diagram for computing the Flow of Water in Open Channels by Bazin's New Formula.

xample. Given the area of the cross section of a stream $(A) = 42.5$ square the mean hydraulic radius $(R) = 4.4$ feet; the slope $(S) = .0015$; and 29. Find the velocity (V), the discharge (Q), and the value of C.

find V. The vertical through $R = 4.4$ cuts the curve for $\Gamma = .29$ at D, roduct line through D is cut at E by a horizontal from $S = .0015$. The sa of E is 11.3, the velocity (V) in feet per second.

find Q. Continue the perpendicular through E to its intersection at F horizontal through 42.5 (A). The product line through this point reads e discharge (Q) in cubic feet per second.

find C. Trace a horizontal through D to I its intersection with a vertical gh 2.097 ($R^{\frac{1}{2}} = 2.097$) a product line IJ through this point reads $C = 140$.

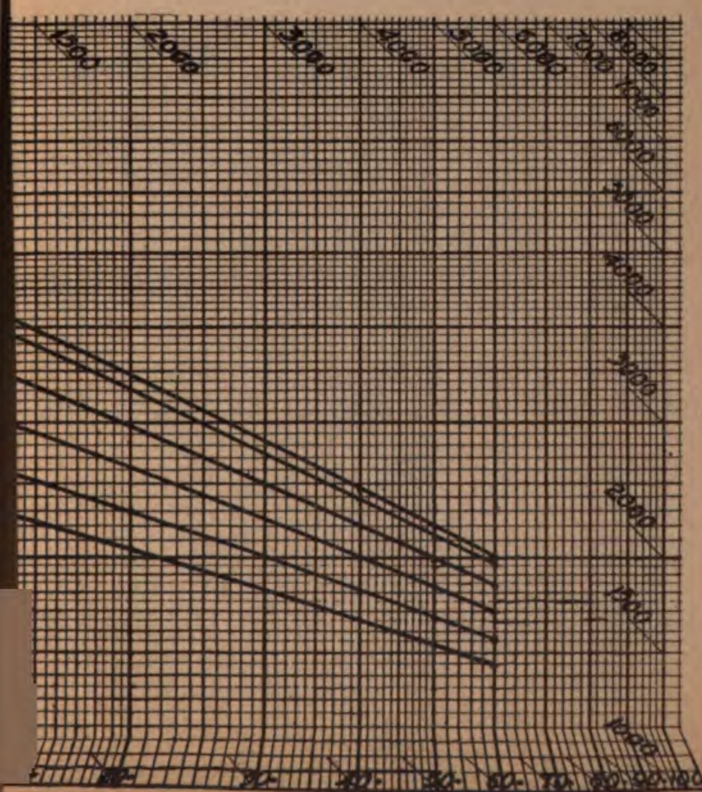
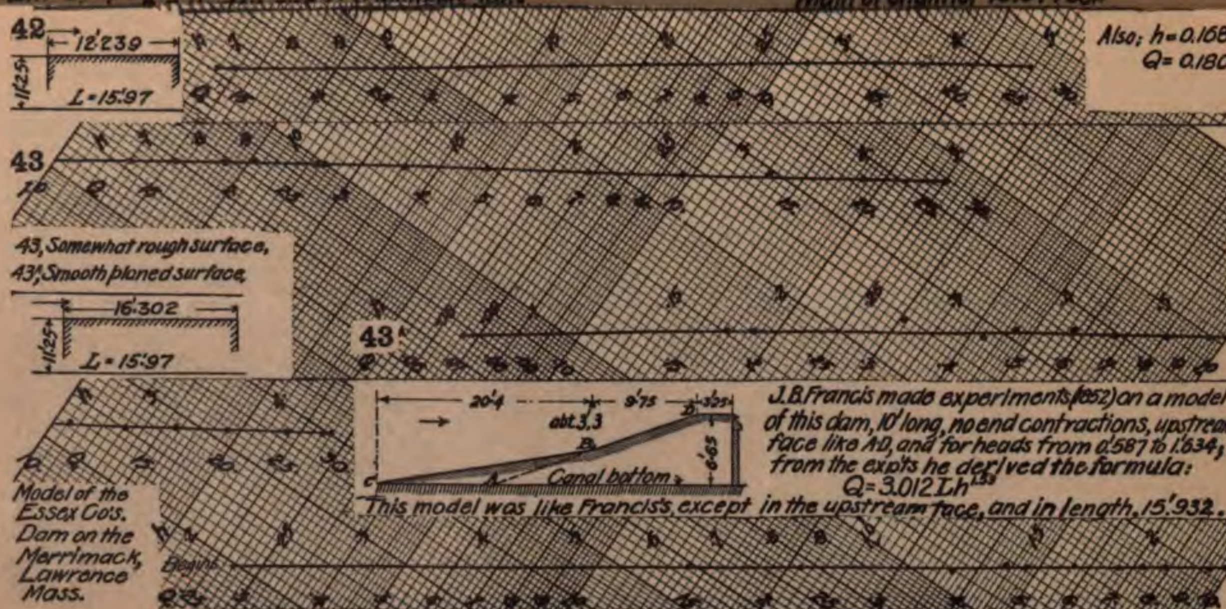


TABLE XXXVIII. WEIRS OF

EXPERIMENTS OF U.S. GEOLOGICAL SURVEY. (SELECTED TYPES)
Width of channel 15.97 feet



Applied Electrochemistry

By M. DE KAY THOMPSON, Ph.D., Assistant Professor of Electrochemistry in the Massachusetts Institute of Technology

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Cloth, 8vo, 329 pages, index, $2.10 net

This book was written to supply a need felt by the author in giving a course of lectures on Applied Electrochemistry in the Massachusetts Institute of Technology. There has been no work in English covering this whole field, and students had either to rely on notes or refer to the sources from which this book is compiled. Neither of these methods of study is satisfactory, for notes cannot be well taken in a subject where illustrations are as important as they are here; and in going to the original sources too much time is required to sift out the essential part. It is believed that, by collecting in a single volume the material that would be comprised in a course aiming to give an account of the most important electrochemical industries, as well as the principal applications of electrochemistry in the laboratory, it will be possible to teach the subject much more satisfactorily.

The plan adopted in this book has been to discuss each subject from the theoretical and from the technical point of view separately. In the theoretical part a knowledge of theoretical chemistry is assumed.

Full references to the original sources have been made, so that every statement can be easily verified. It is thought that this will make this volume useful also as a reference book.

An appendix has been added, containing the more important constants that are needed in electrochemical calculations.

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